Briefly Revisit Kinematic Control of Redundant Manipulators via Constrained Optimization

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Abstract

Redundant manipulators are widely utilized in numerous applications among various areas in industry and service. Redundant manipulators take advantage of their inherent or acquired redundancy to achieve certain benefits in kinematic control. Different from non-redundant manipulators, optimization paradigms are more likely to be established and may be more efficient for kinematic control issues in redundant manipulators. In this paper, we revisit the perspective and methodology on constrained optimization paradigms for kinematic control of redundant manipulators.

1. Introduction

Robotic manipulators nowadays are widely appearing in many fields of scientific research and engineering applications, e.g., assembly [1, 2], painting [3, 4], surgery [5–7]. They possess good flexibility, high accuracy and promising stability for fulfilling complicated tasks and sustaining heavy workloads, releasing much labour force of human beings in a long term. Most robotics manipulators are designed and manufactured as serial articulated robots with the end-effectors freely moving in a workspace [8]. meaning that most of their joints are of revolute type rather than prismatic type, so the number of degree of freedom (DoF) in joint motion can be identical to the number of joints in this sense. Since end-effectors in manipulators can possess six DoFs in Cartesian space at most, when the number of joints is more than six, one can basically suppose the manipulators inherently possess redundancy in the motion mapping between joint space and Cartesian space [9].

Generally, the redundancy for robots/manipulators can be mainly investigated in two aspects, that is, the kinematics redundancy [10, 11] and the dynamics redundancy [12, 13]. Redundancy in kinematics means

that the number of DoF in joint space is larger than that in Cartesian space (e.g., the typical applications such as industrial and medical robots [14, 15]). Redundancy in dynamics means that the number of DoF in actuation space is larger than that in joint space (e.g., over-actuated systems such as human muscular-skeletal systems [16, 17]). Most robots are often under-actuated/fully-actuated systems rather than over-actuated systems as humans, due to design, weight, and cost limitations. So, the redundancy mainly reflects on kinematics rather than dynamics.

Exploiting kinematic redundancy may be beneficial to the kinematic control of redundant manipulators [18, 19]. The extra DoFs in joint space have the potential to offer more choices in inverse kinematic resolutions once the DoFs in Cartesian space are less. This is because the imbalance between the DoFs in joint space and those in Cartesian space exists. The mapping function between joint space and Cartesian space reserves such redundancy. Just like solving undetermined nonlinear/linear equations, the number of variables is larger than that of equations, it indicates that we would have larger chances to get multiple solutions and thus the best/optimal solutions can be selected according to our criteria. So, relying heavily upon redundancy can provide a pathway to find the optimal solutions in kinematic control issues as

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the multiple optional resolutions can be guaranteed. However, if we neglect such benefits from redundancy and try to directly find the analytical resolutions or the local iterative resolutions by numerical approaches in kinematic control, some tough situations may be encountered: the analytical resolutions might not exist at all and the global resolutions by conventional numerical approaches might lose at all [20, 21]. The involvement of redundancy in potential kinematic control methods increases the possibility of finding an optimal strategy of resolution. Motivated by these considerations, developing optimization paradigms for manipulator models that contain such redundancy information seems a feasible and superior manner for kinematic control.

In this paper, we investigate the kinematic control problem for redundant manipulators and summarize the general optimization paradigms which relies on redundancy.

2. Forward Kinematic Modeling

For a manipulator with n joints, its forward kinematics is modelled by the following equation [22]

$$T(\theta) = T_1(\theta_1)T_2(\theta_2)\cdots T_n(\theta_n) \tag{1}$$

where $\theta = [\theta_1, \theta_2, \cdots, \theta_n]^T \in R^n$ denotes the joint angular variable, $T(\theta) \in R^{4\times4}$ denotes the final homogenous matrix between the end-effector coordinate system and the base coordinate system, and $T_i(\theta_i) \in R^{4\times4}$ ($i = 1, 2, \cdots, n$) denotes the homogenous matrix between the coordinate system affiliated at the $i-1$th joint and the coordinate system affiliated at the ith joint. Specifically, the homogenous matrix $T(\theta) \in R^{4\times4}$ can be written as

$$T(\theta) = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} \tag{2}$$

where $R \in R^{3\times3}$ denotes the rotation matrix between the end-effector coordinate system and the base coordinate system, and $o \in R^3$ denotes the position vector between the end-effector coordinate system and the base coordinate system [22].

