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Steps Towards Modeling and Querying Based on Linguistic Fuzzy Graph Database

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Abstract

In this paper, we introduce a method for computing with words on linguistic fuzzy graph database ($\mathbb{LGD}$). Computation consists of two processes: Modeling and Querying. The former models $\mathbb{LGD}$ as a fuzzy graph whose nodes contain linguistic data table and the later queries linguistic data from node's data tables.

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1 Introduction

In everyday life, people use natural language (NL) for analyzing, reasoning, and finally, make their decisions. Computing with words (CWW) [2, 6, 8–11, 17] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy set is fuzzy graph [3, 7, 14, 16], combined fuzzy set with graph theory. Fuzzy graph ($\mathbb{FG}$) has a lots of applications in both modeling and reasoning fuzzy knowledge such as Human trafficking, internet routing, illegal immigration [13] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain , for example, linguistic summarization problems [10]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for computing with words. The remainder of paper is organized as follows: Section 2 reviews some main concepts of computing with words based on $\mathbb{HA}$. Important section 3 studies a graph database to model with words using $\mathbb{HA}$ and its properties. Section 4 outlines conclusions and future work.

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2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and some important knowledge used in the paper.

2.1 Hedge algebra

In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition of an $\mathbb{HA}$ is specified by 3- Tuple $\mathbb{HA} = (X, H, \leq)$ in [6]. In [5], to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+, c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c | c \in G, \delta \in H^*$(hedge string over H)$\}$, $\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \dots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is* Age, $G = \{c^+ = young;\ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable Age X is $\mathbb{L}(X)$ or $\mathbb{L}$ for short:
$\mathbb{L}$ = *{very less young ; less young ; young ; more young ; very young ; very very young . . . }*

Fuzziness properties of elements in $\mathbb{HA}$, specified by fm (fuzziness measure) [5] as follows:

Definition 2 .1. A mapping $fm : \mathbb{L} \to [0, 1]$ is said to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c \in \{c^+, c^-\}} fm(c) = 1,\ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h_i \in H} fm(h_i x) = fm(x),\quad x = h_n h_{n-1} \dots h_1 c,$ the canonical form.

3. $fm(h_n h_{n-1} \dots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x)$.

The truth and meaning are fundamental important concepts in fuzzy logic, artificial intelligence and machine learning. In RCT (restriction-centered theory) [10], truth values are organized as a hierarchy with ground level or first-order and second-order. First order truth values are numerical values whereas second order ones are linguistic truth values. A linguistic truth value, say ℓ, is a fuzzy set. We study linguistic truth values on POSET $\mathbb{L}$ whose elements are comparable [5, 6].

Definition 2.2. A $\mathfrak{L}$ STRUCT[ρ] on relational signature ρ is a tuple:

$$\mathfrak{L} = \langle \mathbb{L},\ f_{a_i}^{\mathfrak{L}},\ c_j^{\mathfrak{L}} \rangle \tag{1}$$

Consists of a universe $\mathbb{L} \neq \emptyset$ together with an interpretation of:

- each constant symbol c_j from ρ as an element $c_j^{\mathfrak{L}} \in \mathbb{L}$
- each a_i-ary function symbol f_{a_i} from ρ as a function:

$$f_i^{\mathfrak{L}} : \mathbb{L}^{a_i} \to \mathbb{L} \tag{2}$$

In $\mathbb{HA}$, $\ell \in \mathbb{L}$ and there are order properties:

Theorem 2.1. In [6], let $\ell_1 = h_n \dots h_1 u$ and $\ell_2 = k_m \dots k_1 u$ be two arbitrary canonical representations of ℓ_1 and ℓ_2, then there exists an index $j \leq \bigwedge\{m, n\} + 1$ such that $h_i = k_j$, for $\forall i < j$, and:

1. $\ell_1 < \ell_2$ iff $h_j x_j < k_j x_j$ where $x_j = h_{j-1} \dots h_1 u$;
2. $\ell_1 = \ell_2$ iff $m = n = j$ and $h_j x_j = k_j x_j$;
3. ℓ_1 and ℓ_2 are incomparable iff $h_j x_j$ and $k_j x_j$ are incomparable;

Example 2. Consider linguistic variables: $\{\mathscr{V}\text{true}, \mathscr{P}\text{true}, \mathscr{L}\text{true}\} \in H$, in which $\{\mathscr{V}\text{true}, \mathscr{P}\text{true}, \mathscr{L}\text{true}\}$ stand for : very true, possible true and less true are linguistic truth values generated from variable truth. Assume propositions p = "Lucie is young is $\mathscr{V}$true" and q = "Lucie is smart is $\mathscr{P}$true", interpretations on H are:

- truth(p) = $\mathscr{V}$true $\in H$, truth is a unary function.
- $p \wedge q = \mathscr{V}\text{true} \wedge \mathscr{P}\text{true} = \mathscr{P}\text{true} \in H$. $\wedge$ is a binary function.
- $p \vee q = \mathscr{V}\text{true} \vee \mathscr{P}\text{true} = \mathscr{V}\text{true} \in H$. $\vee$ is a binary function.

