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A Gentle Introduction to Cognitive Map Based on Input Output Linguistic Variables

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Abstract

This paper studies on feedback graph model of linguistic variables which is generated from Hedge algebra. We also introduce a visual graphic model for input - output cognitive maps.

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1 Introduction

In everyday life, people use natural language (NL) for analyzing, reasoning, and finally, m ake their decisions. Computing with words (CWW) [2, 9, 11–14, 20] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by L. A. Zadeh is an approximate method on interval [0,1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy set is fuzzy graph [3, 10, 17, 18], combined fuzzy set with graph theory. Fuzzy graph ($\mathbb{FG}$) has a lots of applications in both modeling and reasoning fuzzy knowledge such as Human trafficking, in ternet ro uting, il legal im migration [16] on interval [0,1] but not in linguistic values, However, many applications cannot model in numerical domain , for example, linguistic summarization problems [13]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for computing with words. The remainder of paper is organized as follows: Section 2 reviews some main concepts of computing with words based on $\mathbb{HA}$. Important section 3 studies a graph database modeling with words using $\mathbb{HA}$ and its properties. Section 4 outlines conclusions and future work.

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2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and some important knowledge used in the paper.

2.1 Hedge algebra

In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition of an $\mathbb{HA}$ is specified by 3-Tuple $\mathbb{HA} = (X, H, \leq\ )$ in [9]. In [8] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+,\ c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c |\ c \in G,\ \delta \in H^*(\text{hedge string over H})\}$, $\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \dots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is* Age, $G = \{c^+ = young;\ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable *Age* X is $\mathbb{L}(X)$ or $\mathbb{L}$ for short:
$\mathbb{L}$ = *{very less young ; less young ; young ; more young ; very young ; very very young . . . }*

Fuzziness properties of elements in $\mathbb{HA}$, specified by *fm* (fuzziness measure) [8] as follows:

Definition 2 .1. A m apping $fm : \mathbb{L} \rightarrow [0, 1]$ i s s aid to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c\in\{c^+,c^-\}} fm(c) = 1,\ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h_i \in H} fm(h_i x) = fm(x)$, $\quad x = h_n h_{n-1} \dots h_1 c$, $\quad$ the canonical form.

