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Toward Modeling and Reasoning with Words Based on Hedge Algebra

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Abstract

In this paper, we introduce a method for reasoning with words based on hedge algebra using linguistic cognitive map. Our computing method consists of static and dynamic reasoning. In static reasoning, inferring on causal path of graph drives fuzzy linguistic value between any vertices and edges. With dynamic reasoning, system behaves as a dynamical system and convolution as automata property.

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1 Introduction

. In everyday life, people use natural language (NL) for analysing, reasoning, and finally, make their decisions. Computing with words (CWW) [5] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy logic (FL) is fuzzy cognitive map ($\mathbb{FCM}$), introduced by B. Kosko [1], combined fuzzy logic with neural network. $\mathbb{FCM}$ has a lots of applications in both modeling and reasoning fuzzy knowledge [3, 4] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain [5], for example, linguistic summarization problems. To solve this problem, in [9] , we used an abtract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for modeling with words without computing on the graph. In the paper we study two method for reasoning with words on our model.

The remainder of paper is organized as follows. Section 2 reviews some main concepts of computing with words based on $\mathbb{HA}$ in subsection 2.1 and describes several primary concepts for $\mathbb{FCM}$ in next subsection 2.2. In section 3, we review our approach technique to modeling with words using $\mathbb{HA}$. Section 4 presents reasoning with words using hedge algebra. Section 5 outlines conclusion and future work.

2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and $\mathbb{FCM}$ used in the paper.

2.1 Hedge algebra

. In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition of a $\mathbb{HA}$ is specified by 3-Tuple $\mathbb{HA} = (X, H, \leq)$ in [6]. In [7] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+, c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c | \, c \in G, \ \delta \in H^*(\text{hedge string over H})\}$, $\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \ldots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is Age*, $G = \{c^+ = young;\ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable Age X is $\mathbb{L}$(X) or $\mathbb{L}$ for short:
$\mathbb{L}$ = *{very less young ; less young ; young ; more young ; very young ; very very young . . . }*

Fuzziness properties of elements in $\mathbb{HA}$, specified by *fm* (fuzziness measure) [7] as follows:

Definition 2 .1. A mapping $fm : \mathbb{L} \longrightarrow [0, 1]$ is said to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c \in \{c^+, c^-\}} fm(c) = 1$, $fm(0) = fm(w) = fm(1) = 0$.
2. $\sum_{h_i \in H} fm(h_i x) = fm(x)$, $x = h_n h_{n-1} \ldots h_1 c$, the canonical form.
3. $fm(h_n h_{n-1} \ldots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x)$.

2.2 Fuzzy cognitive map

Fuzzy cognitive map ($\mathbb{FCM}$) is feedback dynamical system for modeling fuzzy causal knowledge, introduced by B. Kosko [1]. $\mathbb{FCM}$ is a set of nodes, which present concepts and a set of directed edges to link nodes. The edges represent the causal links between these concepts. Mathematically, a $\mathbb{FCM}$ bis defined by .

Definition 2 .2. A $\mathbb{FCM}$ is a 4- Tuple:

$$\mathbb{FCM} = \{C, E, \mathcal{C}, f\} \tag{1}$$

In which:

1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.
2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \{-1,\ 0,\ 1\}$ is a function associating e_{ij} with a pair of concepts (C_i, C_j), so that e_{ij} = "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N \times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i(t) \in [0, 1], t \in N$
4. With $C(0) = [C_1(0, C_2(0), \ldots, C_n(0)] \in [0, 1]^N$ is the initial vector, recurring transformation function f defined as:

$$C_j(t+1) = f\left(\sum_{i=1}^{N} e_{ij} C_i(t)\right) \tag{2}$$

Example 2. Fig.1 shows a medical problem from expert domain of strokes and blood clotting involving. Concepts C={blood stasis (stas), endothelial injury (inju), hypercoagulation factors (HCP and HCF)} [2]. The conection matrix is:

$$E = (e_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

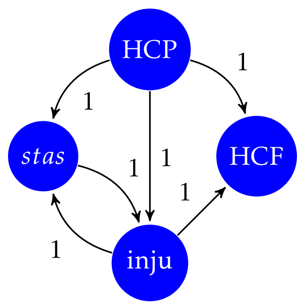

Fig. 1. A simple $\mathbb{FCM}$

$\mathbb{FCM}$s have played a vital role in the applications of scientific areas, including expert system, robotics, medicine, education, information technology, prediction, etc [3, 4].