From (1) and (2), we could know that the mapping from the rotation and position variables $X \in R^6$ of the end-effector to the joint angular variables $\theta \in R^n$ is depicted as a set of nonlinear equations as follows:

$$X = f(\theta) \tag{3}$$

where $f(\cdot): R^n \to R^6$ denotes the coupled nonlinear function array. As both the posture and the position are not simultaneously controlled during the motion process, especially the orientation control of the end-effector is not necessary to be done along the whole process of kinematic control while the position control

of the end-effector may be more necessary to track task paths in most phases. In this circumstance, the nonlinear relationship above often reduces to the following form:

$$r = f(\theta) \tag{4}$$

where $r \in R^3$ denotes the position of the end-effector, and $f(\cdot): R^n \to R^3$ is a reduced coupled nonlinear function array. Such formulation can increase the level of redundancy, as the gap between the number of DoF in joint space and the number of DoF in Cartesian space is enlarged from $n-6$ to $n-3$.

3. Kinematic Control in Joint Angle Level

The kinematic control on such a redundant manipulator essentially needs to inversely solve for reference joint angular variable θ from the known/desired end-effector variable X as the following

$$\theta = f^{-1}(X) \tag{5}$$

where $f^{-1}(\cdot): R^6 \to R^n$ denotes the inverse mapping of the nonlinear function array.

When $n < 6$, the number of equations is greater than the number of variables, indicating that the nonlinear equations can be regarded as over-determined. When $n > 6$, the number of equations is smaller than the number of variables, indicating that the nonlinear equations can be regarded as under-determined. When $n = 6$, the number of equations is equal to the number of variables, implying that unique solution might be obtained. However, the inverse kinematic solution problem might be very complicated because we are facing with nonlinear equation solving. The general analytical solution for the nonlinear equation solving problem usually can not be found, and the iterative numerical methods for the nonlinear equation solving problem might greatly be dependent on the design principle and the initial condition configuration.

Without loss of generality, the kinematic control problem on redundant manipulators can be equivalent to the general inverse kinematic resolution problem, which can be formulated as the following nonlinear optimization problem

$$\min. \quad \|f(\theta) - X\|_2^2 \tag{6}$$

where $\|\cdot\|_2$ denotes the two-norm of a vector. The optimization problem tries to minimize the objective function $\|f(\theta) - X\|_2^2$ with an optimal variable θ finally obtained. Let us expand $\|f(\theta) - X\|_2^2$ and then we get

$$\min. \quad f^T(\theta)f(\theta) - f^T(\theta)X + X^Tf(\theta) + X^TX \tag{7}$$

which is a nonlinear optimization problem whose convexity can not be guaranteed. In this situation, due to potential strong nonlinearity, finding the direct

solution of (6) by related traditional optimization algorithms may be quite tough and the global optimal solution can be even impossible to obtain. Some bio-inspired intelligent algorithms can be considered as alternatives for optimization solvers if the optimization problem (6) is non-convex [23]. Although when $n > 6$, the optimization problem falls into the under-determined case and thus potential multiple/indefinite solutions can be used to find the optimal one. However, because non-convexity may appear in the objective function, the global optimal solution might still be uncertain to solve. Moreover, constraints (e.g., the physical limit constraint $\theta^- < \theta < \theta^+$) in the joint angle/position level increases the difficulty of finding the reasonable global solution [24]. In order to efficiently solve the kinematic control problem based on such inverse kinematic resolution, transferring such difficult nonlinear problems into linear problems may be preferred [25].

4. Kinematic Control in Joint Velocity Level

In order to simplify the inverse kinematic control of redundant manipulators, a typical way is to convert the nonlinear mapping relation in the joint angle and end-effector position level to the following differential kinematics (or termed, velocity kinematics) first [22]:

$$J\dot{\theta} = \dot{X} \qquad (8)$$

where $J \in R^{6 \times n}$ denotes the Jacobian matrix. It can be seen evidently that, such differential kinematics equation is of linear equations. So, when the desired end-effector orientation/position is known, the reference joint angular velocity can be obtained by solving the pseudo-inverse of the Jacobian matrix as follows [26, 27]:

$$\dot{\theta} = J^\dagger \dot{X} \qquad (9)$$

where $J^\dagger$ denotes the pseudo-inverse of the Jacobian matrix J. Specifically, when $n = 6$ holds and the Jacobian matrix J is non-singular, the reference the reference joint angular velocity can be

$$\dot{\theta} = J^{-1} \dot{X} \qquad (10)$$

The resultant reference joint angle can thus be the integration of the resolved joint angular velocity with respect to time t by

$$\theta = \int_0^t \dot{\theta} dt = \int_0^t J^{-1} \dot{X} dt \qquad (11)$$

However, as we already see, such a way of finding the reference joint angle by solving the pseudo-inverse of the Jacobian matrix may suffer from some drawbacks, e.g., the task and physical limits on a joint angle or joint angular velocity are not considered to be involved in

the resolution. Therefore, the resolved joint angle or the joint angular velocity may exceed their safe boundaries and unsafe motion planning and control circumstances can occur. Moreover, redundancy is actually fully made use to obtain the optimal solutions explicitly.