2.2 Linguistic fuzzy graph

The first $\mathbb{FG}$ (fuzzy graph) was introduced in [16], which vertices and edges's values are in unit interval [0, 1]. Many $\mathbb{FG}$'s theories were developed in [12, 13] in which computational phases have a bit complex due to converting from linguistic to number value to compute. To reduce complexity, in [4] by applying computing with word method [10] on $\mathbb{FG}$ to produce $\mathbb{LG}$, in which $\mathbb{L}$ is domain of both vertices $\mathbb{V}$ and $\mathbb{E}$ as in Fig. 1

Definition 2 .3. In [4], a linguistic graph $\mathbb{LG} = (\mathbb{V}, \rho, \delta)$ consists of set $\mathbb{V}$, a fuzzy vertex set ρ on $\mathbb{V}$ and a fuzzy edge set δ on $\mathbb{V}$ so that $\delta(u, v) \leq \rho(u) \wedge \rho(v)$ for every $u, v \in \mathbb{V}$.

$$\mathbb{LG} = \{(\mathbb{V}, \rho, \delta) : \rho \mathrel{\widetilde{\subseteq}} \mathbb{V}; \delta \mathrel{\widetilde{\subseteq}} \mathbb{E}\} \tag{3}$$

Example 3. Fig. 1 shows a simple $\mathbb{LG}$. Let

$$\mathbb{HA} =< \mathcal{X} = \text{truth}; c^+ = \text{true}; \mathcal{H} = \{\mathscr{L}, \mathscr{M}, \mathscr{V}\} > \tag{4}$$

be an $\mathbb{HA}$ with order as $\mathscr{L} < \mathscr{M} < \mathscr{V}$ ($\mathscr{L}$ for less, $\mathscr{M}$ for more and $\mathscr{V}$ for very are hedges).

$$\mathbb{V} = \frac{\mathscr{V}\text{true}}{c_1} + \frac{\mathscr{L}\text{true}}{c_c} + \frac{\mathscr{V}\mathscr{V}\text{true}}{c_3} + \frac{\mathscr{V}\mathscr{M}\text{true}}{c_4}$$

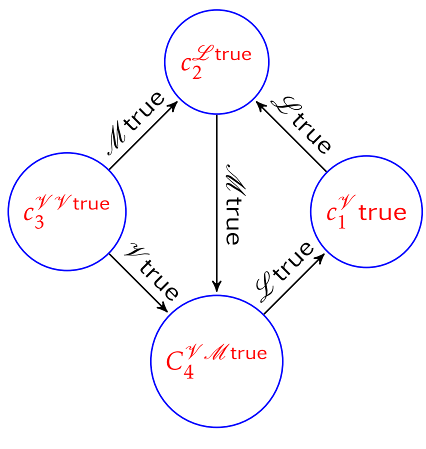

Fig. 1. a simple $\mathbb{LG}$

3 Linguistic fuzzy graph database

Fuzzy graph database ($\mathbb{FGD}$) is a main trend in French research and not yet finished [1, 1 5]. As advance in computing with words on $\mathbb{LG}$ [4], this paper studies the $\mathbb{LGD}$ on linguistic domain $\mathbb{L}$.
Let $\mathbb{Atr}$, $\mathbb{Key}$, $\mathbb{Val}$ be in order to represent for attributes, keys and values in an $\mathbb{LGD}$