3 Modeling with words

Our model, based on linguistic variables, is constructed from linguistic hedge of $\mathbb{HA}$. The following are definitions in our reseach paper.

Definition 3.1 (Linguistic l attice)**.** With $\mathbb{L}$ as in the section 2, set $\{\wedge, \ \vee\}$ are logical operators, defined in [6, 7], a linguistic lattice $\mathcal{L}$ is a tuple:

$$\mathcal{L} = (\mathbb{L}, \vee, \wedge, 0, 1) \tag{3}$$

Property 3.1. The following are some properties for $\mathcal{L}$:

1. $\mathcal{L}$ is a linguistic-bounded lattice.
2. $(\mathbb{L}, \vee)$ and $(\mathbb{L}, \wedge)$ are semigroups.

Proof. Without loss of generality, let $\mathbb{L} = \{\rho\ c^+ | \rho \in (H^+)^* \wedge c^+ \in G\}$. W is the neutral element in $\mathbb{HA}$, we have:

1. $0 < w < c^+ < 1$ and for $\forall\ \rho \in (H^+)^* :\ \rho 0 < \rho w < \rho c^+ < \rho 1$. Because $\rho 0 = 0; \rho w = w; \rho 1 = 1$. This is equivalent to: $0 < \rho c^+ < 1$ or $\mathcal{L}$ is bounded
2. Let $\circ = \wedge$ or $\circ = \vee$ be operators in $\mathbb{HA}$ and $\{p, q, r\} \in X$. Applying definitions of operators $\wedge$ and $\vee$ from [8]: $p \circ (q \circ r) \wedge (\circ = \vee) = max\{p, max\{q, r\}\} = max\{p, q, r\} = (p \circ q) \circ r \wedge (\circ = \vee)$

□

Definition 3.2. A linguistic cognitive map ($\mathbb{LCM}$) is a 4- Tuple:

$$\mathbb{LCM} = \{C, E, \mathcal{C}, f\} \tag{4}$$

In which:

1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.
2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \mathbb{L}; e_{ij} =$ "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N \times N} \in \mathbb{L}^{N \times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i(t) \in \mathbb{L}, t \in N$
4. With $C(0) = [C_1(0), C_2(0), \ldots, C_n(0)] \in \mathbb{L}^N$ is the initial vector, recurring transformation function f defined as:

$$C_j(t+1) = f\left(\sum_{i=1}^{N} e_{ij} C_i(t)\right) \in \mathbb{L} \tag{5}$$

Example 3. Fig. 2 shows a simple $\mathbb{LCM}$. Let

$$\mathbb{HA} =< \mathcal{X} = \text{truth}; c^+ = \text{true}; \mathcal{H} = \{\mathscr{L}, \mathscr{M}, \mathscr{V}\} > \tag{6}$$

be a $\mathbb{HA}$ with order as $\mathscr{L} < \mathscr{M} < \mathscr{V}$ ($\mathscr{L}$ for less, $\mathscr{M}$ for more and $\mathscr{V}$ for very are hedges).

$C = \{c_1, c_2, c_3, c_4\}$ is the set of 4 concepts with corresponding values $\mathcal{C} = \{\text{true}, \mathscr{M}\text{true}, \mathscr{L}\text{true}, \mathscr{V}\text{true}\}$

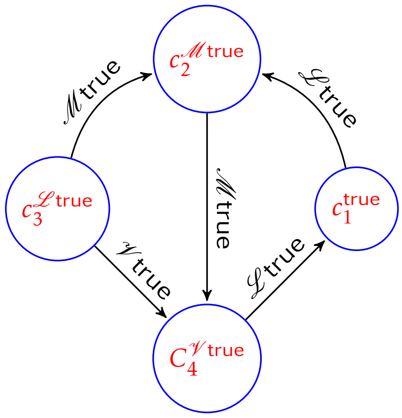

Fig. 2. A simple $\mathbb{LCM}$

Square matrix:

$$M = (m_{ij} \in \mathbb{L})_{4\times 4} = \begin{vmatrix} 0 & \mathscr{L}\text{true} & 0 & 0 \\ 0 & 0 & 0 & \mathscr{M}\text{true} \\ 0 & \mathscr{M}\text{true} & 0 & \mathscr{V}\text{true} \\ \mathscr{L}\text{true} & 0 & 0 & 0 \end{vmatrix}.$$

is the adjacency matrix of $\mathbb{LCM}$. Causal relation between c_i and c_j is m_{ij}, for example if $i = 1, j = 2$ then causal relation between c_1 and c_2 is: " *if* c_1 *is* true *then* c_2 *is* $\mathscr{M}$true *is* $\mathscr{L}$true" or let $\mathcal{P}$="if c_1 is true then c_2 is $\mathscr{M}$true" be a proposition then truth$(\mathcal{P}) = \mathscr{L}$true

Definition 3 .3. A $\mathbb{LCM}$ is called complete if between any two nodes alway having a connected edge (without looping edges).