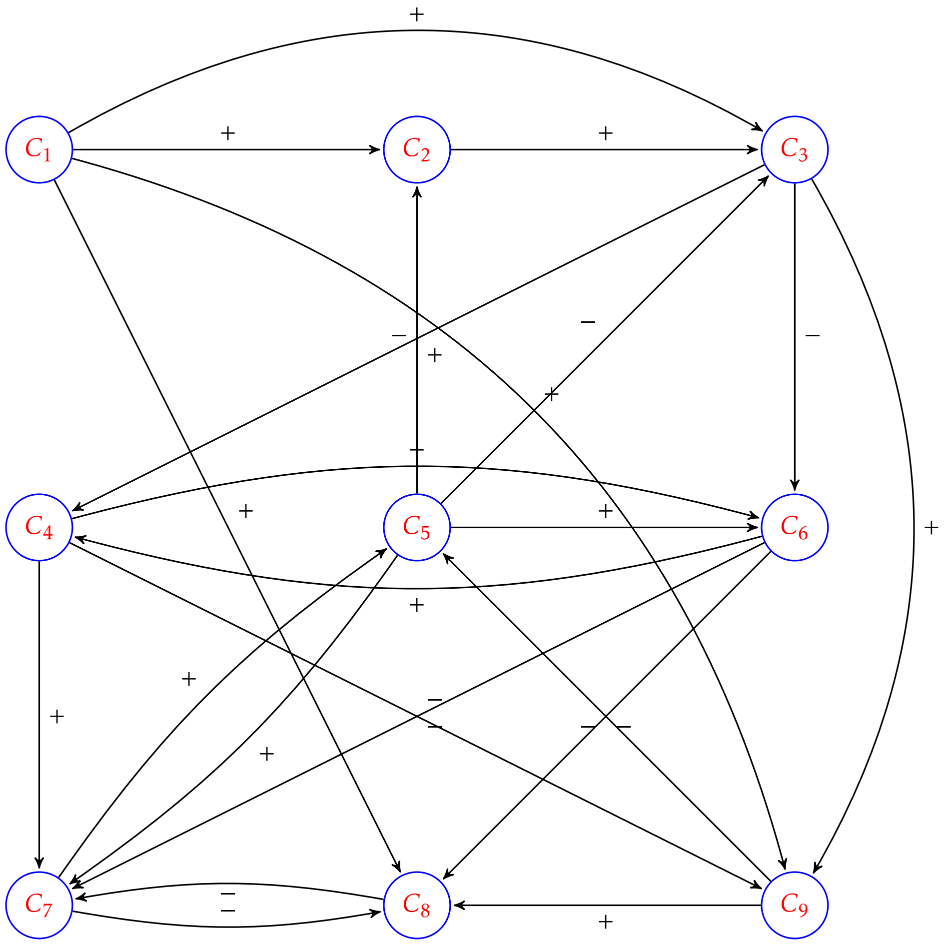

Fig. 1. A simple $\mathbb{FCM}$

3. $fm(h_n h_{n-1} \dots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x)$.

The truth and meaning are fundamental important concepts in fuzzy logic, artificial intelligence and machine learning. In RCT (restriction-centered theory) [13], truth values are organized as a hierarchy with ground level or first-order and second-order. First order truth values are numerical values whereas second order ones are linguistic truth values. A linguistic truth value, say ℓ, is a fuzzy set. We study linguistic truth values on POSET $\mathbb{L}$ whose elements are comparable [8, 9].

Definition 2.2. A $\mathfrak{L}$ STRUCT[ρ] on relational signature ρ is a tuple:

$$\mathfrak{L} = \langle \mathbb{L},\ f_{a_i}^{\mathfrak{L}},\ c_j^{\mathfrak{L}} \rangle \tag{1}$$

Consists of a universe $\mathbb{L} \neq \emptyset$ together with an interpretation of:

- each constant symbol c_j from ρ as an element $c_j^{\mathfrak{L}} \in \mathbb{L}$
- each a_i-ary function symbol f_{a_i} from ρ as a function:

$$f_i^{\mathfrak{L}} : \mathbb{L}^{a_i} \to \mathbb{L} \tag{2}$$

In $\mathbb{HA}$, $\ell \in \mathbb{L}$ and there are order properties:

Theorem 2.1. In [9], let $\ell_1 = h_n \dots h_1 u$ and $\ell_2 = k_m \dots k_1 u$ be two arbitrary canonical representations of ℓ_1 and ℓ_2, then there exists an index $j \leq \bigwedge\{m, n\} + 1$ such that $h_i = k_j$, for $\forall i < j$, and:

1. $\ell_1 < \ell_2$ iff $h_j x_j < k_j x_j$ where $x_j = h_{j-1} \dots h_1 u$;
2. $\ell_1 = \ell_2$ iff $m = n = j$ and $h_j x_j = k_j x_j$;
3. ℓ_1 and ℓ_2 are incomparable iff $h_j x_j$ and $k_j x_j$ are incomparable;

Example 2. Consider linguistic variables: $\{\mathscr{V}\text{true},\ \mathscr{P}\text{true},\ \mathscr{L}\text{true}\} \in H$, in which $\{\mathscr{V}\text{true},\ \mathscr{P}\text{true},\ \mathscr{L}\text{true}\}$ stand for very true, possible true and less true, which are linguistic truth values generated from variable truth. Assume propositions p = "Lucie