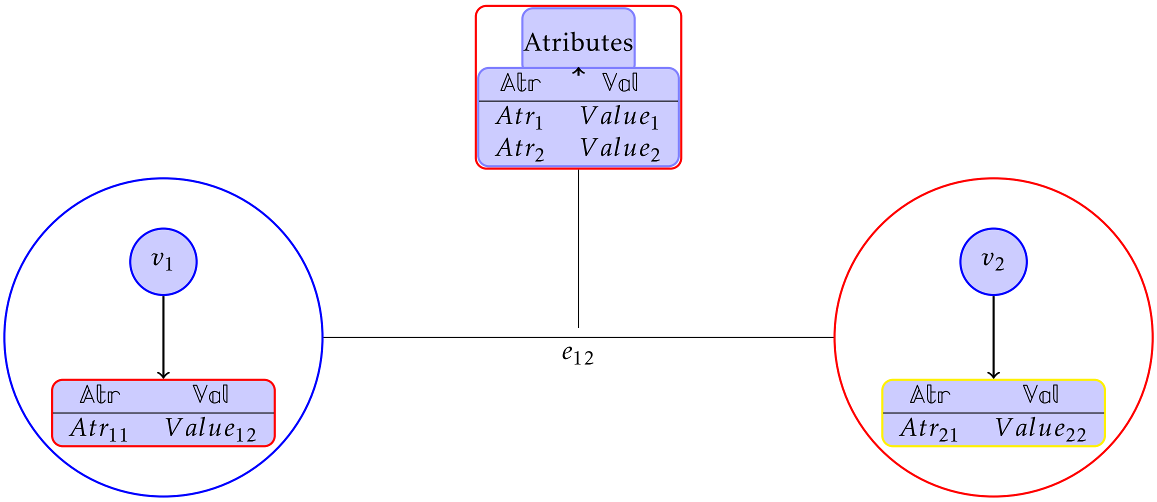

Fig. 2. a simple model for $\mathbb{LGD}$ with two nodes and one edge

Definition 3.1. A linguistic graph database $\mathbb{LGD} = (\mathbb{V}, \mathbb{E}, \rho, \delta, \mathbb{A}\mathrm{tr})$, in which:

1. $\mathbb{V}$ represents for a set of vertices whose attributes are $\mathbb{A}\mathrm{tr}$
2. $\mathbb{E}$ represents for a set of edges whose attributes are $\mathbb{A}\mathrm{tr}$
3. ρ stands for a fuzzy set on $\mathbb{A}\mathrm{tr}$ for vertex's attributes.
4. δ stands for a fuzzy set on $\mathbb{A}\mathrm{tr}$ for edge's attributes.

$$\mathbb{LG} = \{(\mathbb{V}, \mathbb{E}, \rho, \delta, \mathbb{A}\mathrm{tr}) : \rho \,\widetilde{\subseteq}\, \mathbb{A}\mathrm{tr}; \delta \,\widetilde{\subseteq}\, \mathbb{E}\} \quad (5)$$

Fig.2 shows a $\mathbb{LGD}$ with tow nodes $v_1, v_2 \in \mathbb{V}$; $e_{12} \in \mathbb{E}$ is a relation between v_1 and v_2. Attributes for $\mathbb{V}$ and $\mathbb{E}$ are presented in three tables.

Property 3.1. Always modeling a linguistic graph database $\mathbb{LGD}$ from a $\mathbb{FGD}$ to apply advance properties from computing with word methods.

Proof. It is straightforward to prove the property 3.1 by applying domain convergent method [5, 6] □

Table 1 presents a domain convergent from$[0,\ 1]$ to linguistic value in $\mathbb{L}$ with hedges meaning as:

Hedge :	*Meaning*
$\mathscr{V}$	*very*
$\mathscr{W}$	neutral element
$\mathscr{L}$	*less*
$\mathscr{M}$	*more*

Example 4. By using linguistic domain for fuzzy sets ρ and δ, a simple $\mathbb{LGD}$ is illustrated as in Fig. 3.

Range $[-1,\ 1]$	Positive range $[0,\ 1]$	Domain of $\mathbb{L}$
$[-1,\ -0.7)$	$[0,\ 0.15)$	$\mathscr{V}\mathscr{V}$low
$[-0.7,\ -0.4)$	$[0.15,\ 0.3)$	$\mathscr{L}\mathscr{M}$low
$[-0.4,\ -0.1)$	$[0.3,\ 0.45)$	$\mathscr{L}\mathscr{L}$low
$[-0.1,\ 0.1)$	$[0.45,\ 0.55)$	$\mathscr{W}$
$[0.1,\ 0.4)$	$[0.55,\ 0.7)$	$\mathscr{V}\mathscr{L}$high
$[0.4,\ 0.7)$	$[0.7,\ 0.85)$	$\mathscr{L}\mathscr{M}$high
$[0.7,\ 1]$	$[0.85,\ 1]$	$\mathscr{V}\mathscr{V}$high

Table 1. Domains conversion

4 Conclusions and future work

We have introduced a fuzzy graph model so-called $\mathbb{FG}$ with the following two advantages

1. Modeling fuzzy graph uses linguistic variable by applying hedge algebra
2. Computing with words on linguistic variable is not converting to numeric values therefore reducing number of operators for computation phases.