4 Reasoning with words

Give state space:

$$\mathfrak{C} = \{\mathcal{C}\}_0^n = \{\mathcal{C}^0,\ \mathcal{C}^1, \ldots, \mathcal{C}^n\}$$

Where $\mathcal{C}^i = \{\mathrm{C}_1^i,\ \mathrm{C}_2^i, \ldots,\ \mathrm{C}_{\mathrm{N}}^i\}, i = \overline{0, n}$. Inference on $\mathbb{LCM}$ consists of static reasoning $\mathfrak{SR}$ and dynamic reasoning $\mathfrak{DR}$.

Definition 4 .1. A linguistic reasoning $\mathfrak{R}$ in $\mathbb{LCM}$ is a sequence of transitions where the source of each is the destination of the previous one, which can be written as:

$$\mathfrak{R} \triangleq \mathbb{O}_0 \xrightarrow{\ell_1} \mathbb{O}_1 \xrightarrow{\ell_2} \mathbb{O}_2 \ldots \xrightarrow{\ell_n} \mathbb{O}_n \tag{7}$$

The state $\mathfrak{O}_0$ is the *source* of the reasoning $\mathfrak{R}$, and $\mathfrak{O}_n$ its *destination*. The length of reasoning is n.

Static properties allow deduction between concepts in a specific case of $\mathcal{C}^i \in \mathfrak{C}$. In equation (7) by definition (4.1), substitute objects $\mathfrak{O}_i$ with concepts C_i $(C_i \quad \mathfrak{O}_i)$ for $\forall i \in \overline{1,n}$ to drive $\mathfrak{SR}$

Definition 4.2. Static reasoning on state $\mathcal{C}^i \in \mathfrak{C}$ is a process:

$$\mathfrak{SR} \triangleq C_1^i \xrightarrow[\mathbb{LCM}]{\ell_1} C_2^i \xrightarrow[\mathbb{LCM}]{\ell_2} \dots \xrightarrow[\mathbb{LCM}]{\ell_n} C_n^i;\ \ell_i \in \mathbb{L}\ \forall i = \overline{1,n} \tag{8}$$

We write $\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^{\mathbb{N}}(C_1^i,\ C_2^i, \dots,\ C_{\mathbb{N}}^i)$ to mean that causal relation $\varphi^2(C_j^i, C_k^i) \in \mathbb{L}$ for $1 \le j \le k \le \mathbb{N}$ on causal path: $C_1^i \to C_2^i \to \dots \to C_{\mathbb{N}}^i$ by applying $\mathfrak{SR}$ interpretation rules.

Theorem 4.1. For every $C_j^i \in \mathcal{C}^i; 1 \le j \le \mathbb{N}$:

$$\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^2(C_1^i, C_j^i) \in \mathbb{L} \tag{9}$$

Proof. We use inductive mathematical method to prove theorem 4.1. $\mathbb{QED}$ is short for the Latin phrase *quod erat demonstrandum*, which means “which was to be proved”. Justification is placed in $\lhd . \rhd$

Statement

$\lhd$ Justification$\rhd$

1.$\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^2(C_1^i, C_2^i) \in \mathbb{L}$

$\lhd$ By definition 4.2$\rhd$

2.$\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^k(C_1^i, C_2^i, \dots, C_k^i)$

$\lhd$ Premise of inductive hypothesis$\rhd$

3.$\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^2(C_1^i, C_k^i) \in \mathbb{L}$

$\lhd$ Infer from 2$\rhd$

4.$\mathbb{LCM} \vdash_{\mathfrak{SR}} \varphi^2(C_k^i, C_{k+1}^i) \in \mathbb{L}$

$\lhd$ By definition 4.2$\rhd$

5.$\mathbb{QED}$

$\lhd$ 3, 4, Fuzzy hypothetical Syllogism$\rhd$

□

Example 4. Considering the set of fuzzy concepts:

1. If a student studying possible hard or his university is high-ranking, then he will be a good employee is more very true.
2. The university where Mary studies is very high-ranking is possibly very true.
3. Mary is studying very hard is more true.