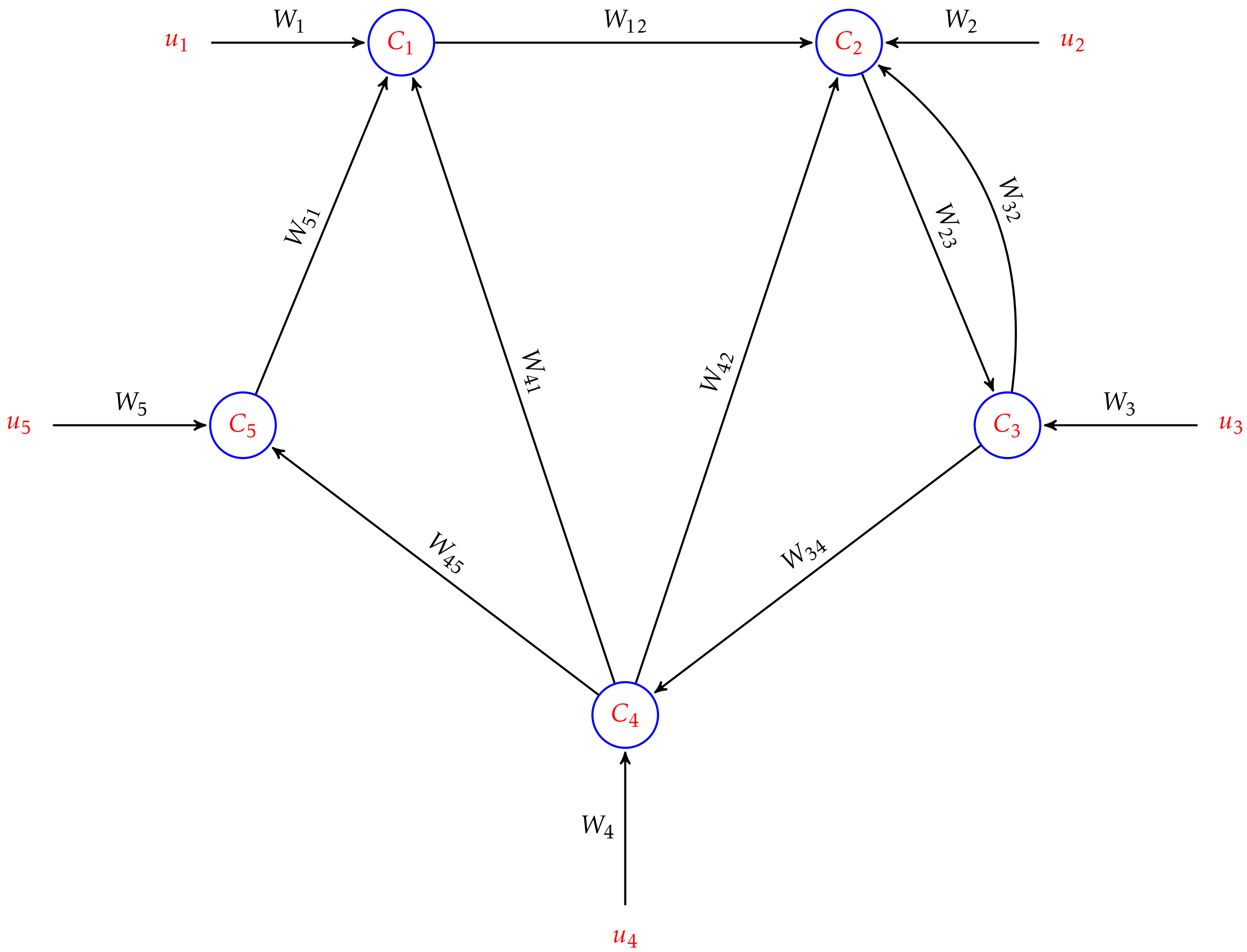

Fig. 2. A new $\mathbb{FCM}$ model

is young is $\mathscr{V}$true" and q = "Lucie is smart is $\mathscr{P}$true", interpretations on H are:

- truth$(p) = \mathscr{V}$true $\in H$, truth is a unary function.
- $p \wedge q = \mathscr{V}$true $\wedge$ $\mathscr{P}$true $= \mathscr{P}$true $\in H$. $\wedge$ is a binary function.
- $p \vee q = \mathscr{V}$true $\vee$ $\mathscr{P}$true $= \mathscr{V}$true $\in H$. $\vee$ is a binary function.

2.2 New model fuzzy cognitive map

The first F uzzy C ognitive M aps ($\mathbb{FCM}$) w as introduced in [10, 15] and fast developed in many applications [3, 17]. Fig.1 is a simple $\mathbb{FCM}$ with its matrix in Fig.3. In [1, 19], that is a new $\mathbb{FCM}$ model with input signals.

Example 3. Fig.2 shows a new model together with state transform equation which presents in Fig.4

Vertices$C_1 - C_9$

$$M = \begin{array}{cc} & \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Fig. 3. Matrix for Fig. 1

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \\ x_5(t+1) \end{bmatrix} = f(\begin{bmatrix} 0 & 0 & 0 & W_{41} & W_{51} \\ W_{12} & 0 & W_{32} & 0 & 0 \\ 0 & W_{23} & 0 & 0 & 0 \\ 0 & 0 & W_{34} & 0 & 0 \\ 0 & 0 & 0 & W_{45} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \begin{bmatrix} W_1 & 0 & 0 & 0 & 0 \\ 0 & W_2 & 0 & 0 & 0 \\ 0 & 0 & W_3 & 0 & 0 \\ 0 & 0 & 0 & W_4 & 0 \\ 0 & 0 & 0 & 0 & W_5 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix})$$

