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Towards Linguistic Fuzzy Topological Spaces Based on Hedge Algebra

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Abstract

This paper studies on linguistic language which is generated from Hedge algebra. We indicate that the language satisfies properties of topology. Both linguistic topology and topological spaces are also studied in the paper.

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1 Introduction

Natural language processing (NL) is very important for analyzing, reasoning and decision-making for Artificial intelligence (AI). Computing with words (CWW [8, 11] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy set is fuzzy graph, fuzzy neural network and machine learning [2, 7, 10], combined fuzzy set with graph theory. Fuzzy graph ($\mathbb{FG}$) has a lots of applications in both modeling and reasoning fuzzy knowledge such as Human trafficking, internet routing, illegal immigration [9] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain, for example, linguistic summarization problems [8]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for studying with words. The remainder of paper is organized as follows: Section 2 reviews some main concepts of modeling with words based on $\mathbb{HA}$. Main section 3 studies properties of topological linguistic spaces. Section 4 outlines conclusions and future work.

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2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and some important knowledge used in the paper.

2.1 Hedge algebra

In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition o f a n $\mathbb{HA}$ i s s pecified by 3- Tuple $\mathbb{HA} = (X, H, \leq\ )$ in [5]. In [4] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+, c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c | c \in G, \delta \in H^*$(hedge string over H)$\}$, $\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \dots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is* Age, $G = \{c^+ = young;\ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable *Age* X is $\mathbb{L}(X)$ or $\mathbb{L}$ for short:
$\mathbb{L}$ = *{very less young ; less young ; young ; more young ; very young ; very very young . . . }*

Fuzziness properties of elements in $\mathbb{HA}$, specified by *fm* (fuzziness measure) [4] as follows:

Definition 2 .1. A m apping $fm : \mathbb{L} \to [0, 1]$ i s s aid to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c \in \{c^+, c^-\}} fm(c) = 1,\ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h_i \in H} fm(h_i x) = fm(x), \quad x = h_n h_{n-1} \dots h_1 c,$ the canonical form.

3. $fm(h_n h_{n-1} \dots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x)$.

The truth and meaning are fundamental important concepts in fuzzy logic, artificial intelligence and machine learning. In RCT (restriction-centered theory) [8], truth values are organized as a hierarchy with ground level or first-order and second-order. First order truth values are numerical values whereas second order ones are linguistic truth values. A linguistic truth value, say ℓ, is a fuzzy set. We study linguistic truth values on POSET $\mathbb{L}$ whose elements are comparable [3].

Definition 2.2. A $\mathfrak{L}$ STRUCT[ρ] on relational signature ρ is a tuple:

$$\mathfrak{L} = \langle \mathbb{L},\ f_{a_i}^{\mathfrak{L}},\ c_j^{\mathfrak{L}} \rangle \tag{1}$$

Consists of a universe $\mathbb{L} \neq \emptyset$ together with an interpretation of:

- each constant symbol c_j from ρ as an element $c_j^{\mathfrak{L}} \in \mathbb{L}$
- each a_i-ary function symbol f_{a_i} from ρ as a function:

$$f_i^{\mathfrak{L}} : \mathbb{L}^{a_i} \to \mathbb{L} \tag{2}$$

In $\mathbb{HA}$, $\ell \in \mathbb{L}$ and there are order properties:

Theorem 2.1. In [5] let $\ell_1 = h_n \dots h_1 u$ and $\ell_2 = k_m \dots k_1 u$ be two arbitrary canonical representations of ℓ_1 and ℓ_2, then there exists an index $j \leq \bigwedge\{m, n\} + 1$ such that $h_i = k_j$, for $\forall i < j$, and:

1. $\ell_1 < \ell_2$ iff $h_j x_j < k_j x_j$ where $x_j = h_{j-1} \dots h_1 u$;
2. $\ell_1 = \ell_2$ iff $m = n = j$ and $h_j x_j = k_j x_j$;
3. ℓ_1 and ℓ_2 are incomparable iff $h_j x_j$ and $k_j x_j$ are incomparable;

Example 2. Consider linguistic variables: $\{\mathcal{V}\text{true}, \mathcal{P}\text{true}, \mathcal{L}\text{true}\} \in H$, in which $\{\mathcal{V}\text{true}, \mathcal{P}\text{true}, \mathcal{L}\text{true}\}$ stand for : very true, possible true and less true are linguistic truth values generated from variable truth. Assume propositions p = "Lucie is young is $\mathcal{V}$true" and q = "Lucie is smart is $\mathcal{P}$true", interpretations on H are:

- truth(p) = $\mathcal{V}$true $\in H$, truth is a unary function.
- $p \wedge q = \mathcal{V}\text{true} \wedge \mathcal{P}\text{true} = \mathcal{P}\text{true} \in H$. $\wedge$ is a binary function.
- $p \vee q = \mathcal{V}\text{true} \vee \mathcal{P}\text{true} = \mathcal{V}\text{true} \in H$. $\vee$ is a binary function.

