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Toward Modeling Linguistic Fuzzy Spanning Trees Based on Hedge Algebra

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Abstract

This paper presents an innovative approach to modeling Linguistic Fuzzy Maximum Spanning Trees (L-FMSTs) using Hedge Algebra (HA). HA provides a robust framework for quantifying linguistic terms, which is essential for handling the vagueness inherent in natural language. By integrating HA with L-FMSTs, we aim to enhance the interpretability and performance of fuzzy systems in applications requiring complex decision-making and optimization.

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1. Introduction

The field of fuzzy systems [4] has evolved to address the complexities and uncertainties associated with linguistic data. Traditional fuzzy systems often struggle with the inherent imprecision of natural language, making it challenging to model and interpret linguistic terms effectively. This has led to the development of advanced methods such as Linguistic Fuzzy Maximum Spanning Trees (L-FMSTs), which aim to optimize and analyze relationships within linguistic data.

Fuzzy Maximum Spanning Trees (FMSTs) [6] are an extension of classical maximum spanning trees (MSTs), leveraging fuzzy logic to handle the vagueness of linguistic variables. These trees are particularly useful in applications where relationships between data points are not precisely defined, such as in social networks, biological systems, and decision-making processes. Hedge Algebra (HA) [2, 3] provides a systematic approach to the quantification of linguistic terms, addressing the limitations of traditional fuzzy systems. HA introduces a structured way to handle linguistic variables by defining algebraic operations on them, which can significantly improve the interpretability and performance of fuzzy systems. By integrating HA with L-FMSTs, it becomes possible to develop a more robust model that can handle complex linguistic data with greater accuracy and efficiency.

This paper proposes a novel model for L-FMSTs based on HA, aiming to enhance the capability of fuzzy systems in dealing with linguistic data. We detail the mathematical foundations of HA and its application to the construction of L-FMSTs. Through extensive simulations and benchmark tests, we demonstrate the superiority of the HA-based L-FMST model in various decision-making and optimization scenarios.

The rest of the paper is organized as follows: Section 2 recalls some of the main foundation concepts of fuzzy graphs and fuzzy spanning trees. Section 3 proposes model of L-FMSTs based on HA and an algorithms for constructing L-FMST. Section 4 summaries outlines the corollary as well as future work.

2. Literature review and preliminary

Introduced by Zadeh in 1965, fuzzy logic provides a framework for dealing with uncertainty and partial truth, unlike classical binary logic. Fuzzy logic is particularly useful in real-world scenarios where data is imprecise, vague, or uncertain. This has significant

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implications for modeling relationships and making decisions based on linguistic variables or uncertain data .

2.1. Fuzzy Spanning Trees

Fuzzy spanning trees [6] extend classical spanning trees by incorporating fuzzy logic to handle the uncertainty and imprecision in edge weights. These extensions allow for a more realistic representation of networks where relationships are not crisply defined. Early work in fuzzy spanning trees focused on adapting classical MST algorithms to fuzzy contexts. This involved defining fuzzy weights for edges and using fuzzy set theory to evaluate the spanning tree. More sophisticated algorithms have been developed to consider the fuzziness of both edges and nodes, improving the accuracy and efficiency of fuzzy spanning tree algorithms. These include fuzzy clustering methods, fuzzy multi-objective optimization, and evolutionary algorithms such as genetic algorithms.

Maximum fuzzy spanning trees (MFSTs) [6] aim to maximize the total fuzzy weight of the spanning tree, unlike traditional MSTs which minimize it. This approach is particularly useful in applications where the goal is to maximize certain criteria, such as network reliability, connectivity strength, or resource utilization under uncertainty. This section summarizes the knowledge related to the article. These include hedge algebra (HA) and fuzzy spanning trees.

2.2. Hedge algebra

Hedge Algebra (HA) [2, 3] is a formal approach for handling linguistic terms and hedges (modifiers like "very," "slightly") in systems dealing with imprecision and uncertainty, such as fuzzy logic systems. It was developed to model the meaning of linguistic terms in a structured way, allowing for ordering, modification, and computation of these terms within an algebraic framework.

- Linguistic Variables: A linguistic variable is one whose values are words or sentences in natural language, such as "low," "medium," "high" for a variable like "temperature." Each linguistic variable X has a domain of linguistic values $V(X)$, often ordered and structured.

- Hedges (Modifiers): Hedges are modifiers that adjust the intensity or degree of a linguistic term. For example, "very" or "slightly" can be applied to base terms like "low" or "high" to create terms like "very low" or "slightly high." These hedges modify the meaning of the original term and are systematically handled within the algebra.

- Algebraic Structure: Hedge Algebra imposes an algebraic structure on linguistic terms and their hedges, allowing for operations like applying hedges to terms, which follow a formal set of rules. Ordering is a fundamental part of the structure: linguistic terms and their hedge-modified versions are ordered in a way that reflects their intensity or strength.