Concepts are written in fuzzy propotions:

C_1 Student studying Possible hard

$\rhd$ stud(x, $\mathscr{P}$ Hard)

C_2 University is high-ranking

$\rhd$ isUniv(x, Hi-rank)

C_3 Good employee

$\rhd$ emp(x, good)

Causal relation between $(C_i, C_j), 1 \le i \ne j \le 3$ are

$$\varphi^2(C_1, C_3) = \mathscr{M}\ \mathscr{V}\ \text{true},\ \ \varphi^2(C_2, C_3) = \mathscr{M}\ \mathscr{V}\ \text{true}$$

Dynamic properties appear between states $\mathcal{C}^i$, for $\forall\ i = \overline{1,n}$ in state space $\mathfrak{C}$ $(\mathcal{C}^i \in \mathfrak{C})$. In equation (7) by definition (4.1), substitute objects $\mathfrak{O}_i$ with concepts C_i $(C_i \leftarrow \mathfrak{O}_i)$ for $\forall i \in \overline{0,n}$ to drive $\mathfrak{DR}$

Definition 4.3. Dynamic reasoning on state space $\mathfrak{C}$ is a sequences:

$$\mathfrak{DR} \triangleq \mathcal{C}^0 \xrightarrow[\mathfrak{C}]{\ell_1} C^1 \xrightarrow[\mathfrak{C}]{\ell_2} C^2 \dots \xrightarrow[\mathfrak{C}]{\ell_n} C^n;\ \ell_i \in \mathbb{L}\ \forall i = \overline{1,n} \tag{10}$$

Example 5. Let us consider the $\mathcal{FP}$ set in example 4, using inverse mapping for normalizing input propositions, say $\mathbb{I}$, to concepts $\mathcal{FP}$:

- ☐ For proposition: “Mary is studying very hard is more true” is formalized as:

  Mary is studying very hard is more true

  $\triangleq$ (stud(Mary, $\mathscr{V}$Hard), $\mathscr{M}$ true)

  = (stud(Mary, $\mathscr{V}^{-}\mathscr{V}$Hard), $\mathscr{V}\mathscr{M}$ true)

  = (stud(Mary, Hard), $\mathscr{V}\mathscr{M}$ true)

  = (stud(Mary,$\mathscr{P}$ Hard), $\mathscr{P}^{-}\mathscr{V}\mathscr{M}$ true)

  = (stud(Mary,$\mathscr{P}$ Hard), $\mathscr{P}\mathscr{V}\mathscr{V}$ true)

  Let $\alpha = \mathscr{P}\mathscr{V}\mathscr{V}$ true be an input then $\alpha \in \mathbb{I}$.

- ☐ Proposition "The university where Mary studies is very high-ranking is possibly very true $\triangleq$ (isUniv(Mary, $\mathscr{V}$Hi-rank),$\mathscr{P}\mathscr{V}$true)"

$$\begin{aligned}&(\text{isUniv(Mary, }\mathscr{V}\text{Hi-rank)},\mathscr{P}\mathscr{V}\text{true})\\ &= (\text{isUniv(Mary, }\mathscr{V}^{-}\mathscr{V}\text{Hi-rank)},\mathscr{V}\mathscr{P}\mathscr{V}\text{true})\\ &= (\text{isUniv(Mary, Hi-rank)},\mathscr{V}\mathscr{P}\mathscr{V}\text{true})\end{aligned}$$

and $\beta = \mathscr{V}\mathscr{P}\mathscr{V}\text{true} \in \mathbb{I}$.

We assume that, concepts are initated to neutral value: $\mathcal{C}^0 = \{C_1^0 = \mathscr{W},\ C_2^0 = \mathscr{W},\ C_3^0 = \mathscr{W}\}$, next state $\mathcal{C}^1$ is in example 6. State space $\mathfrak{C}$ behaves as an automata $\mathscr{A} \triangleq \mathscr{A}(\mathbb{I}, \mathscr{Q}, \mathscr{F})$ in the following property.