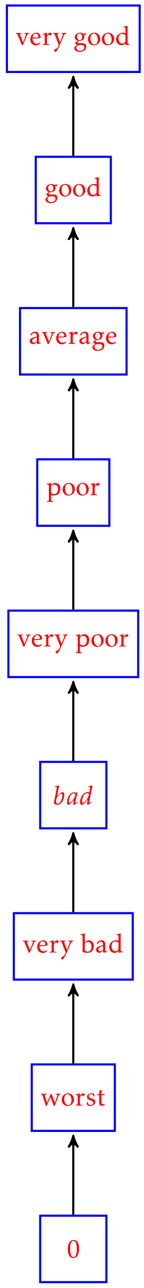

Fig. 1. the order of linguistic space in example 3

2.2 Linguistic topological spaces

Linguistic topological spaces (LTS) was introduced in [6]. In which, let L be a set of adjectives that is describing the situation in a very sensitive way then L is called a linguistic space (LS).

Example 3. In [6], consider a linguistic space which associated with students' performance in an exam with:
Let L={0, good, poor, average, bad, very bad, worst, very poor, very good } be an LS and order of linguistic

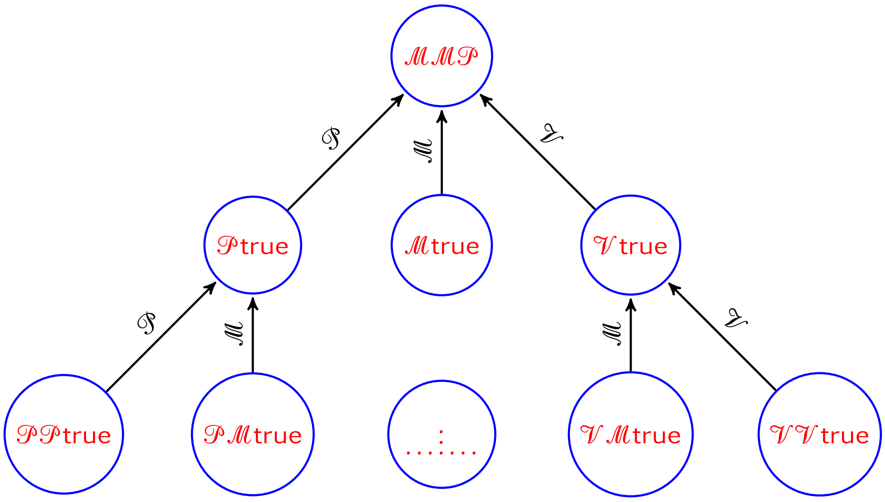

Fig. 2. tree of hedges

terms as in Fig. 1 and from which we have:

$$
\begin{aligned}
\mathscr{Min}\{good, poor\} &= poor \\
\mathscr{Min}\{poor, very\ bad\} &= very\ bad \\
\mathscr{Max}\{poor, good\} &= good \\
\mathscr{Max}\{poor, vevy\ bad\} &= poor \\
\mathscr{Min}\{0, vevy\ good\} &= 0 \\
\mathscr{Max}\{0 very\ good\} &= very\ good
\end{aligned}
$$

Total order $\langle L, \mathscr{Max}, \mathscr{Min}\rangle$ is called fuzzy linguistic chain lattice together fuzzy linguistic space L [6].

LTS is a special case of fuzzy topological spaces which was first defined in [1].

Definition 2.3. A fuzzy topological is a family T of fuzzy sets in X which satisfies the following condition:

1. $\phi, X \in T$,
2. If $A, B \in T$, then $A \cap B \in T$,
3. If $A_i \in T$ for each $i \in I$ then $\bigcup_I A_i \in T$.

T is called a fuzzy topology for X and the pair $\langle X, T\rangle$ is a fuzzy topological space (FTS).

3 Hedge algebra and linguistic topological space

This section presents diagrams for linguistic variables which are generated from $\mathbb{HA}$
Linguistic is a special case of fuzzy set and extends definition 2.3 to have definition of linguistic topology.

Definition 3.1. A linguistic fuzzy topological is a family T of linguistic term sets in L which the following condition holds:

1. $\phi, L \in T$,
2. If $A, B \in T$, then $A \cap B \in T$,
3. If $A_i \in T$ for each $i \in I$ then $\bigcup_I A_i \in T$.

T is called a linguistic fuzzy topology for L.

Let L be a language which is generated from an $\mathbb{HA}$ with m hedges and T be a set of subset of leaf nodes on a complete m-ary tree as in Fig. 2 then we have property

Property 3.1.
1. T is a linguistic topology on L
2. Couple (L, T) is a linguistic topological space.