Fig. 4. Matrix equation for new $\mathbb{FCM}$ model

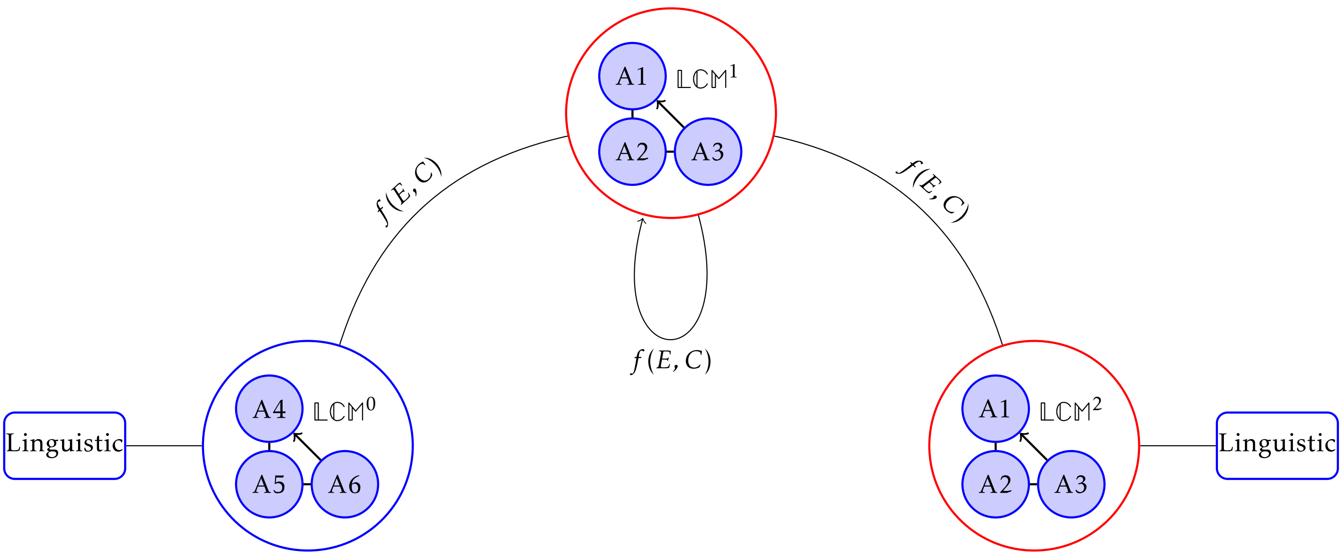

Fig. 5. Diagram of new $\mathbb{LCM}$-computing model

3 Input-Output Linguistic Cognitive maps

Linguistic cognitive maps $\mathbb{LCM}$ have been applying and studying in many areas of artificial intelligence [4–7]

Definition 3.1. . A linguistic cognitive map ($\mathbb{LCM}$) is a 4- Tuple:

$$\mathbb{LCM} = \{C, E, \mathcal{C}, f\} \tag{3}$$

In which:

1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.

2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \mathbb{L}$; e_{ij} = weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N\times N} \in \mathbb{L}^{N\times N}$

3. The map: $\mathcal{C} : C_i \longrightarrow C_i^t \in \mathbb{L}, t \in N$

4. $\mathcal{C}(0) = [C_1^0, C_2^0, \ldots, C_n^0] \in \mathbb{L}^N$ is the initial vector. The recurring transformation function f is defined as:

$$C_j^{t+1} = f(\sum_{i=1}^{N} e_{ij} C_i^t) \in \mathbb{L} \tag{4}$$

In the paper, we modify $\mathbb{LCM}$ to have a new $\mathbb{LCM}$ with input-output linguistic variables. Fig.5 illustrates a new abstract $\mathbb{LCM}$ and Fig.6 shows a simple new $\mathbb{LCM}$ with 5 inputs $\mathcal{I} = \{I_1,\ I_2,\ I_3,\ I_4,\ I_5\}$ and 5 outputs $\mathcal{O} = \{O_1,\ O_2,\ O_3,\ O_4,\ O_5\}$

Property 3.1. A new $\mathbb{LCM}$ runs faster than $\mathbb{LCM}$ if it has the same concept space vertices.