Property 4.1.

$$\mathfrak{C} \vdash \mathscr{A}(\mathbb{I}, \mathscr{Q}, \mathscr{F}) \tag{11}$$

In which:

1. $\mathbb{I}$ is a finite set of input symbols.
2. $\mathscr{Q}$ is the internal states of the system.
3. $\mathscr{F}$ is a state transition function.:

$$\mathscr{F} : \mathscr{Q} \times \mathbb{I} \to \mathscr{Q} \tag{12}$$

Proof. Set $\mathscr{Q} \subseteq \mathfrak{C}$, $\mathbb{I}$ is nomalized concepts as in example 5 and $\mathscr{F} = \{f(\sum_{i=1}^{N} e_{ij} C_i^t)\}$ as in equation (5) then $\mathscr{A}(\mathbb{I}, \mathscr{Q}, \mathscr{F})$ is an automata. □

Example 6. Let input set $\mathbb{I} = \{\alpha,\ \beta\}$, $\mathcal{C}^0$ be in example 5. $f(.) = \bigvee$ is the max function and matrix

$$M = (m_{ij})_{3\times 3} = \begin{vmatrix} 0 & 0 & \mathscr{M}\mathscr{V}\text{true} \\ 0 & 0 & \mathscr{M}\mathscr{V}\text{true} \\ 0 & 0 & 0 \end{vmatrix}.$$

- With input $\{\alpha\}$, $C_1^1 \quad \bigvee\{\alpha, \mathcal{C}^0 \bigvee m_{i1}, i = \overline{1,3}\}$ $=\bigvee\{\mathscr{P}\mathscr{V}\mathscr{V}\ \text{true}, 0\} = \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true}$, drives $\mathcal{C}^1 = \{C_1^1,\ C_2^1,\ C_3^1\} = \{\mathscr{P}\mathscr{V}\mathscr{V}\ \text{true},\ \mathscr{W},\ \mathscr{W}\}$
- With input $\{\beta\}$, $C_2^2 \quad \bigvee\{\beta, \mathcal{C}^1 \bigvee m_{i2}, i = \overline{1,3}\}$ $=\bigvee\{\mathscr{V}\mathscr{P}\mathscr{V}\ \text{true}, \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true}\} = \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true}$, drives $\mathcal{C}^2 =$ $\{C_1^2,\ C_2^2,\ C_3^2\}\{\mathscr{P}\mathscr{V}\mathscr{V}\ \text{true},\ \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true},\ \mathscr{W}\}$

If we write $\mathscr{F} = \{\sigma^{\mathscr{A}}\}_{\sigma\in\mathbb{I}}$:

$$\alpha^{\mathscr{A}} = \begin{pmatrix} \mathscr{W} & \mathscr{W} & \mathscr{W} \\ \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true} & \mathscr{W} & \mathscr{W} \end{pmatrix},$$

$$\beta^{\mathscr{A}} = \begin{pmatrix} \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true} & \mathscr{W} & \mathscr{W} \\ \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true} & \mathscr{P}\mathscr{V}\mathscr{V}\ \text{true} & \mathscr{W} \end{pmatrix}$$

Then automata $\mathscr{A}$ in example 6 becomes:

$$\mathscr{A} = \left(\{\mathcal{C}^0,\ \mathcal{C}^1,\ \mathcal{C}^2\},\ \{\alpha,\ \beta\}, \left\{\alpha^{\mathscr{A}},\ \beta^{\mathscr{A}}\right\}\right)$$

5 Conclusion and future work

We have introduced a new graphical model for representing fuzzy knowledge using linguistic variables from $\mathbb{HA}$. Our model, called $\mathbb{LCM}$, extended from $\mathbb{FCM}$, is a dynamical system with two properties: static and dynamic. Static properties allow forward or what-if inferencing between concepts on linguistic domain. Especially, we indicate inverse proportion relationship between length of hedges string and a number of partitions in representing fuzzy knowledge. Dynamic behaviors are transformation states in state space $\mathcal{C}^n = \{\mathcal{C}\}_0^n = \{\mathcal{C}(0),\ \mathcal{C}(1), \overline{\ldots}, \mathcal{C}(n)\}$, where $\mathcal{C}(i) = \{C_1(i),\ C_2(i), \ldots,\ C_{\mathbb{N}}(i)\}, i = \overline{0, n}$. We also prove the theorem about the number of states in state space is $|\mathcal{C}^n| = |\hbar|^{\mathbb{N}\times|\hbar|}$, this is the important theorem to decide whether or not installable computer programs. Our next study is as follow: Let $A = \{\hbar^n :\ \hbar^n = h_n h_{n-1} \ldots h_1 h_0 \text{ with } h_i \in H,\ i = \overline{0, n}\}$ be a string of hedges. Assume $I = \mathcal{C}(0)$, $T = \mathcal{C}(n)$ and $\mathcal{T} \subset \mathcal{C} \times A \times \mathcal{C}$ in order are initial, final and transition states. We will prove that $\mathbb{LCM}$ actions are fuzzy automata $\mathcal{A} =< A,\ \mathcal{C},\ I,\ \mathcal{T},\ T >$.

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