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Toward Combining Fuzzy Graphs Based on Hedge Algebra

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Abstract

In this paper, we study fuzzy graph properties with combinatorial matrix theory in fuzzy linguistic matrix. We use hedge algebra and linguistic variables for combining and reasoning with words. We figure out theorem of limiting in matrix space. We also discover limit space states of fuzzy graph with a fundamental theorem. This is the important theorem to decide whether automata are finite or not.

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1 Introduction

In everyday life, people use natural language (NL) for analyzing, reasoning, and finally, make their decisions. Computing with words (CWW) [5] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy logic (FL) is fuzzy cognitive map ($\mathbb{FCM}$), introduced by B. Kosko [1], combined fuzzy logic with neural network. $\mathbb{FCM}$ has a lots of applications in both modeling and reasoning fuzzy knowledge [3, 4] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain [5], for example, linguistic summarization problems [6]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for computing with words. The remainder of paper is organized as follows. Section 2 reviews some main concepts of computing with words based on $\mathbb{HA}$ in subsection 2.1 and describes several primary concepts for $\mathbb{FCM}$ in subsection 2.2. Section 3 reviews modeling with words using $\mathbb{HA}$. Important section 4 proves two theorems, the center of paper. Section 6 outlines conclusions and future work.

2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and $\mathbb{FCM}$ used in the paper.

2.1 Hedge algebra

. In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition of a $\mathbb{HA}$ is specified by 3-Tuple $\mathbb{HA} = (X, H, \leq\ )$ in [7]. In [8] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+,\ c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c | \ c \in G,\ \delta \in H^*(\text{hedge string over H})\}$,

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$\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \dots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is* Age, $G = \{c^+ = young; \ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable Age X is $\mathbb{L}$(X) or $\mathbb{L}$ for short:
$\mathbb{L}$ = *{very less young ; less young ; young ; more young ; very young ; very very young . . . }*

Fuzziness properties of elements in $\mathbb{HA}$, specified by *fm* (fuzziness measure) [8] as follows:

Definition 2.1. A mapping $fm : \mathbb{L} \to [0, 1]$ is said to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c\in\{c^+,c^-\}} fm(c) = 1, \ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h_i\in H} fm(h_i x) = fm(x), \ \ x = h_n h_{n-1} \dots h_1 c,$ the canonical form.
3. $fm(h_n h_{n-1} \dots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x).$

2.2 Fuzzy cognitive map

Fuzzy cognitive map ($\mathbb{FCM}$) is feedback dynamical system for modeling fuzzy causal knowledge, introduced by B. Kosko [1]. $\mathbb{FCM}$ is a set of nodes, which present concepts and a set of directed edges to link nodes. The edges represent the causal links between these concepts. Mathematically, a $\mathbb{FCM}$ is defined by

Definition 2.2. A $\mathbb{FCM}$ is a 4- Tuple:

$$\mathbb{FCM} = \{C, E, \mathcal{C}, f\} \tag{1}$$

In which:

1. $C = \{C_1, C_2, \dots, C_n\}$ is the set of N concepts forming the nodes of a graph.
2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \{-1, \ 0, \ 1\}$ is a function associating e_{ij} with a pair of concepts (C_i, C_j), so that e_{ij} = weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N\times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i(t) \in [0, 1], t \in N$
4. $C(0) = [C_1(0, C_2(0), \dots, C_n(0)] \in [0, 1]^N$ is the initial vector, recurring transformation function f is defined as:
$$C_j(t + 1) = f(\sum_{i=1}^{N} e_{ij} C_i(t)) \tag{2}$$

Example 2. Fig.1 shows a medical problem from expert domain of strokes and blood clotting involving. Concepts C={blood stasis (stas), endothelial injury (inju), hypercoagulation factors (HCP and HCF)} [2]. The connection matrix is:

$$E = (e_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

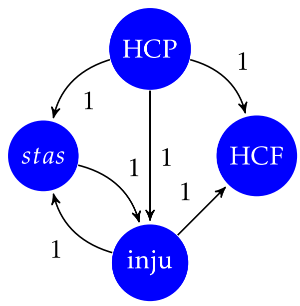


Fig. 1. A simple $\mathbb{FCM}$

$\mathbb{FCM}$s have played a vital role in the applications of scientific areas, including expert system, robotics, medicine, education, information technology, prediction, etc [3, 4].

3 Modeling with words [10]

Fuzzy model, based on linguistic variables, is constructed from linguistic hedge of $\mathbb{HA}$.