Example 1. Fuzzy subset X *is Temperature*, $G = \{c^+ = \text{high}; c^- = \text{low}\}$, $H = \{slightly, absolutely\}$ so term-set of linguistic variable Temprature X is $\mathscr{Words}$(X) or $\mathscr{Words}$ for short:

$$\begin{aligned}\mathscr{Words} = \{\ldots\\ &slightly\ absolutely\ \text{low}\\ &slightly\ slightly\ \text{low}\\ &\ldots\\ &\mathbb{W}\\ &slightly\ slightly\ \text{high}\\ &slightly\ absolutely\ \text{high}\\ &\ldots\end{aligned}$$

2.3. Linguistic fuzzy graphs

Hedge Algebra offers several advanced advantages when applied to fuzzy graphs, which are mathematical structures used to represent relationships and connections between elements with varying degrees of uncertainty or fuzziness.

A fuzzy graph [5, 7] is an extension of a classical graph, where each edge has a membership value indicating the strength of the relationship between nodes. When combined with hedge algebra, fuzzy graphs become even more powerful and flexible in handling linguistic uncertainty and imprecision. Below are some key advantages of applying Hedge Algebra in fuzzy graphs [1]:

Linguistic fuzzy graph is referred to as $\mathbb{LFG}$, where $\mathscr{Words}$ represents the domain of both the vertex set $\mathbb{V}$ and the edge set $\mathbb{E}$, as shown in Fig. 1.

Definition 1. A linguistic graph $\mathbb{LFG} = (\mathbb{V}, \rho, \delta)$ consists of the set $\mathbb{V}$, a fuzzy vertex set ρ on $\mathbb{V}$, and a fuzzy edge set δ on $\mathbb{V}$, such that $\delta(u, v) \leq \rho(u) \wedge \rho(v)$ for every $u, v \in \mathbb{V}$.

$$\mathbb{LFG} = \{(\mathbb{V}, \rho, \delta) : \rho \widetilde{\subset} \mathbb{V}; \delta \widetilde{\subset} \mathbb{E}\} \tag{1}$$