To remedy this, the following constrained optimization paradigm can be [28]

$$
\begin{aligned}
\min. \quad & \|J\dot{\theta} - \dot{X}\|_2^2 \\
\text{s.t.} \quad & \dot{\theta}^- < \dot{\theta} < \dot{\theta}^+ \\
& g(\dot{\theta}) \le 0
\end{aligned} \qquad (12)
$$

where $\dot{\theta}^-$ and $\dot{\theta}^+$ respectively denote the physical lower and upper limits for joint velocity $\dot{\theta}$, and $g(\cdot) : R^n \to R^m$ denotes a nonlinear or linear mapping on $\dot{\theta}$ to construct the task constraint. However, such optimization paradigm still can not be guaranteed strictly convex as the Jacobian matrix might become singular at some time instant t, and improved optimization paradigm is yet to be proposed. We still reserve the possibility to find the convex optimization paradigm and expect to achieve global solutions.

5. General Optimization Paradigm

In this paper, the following general optimization paradigm for kinematic control of redundant manipulators is addressed:

$$
\begin{aligned}
\min. \quad & \|\dot{\theta}\|_p^p \\
\text{s.t.} \quad & J\dot{\theta} = \dot{X} \\
& \dot{\theta}^- < \dot{\theta} < \dot{\theta}^+ \\
& g(\dot{\theta}) \le 0
\end{aligned} \qquad (13)
$$

where $\| \cdot \|^p$ ($0 \le p \le 2$) denotes the p-norm of a vector. Utilization of such p-norm ($0 < p \le 2$) in the objective function can guarantee strict convexity. The differential kinematics $J\dot{\theta} = \dot{X}$ here is treated as one necessary constraint. Such way of processing can get rid of non-convexity modeling and global solutions are expected to get theoretically. However, the number of the constraints is increased and the manipulator has to possess stronger redundancy to guarantee all of the constraints are satisfied. It means that the redundancy has to leave enough margins for finding the optimal solutions. In this situation, solutions seems global and some of the constraints might be not satisfied. All of these indicates that a highly-redundant manipulator is preferred to fulfill more complicated tasks with a lot of task and physical constraints.

5.1. Non–sparse l_2–norm based optimization

When $p = 2$, the p-norm becomes a two-norm. The objective function thus becomes a form like sum of

squares, and the optimization paradigm becomes

$$\begin{aligned} \min. \quad & \|\dot{\theta}\|_2^2 \\ \text{s.t.} \quad & J\dot{\theta} = \dot{X} \\ & \dot{\theta}^- < \dot{\theta} < \dot{\theta}^+ \\ & g(\dot{\theta}) \le 0 \end{aligned} \tag{14}$$

Two-norm based optimization is widely used in the kinematic control and redundancy resolution issue depending on different types of task constraint $g(\dot{\theta}) \le 0$. These are the relevant literature based on the non-sparse l_2-norm based optimization in addition to physical limits and the differential kinematics as the basic constraints, the task constraints can be the repetitive motion planning [29, 30], obstacle avoidance [31, 32], joint drift elimination [33–36], RCM (remote center of motion) based kinematic control [37–40], and fault-tolerant motion planning and control [41–43].

5.2. Sparse l_p-norm based optimization

When $0 \le p < 2$, the following p-norm based optimization can be used for kinematic control in a sparse perspective:

$$\begin{aligned} \min. \quad & \|\dot{\theta}\|_p^p \\ \text{s.t.} \quad & J\dot{\theta} = \dot{X} \\ & \dot{\theta}^- < \dot{\theta} < \dot{\theta}^+ \\ & g(\dot{\theta}) \le 0 \end{aligned} \tag{15}$$

However, when $0 < p < 1$, it is very difficult to perform computation of optimization as it is closer to a N-P hard computation issue when p approaches zero more. To make the computation feasible, $1 \le p < 2$ is chosen for sparse l_p-norm based optimization. To evaluate the sparsity of the optimization in terms of different p in the velocity level, the following metrics can be considered [44]:

$$\text{Sparsity} = -\|\dot{\theta}\|_q \quad q > 0 \tag{16}$$

Currently, most related works proposed and developed were based on the two-norm non-sparse optimization, there are few sparse optimization based works on kinematic control issues [45], e.g., l_1-norm based optimization [46] and l_p-norm based optimization with simultaneous minimizing potential energy [47]. Usage of sparse optimization can "compress" the joint motion information like the way of compress sensing. The resolved joint velocity might present their amplitudes in a lower level but still the kinematic control can be successfully guaranteed. It may be beneficial to decrease the kinetic energy of the joints and links when doing the same path tracking task at the end-effector level. When the redundancy of the manipulator is higher, such benefits can be more evident in sparse optimization based kinematic control.