Definition 3.1 (Linguistic lattice)**.** With $\mathbb{L}$ as in the section 2.1, set $\{\wedge, \ \vee\}$ is logical operators, defined in [7, 8], a linguistic lattice $\mathcal{L}$ is a tuple:

$$\mathcal{L} = (\mathbb{L}, \vee, \wedge, 0, 1) \tag{3}$$

Property 3.1. The following is some properties for $\mathcal{L}$:

1. $\mathcal{L}$ is a linguistic-bounded lattice.
2. $(\mathbb{L}, \ \vee)$ and $(\mathbb{L}, \ \wedge)$ are semigroups.

Definition 3.2. A linguistic cognitive map ($\mathbb{LCM}$) is a 4- Tuple:

$$\mathbb{LCM} = \{C, E, \mathcal{C}, f\} \tag{4}$$

In which:

1. $C = \{C_1, C_2, \dots, C_n\}$ is the set of N concepts forming the nodes of a graph.

2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \mathbb{L}$; e_{ij} = weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N\times N} \in \mathbb{L}^{N\times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i^t \in \mathbb{L}, t \in N$
4. $\mathcal{C}(0) = [C_1^0, C_2^0, \dots, C_n^0] \in \mathbb{L}^N$ is the initial vector, recurring transformation function f is defined as:

$$C_j^{t+1} = f(\sum_{i=1}^{N} e_{ij} C_i^t) \in \mathbb{L} \quad (5)$$

Example 3. Fig. 2 shows a simple $\mathbb{LCM}$. Let

$$\mathbb{HA} =< \mathcal{X} = \text{truth}; c^+ = \text{true}; \mathcal{H} = \{\mathscr{L}, \mathscr{M}, \mathscr{V}\} > \quad (6)$$

be a $\mathbb{HA}$ with order as $\mathscr{L} < \mathscr{M} < \mathscr{V}$ ($\mathscr{L}$ for less, $\mathscr{M}$ for more and $\mathscr{V}$ for very are hedges). $C = \{c_1, c_2, c_3, c_4\}$ is the set of 4 concepts with corresponding values $\mathcal{C} = \{\text{true}, \mathscr{M}\text{true}, \mathscr{L}\text{true}, \mathscr{V}\text{true}\}$

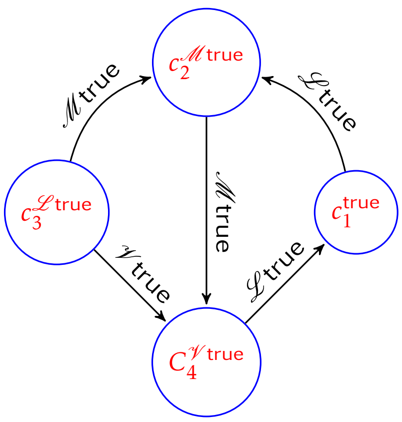


Fig. 2. A simple $\mathbb{LCM}$

Square matrix:

$$M = (m_{ij} \in \mathbb{L})_{4\times 4} = \begin{pmatrix} 0 & \mathscr{L}\text{true} & 0 & 0 \\ 0 & 0 & 0 & \mathscr{M}\text{true} \\ 0 & \mathscr{M}\text{true} & 0 & \mathscr{V}\text{true} \\ \mathscr{L}\text{true} & 0 & 0 & 0 \end{pmatrix}.$$

is the adjacency matrix of $\mathbb{LCM}$. Causal relation between c_i and c_j is m_{ij}, for example if $i = 1, j = 2$ then causal relation between c_1 and c_2 is: " *if* c_1 *is* true *then* c_2 *is* $\mathscr{M}$true *is* $\mathscr{L}$true" or let $\mathcal{P}$="if c_1 is true then c_2 is $\mathscr{M}$true" be a proposition then truth($\mathcal{P}$) = $\mathscr{L}$true

Definition 3.3. [11] A $\mathbb{LCM}$ is called complete if between any two nodes always having a connected edge (without looping edges).

4 Combining $\mathbb{LCM}$

In many learning algorithms, which use Hebbian rule [3, 4], as time t, the weight of every edges will always be updated $\Delta e_{ij} = f(\sum_k e_{jk} \times c_k \times c_j)$. Let $^{\hbar}M(n)$, $2 \le n \le \mathbb{N}$ be total connection matrices with $\mathbb{N}$ vetices. Fuzzifying edge set uses $\hbar$ hedges. The following theorem figures out side of the connection matrix.

