

The application of a optimal controller for the flexible joint robot

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Abstract

This paper describes the operation of a single Degree of Freedom (DOF) flexible robotic arm by the optimal controller. The derived model is based on Euler- Lagrange approach, while the first and second-order (super twisting) Sliding Mode Control (SMC) is proposed as a nonlinear control strategy. Exploration of a flexible robotic arm with using state-of-the-art controllers is essential for practical applications. This is even more refreshing when these arms have joints that work independently of each other to create a smooth connection between the joints, but it still ensures the natural properties like a real human arm. This system has many similarities with the flexible link system of the MIMO model in its operational state analysis. Control laws must be followed by logical rules in a coherent whole. The next step is to design a controller to fit the structure of the system. The author also compared the use of the above controller with a system without using any controller to determine the effectiveness of using the above controller.

Keywords: flexible joint, optimal controller, joint robot.

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1. Introduction

The use of robots to replace human manual labor has been practiced since very early, since the research on robotics has been promising since the 16th century [1]. The field of electronic devices such as micro-controllers has developed rapidly [2], thanks to which people have access to the most advanced technologies. Electronic devices can be seen as the brains of a robot, as they control the robot's activities [3], including the gestures of the robot's arms [4]. The application of information technology to the control system has also been implemented [5]. Connecting a control system to the human brain [6] to execute human intentions on the system, it will make things easier when adverse factors can occur, such as extreme climate or long distances, etc. Some sensors can alter emotions like humans [7]. Mechanical engineering and electronic engineering in combination to form a new field in the industry specialized in manufacturing consumer products [8]. Besides, a control system is applied in biomedical electronics [9] or any other

field. These areas include the soft movements of the fingers in a robotic hand [10], a lithe gait of a robot similar to the graceful gait of a maiden[11], which could be implemented in the future. Another challenge is that robots must have a high enough awareness to handle the serious problems of dealing with nuclear radiation hazards [12]. Robots could replace astronauts in the future, when the gravity factors of a robot are taken into account[13]. There are many types of robots that are designed according to separate structures and their operating functions are different, depending on the purpose of use[14]. A robot that acts as a mobile phone [15] is a new topic. Automating operations to the program language will be the challenge of the time[16]. Using the techniques in [17], [18] based on artificial neural network is an interesting idea to study it. A robot searching for objects in the sea that it can withstand at the deepest possible depth is a topic of interest[19]. The use of a robot to track targets based on strategies [20] is a novel topic. Computer vision has inspired the detection of distant targets in outer space according to [21] that need further attention. Tethered robotic systems [22] located in the depths of the sea is a fascinating research topic. Designing the optimal controller

[23] for a robot operating in the deep sea that can withstand great pressure needs more attention in research. Human foot compatible underactuated exoskeleton robotic system based on [24] is a future work. Similarly, this is done for [25] with a human foot. The use of algorithms in [26] for the case management of comatose patients before entering the hospital is a unique idea. The mobility of the patient [27] in the home is necessary to be taken into account. Mobility control of elbow joints for robots is based on [28] to form a research paper. The reference in [29] to investigate the mobility at the knee of the robot's leg is a novel idea. The performance of the wheels can be tested on any surface [30-33], which will give a satisfactory result. The formation of a multilink mobile robot from previous studies [30-33] based on [34] is very useful. The positive control for associations [35] of emotional robots may be investigated in the future. For this type of robot, the control is in the linear approach [36, 37] or in the linear approach [38, 39]. Based on [40] to model a robot foot. The focus of this paper is on the working mechanism of a flexible joint controller. In this controller, there is a rigid arm that is padded with springs to give the system elasticity and flexibility in all its movements. The main contribution of this paper is investigation of the system stability through the optimal controller, which was previously rare for a detailed study of the robotic arm category. At the same time, through this article, readers can evaluate the properties of the systems without using any control algorithm. Figure 1 is such a depicted model. It was developed by Quanser [41] on the basis of: an initially a static period for the rotational platform to be executed with a flexible controller mounted in it. The use of modern controllers to keep systems stable [42], [43] is a refreshing experience. The positive effects of academic problems are the innovations in control methods mentioned in this paper. These innovations give impetus to other researches, especially those of control algorithms. They also impact other areas related to other flexible robotic systems such as biomedical flexible robotic systems. Negative effects are rare. Reliability of this paper: Recent references have shown that surveys have not been conducted on this flexible model with using optimal control methods.