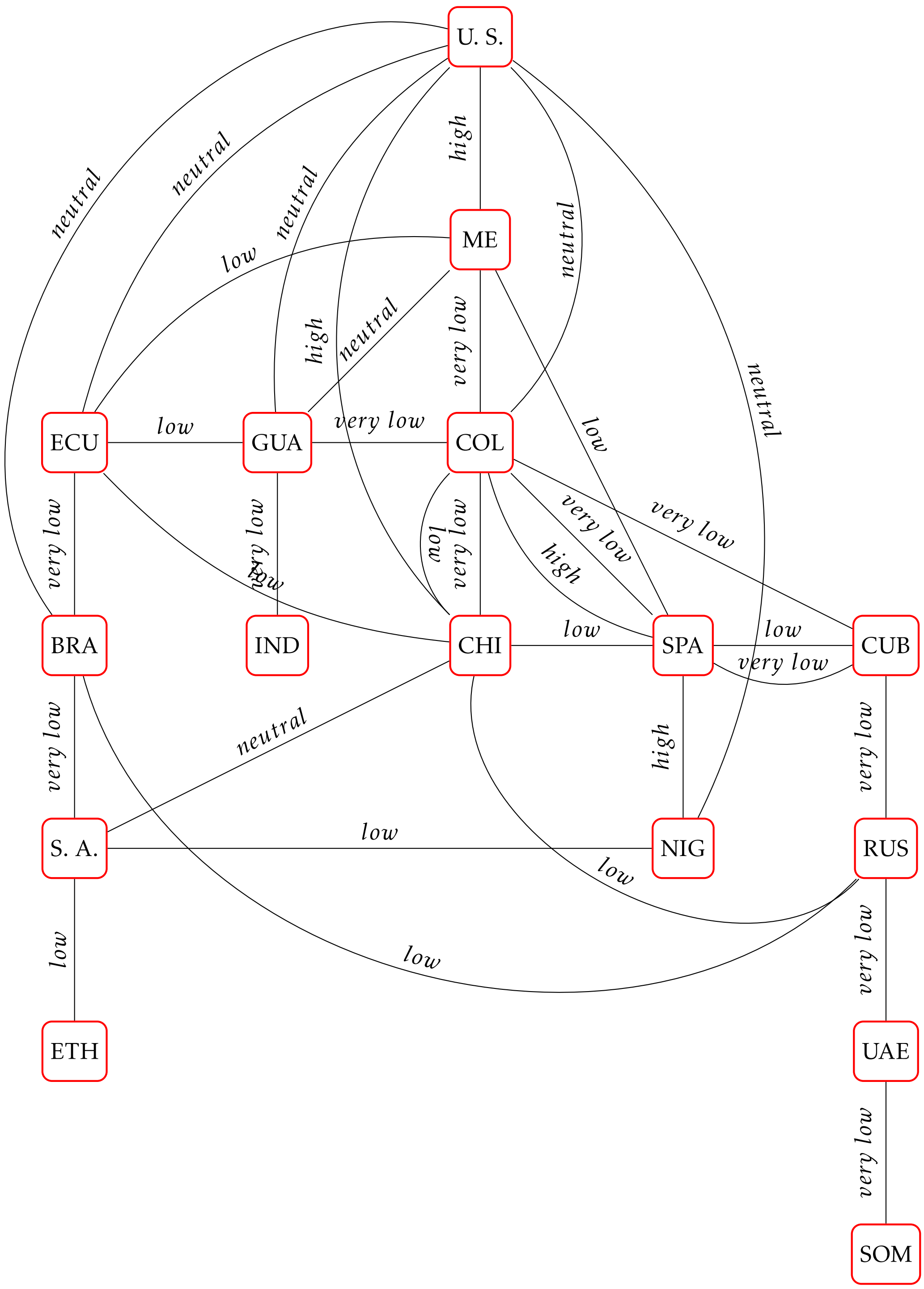

Figure 1. Linguistic fuzzy graph models Immigration problem
Where U. S: United States; ME: Mexico; COL: Columbia; GUA: Guatemala; ECU: Ecuador; CHI: China; IND: India; BRA: Brazil; S. A. : S.Africa; ETH: Ethiopia; SPA: Spain; CUB: Cuba; NIG: Nigeria; RUS: Russia; UAE: United Arab Emirates; SOM: Somalia

3. Towards maximum linguistic fuzzy spanning tree

The Linguistic Fuzzy Spanning Tree (LFST) is an extension of the fuzzy spanning tree concept from graph theory, integrating linguistic fuzzy logic to handle uncertainty and imprecision in real-world problems [6]. This makes the LFST highly valuable in areas where relationships between nodes.

Maximum Linguistic Spanning Tree (MLST), on the other hand is a powerful tool for optimizing complex networks with fuzzy and uncertain relationships. It provides greater flexibility, interpretability, and precision by incorporating linguistic fuzzy logic, making it highly valuable in decision-making systems, network design, and dynamic environments where relationships between nodes are qualitative or uncertain.

Algorithm 1 is the pseudo code to find MLST in a linguistic fuzzy graph $\mathbb{LFG} = (\mathbb{V}, \rho, \delta)$

Algorithm 1 Maximum linguistic spanning tree algorithm

Input: Graph $\mathbb{LFG} = (\mathbb{V}, \rho, \delta)$
Output: $MLST$
MLST $\leftarrow \emptyset$
1: **for** $i \leftarrow 1$ to $|\mathbb{V}| - 1$ **do**
2: $e \leftarrow \bigvee_i e_i \in \mathbb{E}$
3: **if** MLST $\cup\, e_i$ does not construct a simple circuit **then**
4: MLST $\leftarrow$ MLST $\cup\, e_i$
5: **end if**
6: **end for**
7: **return** MLST
▷ MLST is the maximum linguistic spanning tree of linguistic graph $\mathbb{LFG}$

Property 1. Give a graph $\mathbb{LFG} = (\mathbb{V}, \rho, \delta)$, the complexity for constructing maximum linguistic spanning tree of Algorithm 1 is $\mathcal{O}(|\mathbb{V}|)$.

Proof. The overall complexity of Algorithm 1 is primarily influenced by the frequency and volume of $\leftarrow$ assignment operations that occur within the iteration loops. Specifically, the computational burden scales with the number of such assignments, which play a pivotal role in determining the algorithm's performance and efficiency during execution. These assignment operations are repeated across the iterations, directly affecting the processing time and resource utilization as the algorithm progresses.

Lines	Maximum number of assignment operations
Line 1 to 6	$\sum_{i=1}^{\mathbb{V}-1} 2 = 2 \times (\lvert\mathbb{V}\rvert - 1)$

The total number of assignments of the loops is $2 \times |\mathbb{V}| - 2$ so the complexity of the Algorithm 1 is $\mathcal{O}(|\mathbb{V}|)$ □

4. Conclusion and forthcoming study

In this paper, we have proposed a novel approach for modeling Linguistic Fuzzy Spanning Trees (LFST) based on Hedge Algebra. This framework addresses the challenge of handling linguistic uncertainty in spanning tree problems by utilizing the fuzzy linguistic terms and hedge algebra to represent and manipulate imprecise relationships between nodes in a graph. By integrating hedge algebra, we were able to refine linguistic terms and add nuance to the modeling of edge weights, offering a more flexible and human-friendly representation of real-world systems.

The Maximum Linguistic Spanning Tree (MLST) algorithm presented in this work provides a systematic way to identify the optimal set of connections between nodes in a network, ensuring that the linguistic strength of relationships is maximized.

In the future, several directions for future research can be pursued to expand and improve upon the foundations established in this paper:

- Enhancing the computational efficiency of the MLST algorithm. While the current implementation provides flexibility in handling fuzzy linguistic terms, optimizing the algorithm for larger-scale networks will enable its application in more complex and real-time systems, such as smart cities or large-scale communication infrastructures.

- Applying the proposed model and algorithm to specific real-world case studies would help validate its practical utility. Domains such as transportation planning, telecommunication network design, and social network analysis could benefit significantly from the use of linguistic fuzzy models to optimize complex systems under uncertain conditions.

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