Theorem 4.1. Linguistic connection matrix $M = (m_{ij} \in \mathbb{L})_{\mathbb{N}\times\mathbb{N}}$ constructing on $\hbar$ hedges has size:

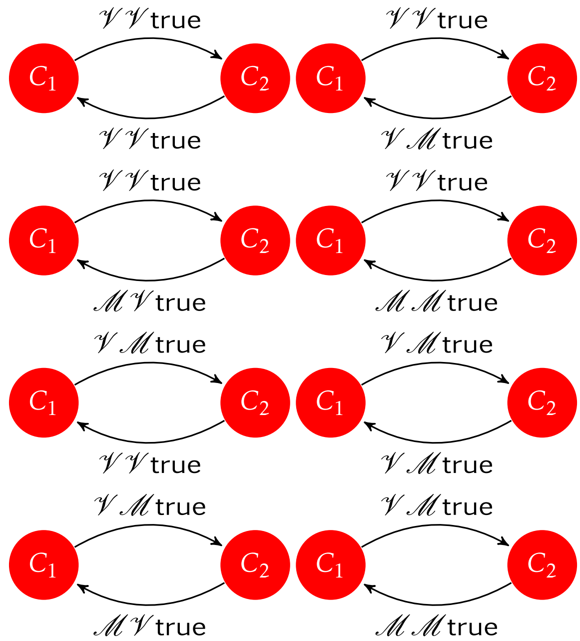
$$^{\hbar}M(\mathbb{N}) = (\hbar^{\hbar})^{2\times\binom{\mathbb{N}}{2}} \quad (7)$$

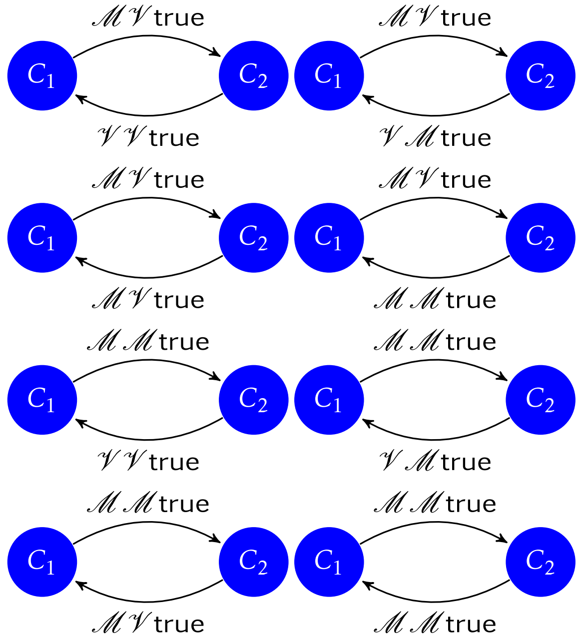
We proof theorem (4.1) by using induction method on number of vetices n, $2 \le n \le \mathbb{N}$. This process follows two steps. First, set $n = 2$ and check to see if the $^{\hbar}M(2)$ is true. Next, assume $^{\hbar}M(n)$ is true, we have to prove $^{\hbar}M(n+1)$ is true as in logical expression:

$$^{\hbar}M(2) \wedge (^{\hbar}M(n) \rightarrow {}^{\hbar}M(n+1)) \rightarrow \forall n\, ^{\hbar}M(n) \quad (8)$$

Proof. Without loss of generality, we set $\hbar = 2$ and induction on n. with $\hbar \ge 3$, the process is the same.

1. To prove $^2M(2)$ is true, we must indicate that: $^2M(2) = (2^2)^{2\times\binom{2}{2}} = 16$. This is done as in the following 16 figures, in which hedges $\{\mathscr{V}$ = *very*, $\mathscr{M}$ = *more*$\}$





2. Now, assume ${}^2M(n)$ is true on n vetices $c_1, c_2, \ldots, c_n$, that is ${}^2M(n) = (2^2)^{2\times\binom{n}{2}}$. We must prove ${}^2M(n+1) = (2^2)^{2\times\binom{n+1}{2}}$ is true. consider on vertex c_{n+1}, there have:

   - n edges from n vetices $c_1, c_2, \ldots, c_n$ go in c_{n+1}
   - n edges from c_{n+1} go out to n vetices $c_1, c_2, \ldots, c_n$
   - Total $2 \times n$ edges connected to c_{n+1} which generates $(2^2)^{2\times n}$ difference combinatories.