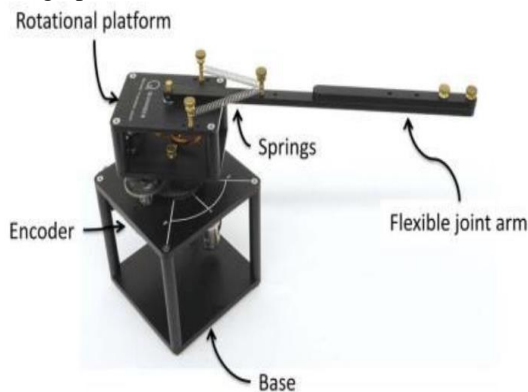


Fig 1. Flexible joint robot arm by Quanser [41]

The arm used in this model is a rotary joint linked to an arm and has two Degrees Of Freedom (DOF). The base is combined with a set of springs that reduce out-of-bounds vibrations in the joints. This type of arm is one of Quanser's products that will be the premise for research into devices with multiple arms. Thus it is very useful for robotics. Figure 2 shows the arm in a position where the spring sets have held it fixedly on a plane viewed from above by the observer. The Euler-Lagrange equation of motion is applied to this system to obtain the mathematical model of the arm described below. This mathematical equation is also used in the flexible linkage model of the MIMO system. The parameters of this system are used to create the model shown in Table 1. The coordinates contained in the platform according to the model [41] are described as follows: the rapid movement of the platform around a vertical axis will form an angle between -45 degrees and 45 degrees, another angle formed by the displacement of the flexible joints when the rotating platform stops moving at a specified position. This is detailed in Figure 2. The Euler-Lagrange's equation (3) 'L' requires the total kinetic and potential energies. The total potential energy 'PTotal' is the sum of the spring's stored energy at the joint and gravity given by (1). The sum of the kinetic energies of the rotational platform and the flexible link manipulator constitutes 'KTotal', which is given by (2)

$$P_{Total} = \frac{1}{2} K_s \alpha^2 + mgh \cos(\theta + \alpha) \quad (1)$$

$$K_{Total} = \frac{1}{2} J_h (\dot{\theta}_2) + \frac{1}{2} J_l (\dot{\theta} + \dot{\alpha})^2 \quad (2)$$

$$L = K_{Total} + P_{Total} \quad (3)$$

The Euler-Lagrange equation of motion (4) is used to get the mathematical equations for the rotational acceleration of rotational platform and flexible joint are given by (5) and (6) to form the corresponding motion mechanisms. In (4) the torque is represented by 'τ' and 'qi' is the variable of differentiation i.e. 'θ' or 'α'. Schematic diagram of flexible joint robotic arm (Fig. 2) is shown below.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau \quad (4)$$

$$\ddot{\theta} = \frac{1}{J_h} (\tau + K_s \alpha) \quad (5)$$

$$\ddot{\alpha} = -K_s \alpha \left(\frac{1}{J_h} + \frac{1}{J_l} \right) + \frac{1}{J_l} mgh \sin(\theta + \alpha) - \tau \quad (6)$$

$$\begin{cases} \ddot{\theta} = \frac{1}{J_h} (\tau + K_s \alpha) \\ \ddot{\alpha} = -K_s \alpha \left(\frac{1}{J_h} + \frac{1}{J_l} \right) + \frac{1}{J_l} mgh \sin(\theta + \alpha) - \tau \end{cases} \quad (7)$$

2. Modelling of flexible joint robot

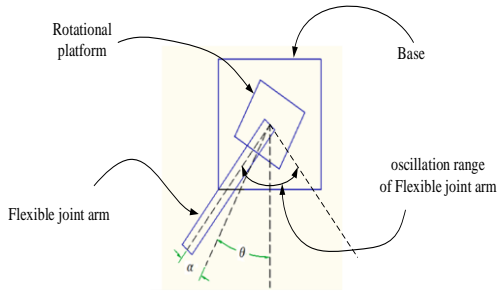


Fig 2. Schematic diagram of flexible joint robotic arm.