Example 4. Give an $\mathbb{HA}$:

$$\mathbb{HA} = \langle \mathscr{X} = \text{truth}; c^+ = \text{true}; \mathscr{H} = \{\mathscr{P}, \mathscr{M}, \mathscr{V}\}\rangle \tag{3}$$

be an $\mathbb{HA}$ with order as $\mathscr{P} < \mathscr{M} < \mathscr{V}$ ($\mathscr{P}$ for possible, $\mathscr{M}$ for more and $\mathscr{V}$ for very are hedges). Let $\{h_i, h_j, h_i \in \mathscr{H} \cup W\}$ in which W is the neutral element, that is W$c^+ = c^+$ ...then language L which generated from linguistic variable $\mathscr{X}$ is as follow, see Fig. 3:

$$
\begin{aligned}
L =&\{h_i h_j h_k \text{true} | h_i \neq h_j \wedge h_i \neq h_k \wedge h_j \neq h_k\} \\
=&\{\mathscr{P}\text{true}, \mathscr{M}\text{true}, \mathscr{V}\text{true}, \\
&\mathscr{PM}\text{true}, \mathscr{MP}\text{true}, \mathscr{PV}\text{true}, \\
&\mathscr{VP}\text{true}, \mathscr{MV}\text{true}, \mathscr{VM}\text{true}, \\
&\mathscr{PMV}\text{true}, \mathscr{VPM}\text{true}, \mathscr{PVM}\text{true}, \\
&\mathscr{VMP}\text{true}, \mathscr{MVP}\text{true}, \mathscr{MPV}\text{true}\}
\end{aligned}
$$

Using $\mathbb{HA}$, any real interval segment can be converted to linguistic domain [4, 5].

Example 5. Linguistic variable SPEED can be modeled in an $\mathbb{HA}$ as:

$$\mathbb{HA} = \langle SPEED, \{\mathscr{V}, \mathscr{W}, \mathscr{L}, \mathscr{M}\}, \{high, low\}\rangle$$

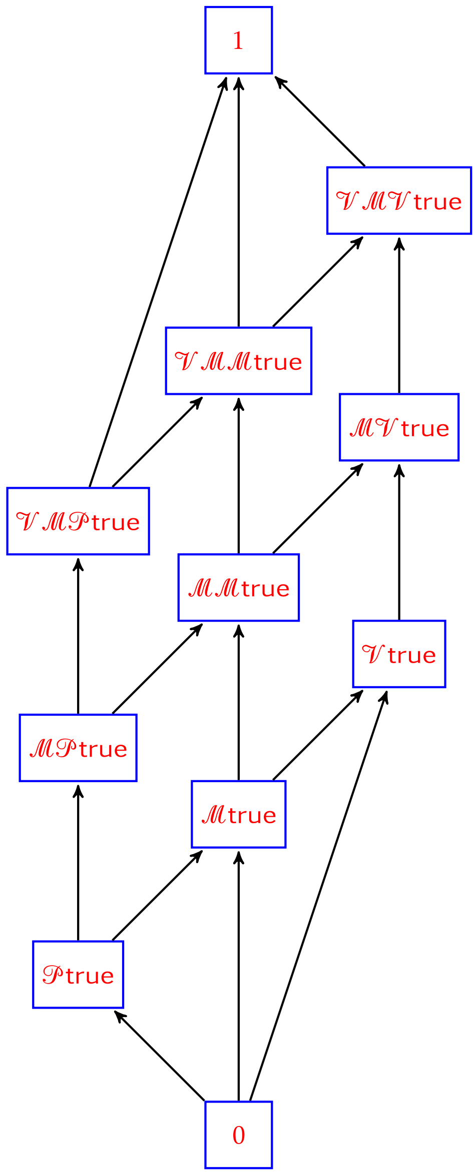

Fig. 3. Hasse diagram

Table 1 presents a domain convergent from$[0,\ 1]$ to linguistic value in $\mathbb{L}$ with hedges meaning as:

$Hedge:$	$Meaning$
$\mathscr{V}$	$very$
$\mathscr{W}$	neutral element
$\mathscr{L}$	$less$
$\mathscr{M}$	$more$

In which components $c^+ = high$ and $c^- = low$ are generating elements.

Range $[-1,\ 1]$	Positive range $[0,\ 1]$	Domain of $\mathbb{L}$
$[-1,\ -0.7)$	$[0,\ 0.15)$	$\mathscr{V}\mathscr{V}$low
$[-0.7,\ -0.4)$	$[0.15,\ 0.3)$	$\mathscr{L}\mathscr{M}$low
$[-0.4,\ -0.1)$	$[0.3,\ 0.45)$	$\mathscr{L}\mathscr{L}$low
$[-0.1,\ 0.1)$	$[0.45,\ 0.55)$	$\mathscr{W}$
$[0.1,\ 0.4)$	$[0.55,\ 0.7)$	$\mathscr{V}\mathscr{L}$high
$[0.4,\ 0.7)$	$[0.7,\ 0.85)$	$\mathscr{L}\mathscr{M}$high
$[0.7,\ 1]$	$[0.85,\ 1]$	$\mathscr{V}\mathscr{V}$high

Table 1. domains conversion

4 Conclusions and future work

We have introduced a visualization method to represent language which uses a complete m-ary tree fuzzy graph model and Hasse diagram.
Our next study will investigate homotopic relation on language L and we will also indicate this relation to be an equivalence relation.

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