   Applying product rule:

$$\begin{aligned} {}^2M(n+1) &= {}^2M(n) \times (2^2)^{2\times n} \\ &= (2^2)^{2\times\binom{n}{2}} \times (2^2)^{2\times n} \\ &= (2^2)^{2\times\binom{n+1}{2}} \\ &\mathcal{QED}. \end{aligned}$$

By using the counting method, it is straightforward to prove theorem (4.1) in the case of complete $\mathbb{LCM}$ □

Theorem 4.1 is important in counting the connection matrices. On the other hand, let ${}^{\hbar}\mathbb{LCM}(n)$ be total $\mathbb{LCM}$ which generate from $\mathbb{N}$ vertices. We want to know whether ${}^{\hbar}\mathbb{LCM}(n)$ finite or infinite. Finding ${}^{\hbar}\mathbb{LCM}(n)$ helps to limit searching space in many cases.

Theorem 4.2. Fuzzifying $\mathbb{N}$ vertices and $2\times\binom{\mathbb{N}}{2}$ edges use $\hbar$ hedges generated a state space ${}^{\hbar}\mathbb{LCM}(n)$:

$$ {}^{\hbar}\mathbb{LCM}(\mathbb{N}) = (\hbar^{\hbar})^{\mathbb{N}^2} \tag{9}$$

Proof. It is straightforward to prove theorem 4.2 by using combinatory algebra.

- $\mathbb{N}$ vetices with $\hbar^{\hbar}$ cases for each vertex which produce $(\hbar^{\hbar})^{\mathbb{N}}$
- Applying result from theorem 4.1:

$$\begin{aligned} {}^{\hbar}\mathbb{LCM}(\mathbb{N}) &= {}^{\hbar}M(\mathbb{N}) \times (\hbar^{\hbar})^{\mathbb{N}} \\ &= (\hbar^{\hbar})^{2\times\binom{\mathbb{N}}{2}} \times (\hbar^{\hbar})^{\mathbb{N}} \\ &= (\hbar^{\hbar})^{\mathbb{N}^2} \\ &\mathcal{QED}. \end{aligned}$$

□

5 Combining $\mathbb{LCM}$

. Linguistic Cognitive Maps allow for a simple aggregation of knowledge which obtained from experts. The combination will improving reliability of the final model. $\mathbb{LCM}$ linguistic matrices additively combine to form new $\mathbb{LCM}$s.

Definition 5.1. Let $\mathbb{LCM}_{\sum}$ with connection matrix $M_{\sum\times\sum}$ be combination from $\mathbb{LCM}_k$ with connection matrix $M_{n_k\times n_k}$. k=1, 2, .. Then:

$$M_{\sum\times\sum} = \bigvee_k M^k_{n_k\times n_k} \tag{10}$$

Example 4. $\mathbb{LCM}$ in Fig. 5 combines two $\mathbb{LCM}$s in Fig. 3 and Fig. 4. In which:
For Fig. 3, square matrix:

$$M^1_{4\times4} = \begin{pmatrix} 0 & \mathscr{M}\mathscr{V}\text{true} & \mathscr{V}\mathscr{V}\text{true} & 0 \\ 0 & 0 & \mathscr{V}\mathscr{V}\text{true} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathscr{V}\mathscr{V}\text{true} & 0 & 0 \end{pmatrix}.$$

For Fig. 4, square matrix:

$$M^2_{4\times4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathscr{V}\mathscr{V}\text{true} & 0 & \mathscr{V}\mathscr{V}\text{true} \\ 0 & \mathscr{V}\mathscr{V}\text{true} & 0 & \mathscr{V}\mathscr{V}\text{true} \\ 0 & 0 & \mathscr{V}\mathscr{M}\text{true} & 0 \end{pmatrix}.$$

For Fig. 5, square matrix $M_{\sum\times\sum}$ is:

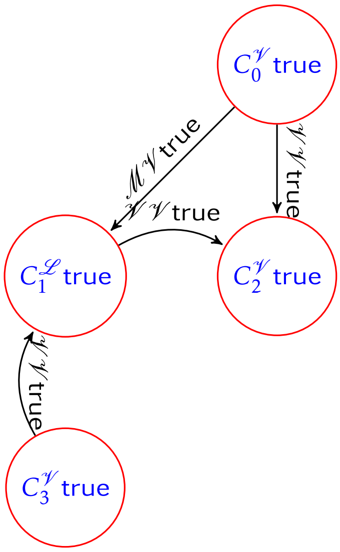


Fig. 3. A first simple $\mathbb{LCM}$

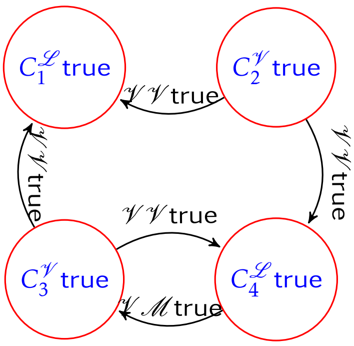


Fig. 4. A second simple $\mathbb{LCM}$

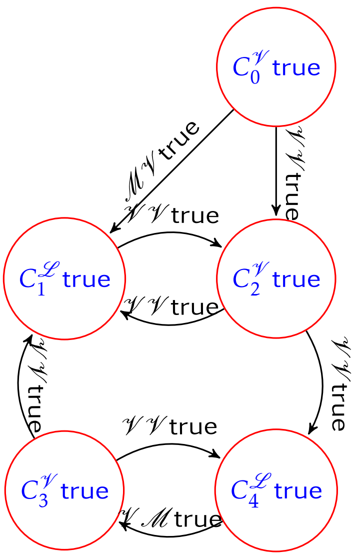


Fig. 5. The combination $\mathbb{LCM}$

$$M_{\Sigma\times\Sigma} = \begin{pmatrix} 0 & 0 & \mathscr{V}\mathscr{V}\,\text{true} & \mathscr{V}\mathscr{V}\,\text{true} & 0 \\ 0 & 0 & \mathscr{V}\mathscr{V}\,\text{true} & 0 & 0 \\ 0 & \mathscr{V}\mathscr{V}\,\text{true} & 0 & 0 & \mathscr{V}\mathscr{V}\,\text{true} \\ 0 & \mathscr{V}\mathscr{V}\,\text{true} & 0 & 0 & \mathscr{V}\mathscr{V}\,\text{true} \\ 0 & 0 & 0 & \mathscr{V}\mathscr{M}\,\text{true} & 0 \end{pmatrix}.$$

Property 5.1. The $\mathbb{LCM}$ combination operator defined in definition (5.1) preserved causal relation properties

6 Conclusions and future work

We have proved two important theorems in combining fuzzy graphs. First theorem verified connection matrix is limited by expression $(\hbar^{\hbar})^{2\times\binom{\mathbb{N}}{2}}$. We also demonstrated the theorem about the whole state space is $(\hbar^{\hbar})^{\mathbb{N}^2}$. This is the important theorem to indicate that graph state space is finite and therefore automata are finite.

Our next study is as follows:

Given $\mathbb{HA} = \{X,\ \mathcal{H},\ \{c^+,\ c^-\},\ \{0,\ W,\ 1\},\ \leq\}$ and let

$$\mathcal{H}^* = \{h_n h_{n-1} \dots h_0\ c^+ |\ \forall h_i \in \mathcal{H}; i \geq 0\} \tag{11}$$

be an alphabet of node and edge labels. A graph over $\mathcal{H}$ is a tuple:

$$\mathbb{LCM} = \langle V_{\mathbb{LCM}},\ edg_{\mathbb{LCM}},\ lab_{\mathbb{LCM}} \rangle \tag{12}$$

In which $V_{\mathbb{LCM}}$ is the finite set of *vertices*; Binary relation $edg_{\mathbb{LCM}} \subseteq V_{\mathbb{LCM}} \times \mathcal{H} \times V_{\mathbb{LCM}}$ saying if two vertices are linked by an edge with label in $\mathcal{H}$. Total map $lab_{\mathbb{LCM}} :\ V_{\mathbb{LCM}} \to \mathcal{H}$ assigning a label in $\mathcal{H}$ to each vertex of $\mathbb{LCM}$.

The set of all $\mathbb{LCM}$ over $\mathcal{H}$ is denoted by $\mathbb{LCM}_{\mathcal{H}}$, and the set of all graphs isomorphic to $\mathbb{LCM}$ is denoted by $[\mathbb{LCM}_{\mathcal{H}}]$. A graph language $\mathscr{L}$ is a subset $\mathscr{L} \subset [\mathbb{LCM}_{\mathcal{H}}]$.

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