Table 1. Parameters of flexible joint robot

| Symbol | Description | Value | Units |
|--------|--|----------|------------------|
| K_s | Spring stiffness | 5.468 | N/m |
| m | Link mass | 0.1 | kg |
| J_h | Inertia of rotational platform | 0.00035 | Kgm ² |
| J_l | Inertia of flexible manipulator | 0.003882 | Kgm ² |
| h | Distance of center of gravity of rotational platform | 0.06 | m |
| g | Gravitational acceleration | -9.81 | N/m |
| τ | Torque applied to Active joint | 0.0134 | N-m/A |

The author had set state and output variables for the system as follows:

$$\begin{cases} x_1 = \theta, x_2 = \alpha, x_3 = \dot{\theta}, x_4 = \dot{\alpha}, x_5 = \arcsin z \\ y_1 = x_1, y_2 = x_2 \end{cases} \quad (8)$$

The author combined (7) and (8) to obtain the system of state equations describing the system :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{K_s}{J_h} & 0 & 0 & 0 \\ 0 & -K_s \left(\frac{1}{J_h} + \frac{1}{J_l} \right) & 0 & 0 & \frac{1}{J_l} mgh(\arcsin z) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\tau}{J_h} \\ -\tau \\ 0 \end{bmatrix} \quad (9)$$

3. Robust optimal controller design

A control system designed in the best working mode is always in the optimal state according to a certain quality (the standard extreme value is reached). Whether or not it is stable depends on the required quality and understanding of the object and its impacts, based on the working condition of the control system ... In the presentation, the author designed the controller to operate in the optimal state according to the J quality criteria function as required by the problem but that the traditional model without using the optimal controller has not been achieved. Substituting values from table 1 into (8), (9) is shown: $z=0.866$ has been selected:

$$\begin{cases} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{cases} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1562285 & 0 & 0 & 0 \\ 0 & -17031.4 & 0 & 0 & -2.39 \times 10^{-4} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 38.28 \\ -0.0134 \\ 0 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

The transfer function of the system:

$$G(s) = \frac{38.28s^2 - 2.167 \times 10^{-13}s + 6.518 \times 10^5}{s^4 + 1.703 \times 10^4 s^2} \quad (11)$$

The equation of state for G(s) is shown below:

$$A = \begin{bmatrix} 0 & -133 & 0 & 0 \\ 128 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 64 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 0.004673 \quad -2.645 \times 10^{-17} \quad 79.57]$$

The author considered the system to have an external impact:

$$\dot{x} = Ax + Bu \quad (12)$$

The author found the value of the matrix (this value is denoted by 'K') of the optimal control vector: $u(t) = -K \cdot x(t)$ satisfy the quality index value 'J' and 'J' must reach the minimum value:

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (13)$$

where Q is a positive determinis tic matrix, R is a positive deterministic matrix.

The value of the matrix is determined from Riccati's equation of the form: (This value is denoted by 'K')

$$K = R^{-1} B^T P \quad (14)$$

The state feedback control structure is shown below (Fig. 3).

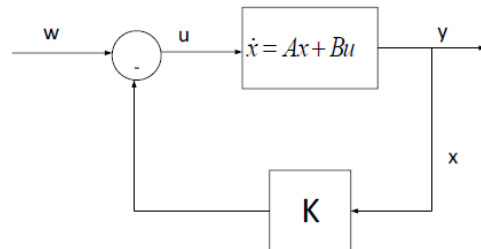


Fig 3. State feedback control structure

Thus, the optimal control law for an optimal control problem with quality criteria is a linear equation and it has the form:

$$u(t) = -K(t) = -R^{-1} B^T P x(t) \quad (15)$$

The value of this matrix is 'P', 'P' must be satisfied the equation:

$$PA + A^T P + Q - PBR^{-1}B^T P = \dot{P} \quad (16)$$

Equation (16) is known as Riccati's equation. The author chose the values of the matrix 'Q' and the values of the matrix 'R' as follows:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = 1$$

The author calculated the value of the matrix 'K' through Matlab software: $K = \text{lqr}(A, B, Q, R) = [1.3989 \ 0.2393 \ 2.3738 \ 1.0000]$ Figures 4, 5, and 6 are shown below.

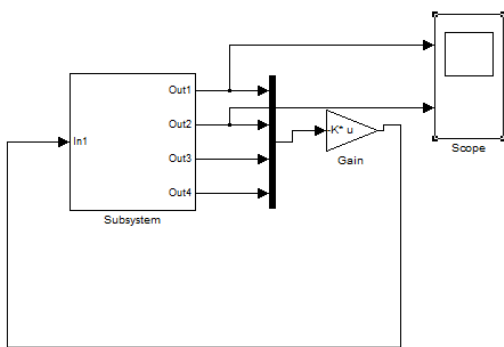


Fig 4. Applying the optimal LQR control method for the system G(s).