Our next study will investigate algorithms to construct and compute $\mathbb{LG} = (\mathbb{V}, \rho, \delta)$

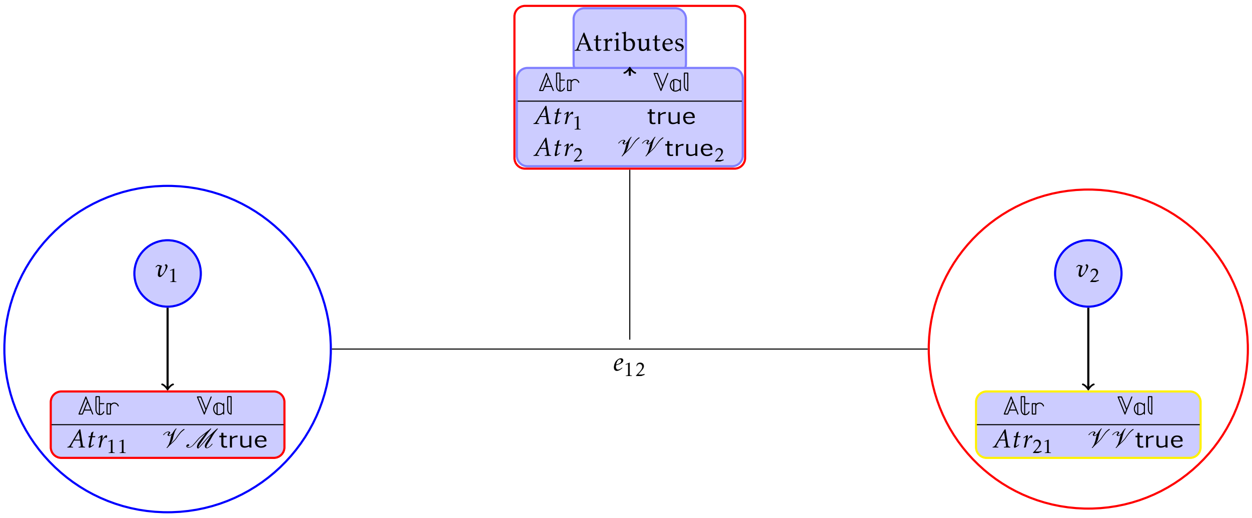

Fig. 3. a simple $\mathbb{LGD}$ with fuzzy $\mathbb{Atr}$

References

[1] Arnaud Castelltort and Anne Laurent. Representing history in graph-oriented nosql databases: A versioning system. *IEEE*, 2013.

[2] Mabel. Frias, Filiberto Yaima, Gonzalo Nápoles, Koen. Vahoof, and Rafael. Bello. Fuzzy cognitive maps reasoning with words: An ordinal approach. *In ISFUROS*, 2017.

[3] Michael Glykas. *Fuzzy Cognitive Maps, Advances in Theory, Tools and Applications*. Springer, 2010.

[4] Nguyen Van Han and Phan Cong Vinh. Toward computing linguistic fuzzy graphs and applying to illegal immigration problem. *EAI Endorsed Trans. Context-aware Syst. & Appl*, 7(22):166361, 2020.

[5] Nguyen Cat Ho, Tran Thai Son, Tran Dinh Khang, and Le Xuan Viet. Fuzziness measure, quantified semantic mapping and interpolative method of approximate reasoning in medical expert systems. *Journal of Computer Science and Cybernetics*, 18(3):237–252, 2002.

[6] Nguyen Cat Ho and W.Wechler. Hedge algebras: An algebraic approach to structure of sets of linguistic truth values. *Fuzzy Sets and Systems*, 35(3):281–293, 1990.

[7] B. Kosko. Fuzzy cognitive mpas. *Internation Journal of Man-Machine Studies*, 24:65–75, 1986.

[8] L.A.Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences*, 8(3):199–249, 1975.

[9] L.A.Zadeh. Fuzzy-set-theoretic interpretation of linguistic hedges. *Journal of Cybernetics*, 2:4–34, 1977.

[10] L.A.Zadeh. *Computing with words - Principal Concepts and Ideas. Studies in Fuzziness and Soft Computing*. Springer, 2012.

[11] E. H. Mmdani. Application of fuzzy logic to approximate reasoning using linguistic synthesis. *IEEE Transactions on Computers*, C-26(12):1182–1191, 1977.

[12] J.N. Mordeson and P.S. Nair. *Fuzzy Graphs and Fuzzy Hypergraphs*. Physica-Verlag, Heidelberg, 2000.

[13] John N. Mordeson and Sunil Mathew. *Advanced Topics in Fuzzy Graph Theory*. Springer Nature Switzeland AG, 2019.

[14] Elpiniki I. Papageorgiou. *Fuzzy Cognitive Maps for Applied Science and Engineering From Fundamentals to Extensions and Learning Algorithms*. Springer-Verlag Berlin Heidelberg, 2014.

[15] Olivier Pivert. *NoSQL Data Models: Trends and Challenges*. Kindle Edition, 2018.

[16] A Rosenfeld. Fuzzy graphs. *Fuzzy Sets and Their Applications*, pages 77–95, 1975.

[17] Lotti A. Zadeh and Janusz Kacprzyk. *Computing with Word in Information Intelligent System 1*. Springer-Verlag BBerlin Heidelberg GmbH, 1999.