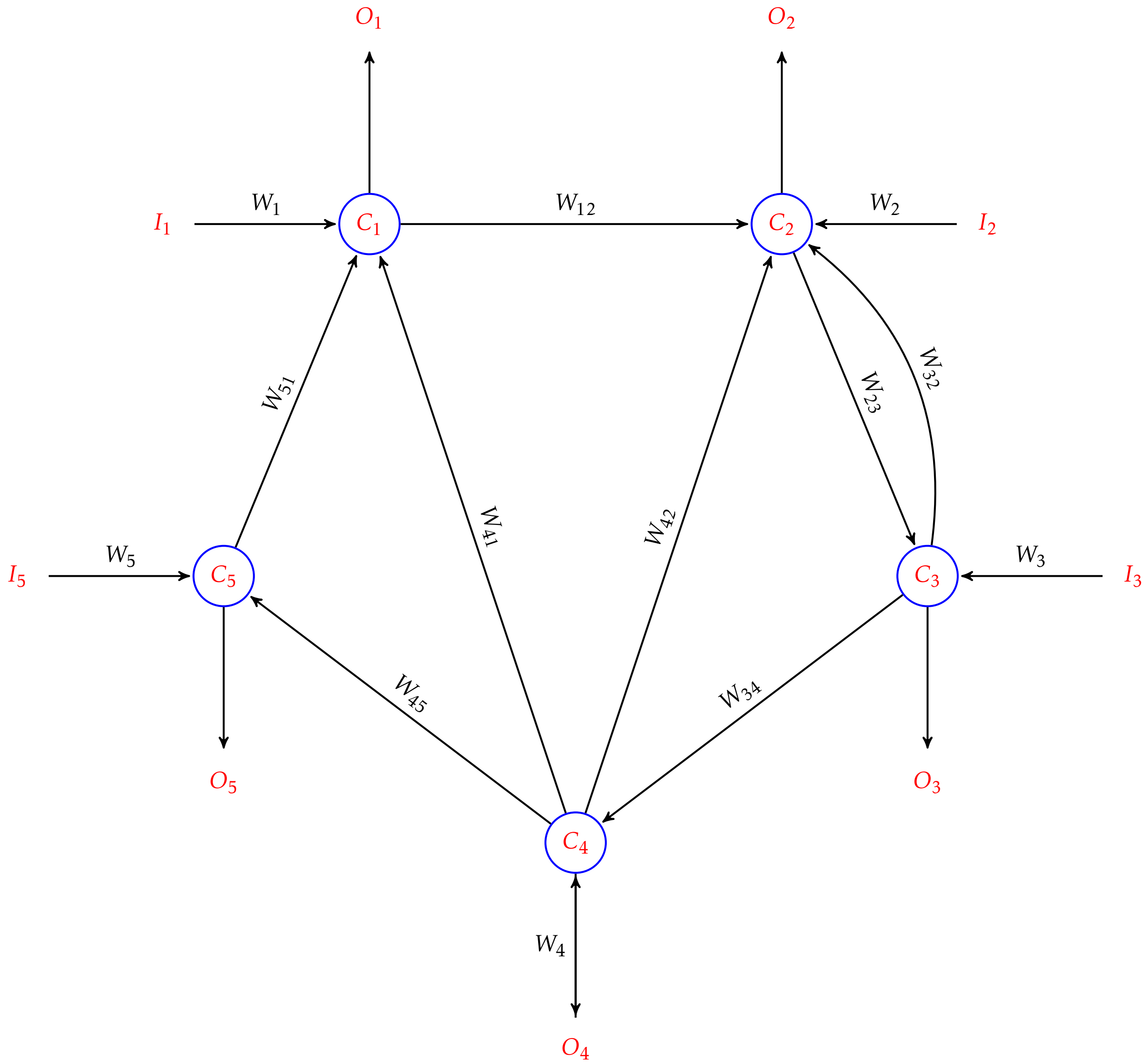

Fig. 6. A new $\mathbb{LCM}$

4 Conclusions and future work

We have introduced a visual model so-called new $\mathbb{LCM}$ with the following two advantages

1. Inputs and Outputs use linguistic variable by applying hedge algebra

2. Computing with words on linguistic variable is not converting to numeric values therefore reducing number of operators for computation phases.

Our next study will investigate algorithms to construct and compute state space for new $\mathbb{LCM}$.

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