5.3. Optimization Solver

In order to solve aforementioned optimization problems both in the non-sparse and sparse manners, as the objective function is always convex, one can directly define the Lagrange function as follows according to the well-known KKT conditions

$$L(\dot{\theta}, \lambda_1, \lambda_2) = \|\dot{\theta}\|_p^p / p + \lambda_1^T (J\dot{\theta} - \dot{X}) + \lambda_2^T g(\dot{\theta}) \tag{17}$$

The final steady-state optimal solutions can be obtained by letting the following gradients to to be zero

$$\begin{aligned} \frac{\partial L(\dot{\theta}, \lambda_1, \lambda_2)}{\partial \dot{\theta}} &= 0 \\ \frac{\partial L(\dot{\theta}, \lambda_1, \lambda_2)}{\partial \lambda_1} &= 0 \\ \frac{\partial L(\dot{\theta}, \lambda_1, \lambda_2)}{\partial \lambda_2} &= 0 \end{aligned} \tag{18}$$

However, directly finding the analytical solutions for the optima would be very difficult. A better methodology that had been frequently used is primal dual neural network (PDNN) [48]. The idea of design principle is to firstly involve the time-derivatives of $\dot{\theta}$, λ_1 and λ_2 to establish a state-space dynamic model. Once the optimal solutions are solved in the steady state, the equilibrium points of the model are achieved. The specific dynamic equations of the PDNN solver are as follows.

$$\begin{aligned} \ddot{\theta} &= \gamma[P_\Omega(u - \frac{\partial L(\dot{\theta}, \lambda_1, \lambda_2)}{\partial \dot{\theta}}) - u] \\ \dot{\lambda}_1 &= \gamma(J\dot{\theta} - \dot{X}) \\ \dot{\lambda}_2 &= \gamma g(\dot{\theta}) \end{aligned} \tag{19}$$

where $\gamma > 0$ scales the convergence rate of the dynamic optimization solver, $P_\Omega(\cdot)$ denotes the piecewise linear projection function array with a solution set Ω, i.e.,

$$P_\Omega(z) = \begin{cases} z^+; & z \ge z^+ \\ z; & z^- < z < z^+ \\ z^-; & z \le z^- \end{cases} \tag{20}$$

with the lower and upper bounds being z^- and z^+ respectively and the input variable being z.

On convergence properties of the constrained optimization solver based on the PDNN for kinematic control, please refer to related works [48–50].

Another way to solve the constrained optimization for kinematic control is the BAS algorithm, refer to [51, 52] for details. Some other works on zeroing neural networks can also be excellent optimization solvers.

5.4. Benefits of Redundancy with Optimization

As redundant manipulators possess inherent redundancy in robotic mechanisms, i.e., the number of DoF

in joint space can be configured larger than that of DoF in Cartesian space. It provides possibility multiple alternative solutions for finding the optimal one subject to the constraints. By designing convex optimization paradigms, joint angular variables can be resolved globally. Such redundancy with optimization offers the following potential benefits for redundant manipulators in kinematic control:

- Redundant manipulators have extra DoFs for fulfilling the secondary task in addition to the end-effector path tracking task;

- Additional constraints such as physical limit constraints, task constraints and performance index constraints can be added into optimization to complete more comprehensive tasks which attain multiple requirements until such redundancy could not afford to find the optimal solutions with all these constraints satisfied;

- Sparsity-based optimization paradigms which preserve convexity can be designed for sparse redundancy resolution, and it may lead to less kinetic energy variations for links and joints;

- Redundant manipulators have additional normal joints to proceed kinematic control tasks even when some joint(s) are falling into fault states such as a joint mechanical lock or joint velocity zeroing, they have such fault-tolerant ability to some extent when facing kinematic failures as the fault joints can generate another special constraint.

6. Conclusions

Redundant manipulators have been widely applied in many industrial and human-centred applications. Redundant manipulators can make use of their redundancy to achieve certain benefits in kinematic control with multiple tasks and indices. In this paper, we have revisited and summarized current constrained optimization paradigms for kinematic control of redundant manipulators, pointed out the feasible optimization paradigms and addressed benefits from such redundancy with optimization exerted.

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