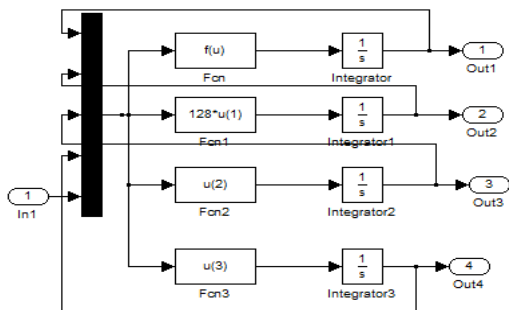


Fig 5. The system with the optimal controller G(s).

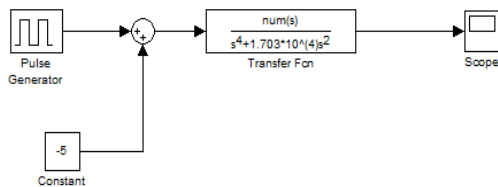


Fig 6. The model without using any algorithm.

4. Simulation results and discussions

Simulations of the system are shown in Figures 7, 8, 9.

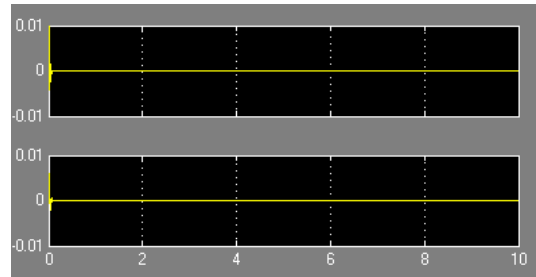


Fig 7. The simulation result using optimal control. Initial condition: $x_1=0.01, x_2=0, x_3=0, x_4=0$.

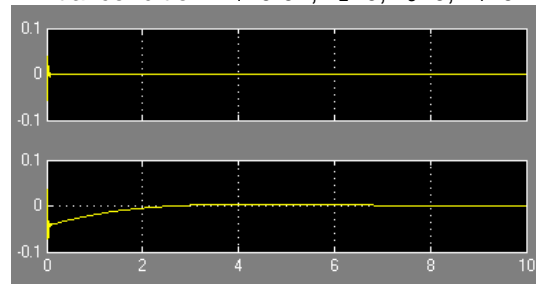


Fig 8. The simulation result using optimal control. Initial condition: $x_1=0.04, x_2=0.03, x_3=0.03, x_4=0.03$.

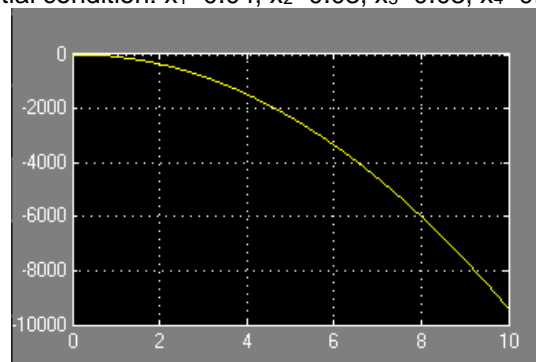


Fig 9. The simulation result without using any algorithm G(s).

Figure 9 shows that the amplitude of oscillation is large. My comments: Figure 7 and Figure 8 of the simulation results show me with the optimal controller, it helps stabilize the output signal of the system. However, if the input signal (This signal is caused by different factors) is large (Fig. 8), then the oscillation range of the signal amplitude at the output is high before it is stabilized. My comments: The result in Figure 7 is better than in Figure 8. The simulation results accurately reflect the problem: the optimal controller performs better when there is no external force applied to the system, for example: noise levels of other signals. The optimal controller used in this case is more efficient than the simulation result without using any algorithms.

5. Conclusions

In this paper, it is proposed to investigate the operation of a flexible joint robot with the simulation results to help the system achieve the desired functions. Besides, the simulation helps me to determine the values of the parameters to have the basis to control the operation of the system to suit the requirements. This also shows the

flexibility in adjusting the parameters of modern controllers as it is applied to the above model. Through this survey, the research achievements on modern control theory for systems with relatively complex structure like the above system will be applied in the future. For complex structured systems, I can use control methods for each of the transfer functions in the host system. From there, I took insight into the problem and I was able to evaluate the host system's properties most accurately. In the future, I can apply modern control algorithms like neural control algorithm in the above model. New control algorithms will later be used for the models to show that the control features are diverse and rich.

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