EAI Endorsed Transactions
on Digital Transformation of Industrial Processes
Research Article EAI.EU

# Analysis of the Effect of Random Numbers in Stochastic Heuristics for Solving Operations Research Problems

Baskar A.[1], Justine Yasappan[1], Anthony Xavior M.[2*], Anna Burduk[3] and Suthep Butdee[4]

[1]Loyola-ICAM College of Engineering and Technology (LICET), Chennai, India
[2]Vellore Institute of Technology, Vellore - 632014, Tamil Nadu, India
[3]Wroclaw University of Science and Technology, Poland
[4]Rajamangala University of Technology Krungthep (RMUTK), Thailand

## Abstract

INTRODUCTION: Random numbers are widely employed in stochastic and randomized heuristics to solve diverse Operations Research problems by facilitating the exploration and exploitation of new solution spaces.
OBJECTIVES: This paper examines various stochastic randomized heuristics, including Lévy-based heuristics, for solving the Travelling Salesman Problem (TSP), Facility Location Problem (FLP), Permutation Flowshop Scheduling Problem (PFSSP), and real-world engineering problems. For the TSP and PFSSP, the insertion technique of the Nawaz–Enscore–Ham (NEH) algorithm is integrated with random numbers to generate approximate solutions, while the FLP and real-world engineering problems are addressed by executing the randomized algorithm across multiple trials.
METHODS: The proposed approaches are evaluated using 12 TSP benchmark instances, including instances with up to 783 nodes, a real-world TSP involving 20 major cities in Slovakia, geodetic coordinates of 79 district headquarters in Slovakia for location science, 90 Taillard instances for scheduling problems, and 10 constrained real-world engineering problems. Up to 20,000 trials are considered, and the minimum, mean, and standard deviation of the resulting solutions are computed.
RESULTS: The findings demonstrate that appropriately designed randomized heuristics can achieve promising approximate solutions across diverse Operations Research problems. The statistical metrics are computed and results across all problem classes are compared with algorithms reported in the literature.
CONCLUSION: The study highlights the potential of stochastic and randomized approaches as effective computational methods for addressing complex optimization problems.

**Keywords:** Travelling Salesman, Permutation Flowshop Scheduling, Location Science, Stochastic Heuristics, Constrained Real-World Engineering problems, Insertion Heuristics.

Received on 28 September 2026, accepted on 10 November 2026, published on 12 November 2026

Copyright © 2026 Baskar A. *et al.*, licensed to EAI. This is an open access article distributed under the terms of the CC BY-NC-SA 4.0, which permits copying, redistributing, remixing, transformation, and building upon the material in any medium so long as the original work is properly cited.

doi: 10.4108/dtip.15352

*Corresponding author. Email: manthonyxavior@vit.ac.in

## 1. Introduction

Operations Research (OR) is an important domain in optimization that uses advanced analytical techniques to solve complex problems. Operations research comprises of a set of tools used for making organizational decisions. These problems often include resource allocation, scheduling, assignment, location science and engineering optimization. The goal of any such problem is to optimise an objective, such as profit, cost, or distance. In general, any OR problem is initially formulated as a mathematical equation in such a way that the optimization, maximization or minimization, could be effectively carried out using a particular type of algorithm. That is, the foremost characteristic of any OR problem is that a mathematical model is constructed to analyse the specific problem. These models typically take the form of an objective function that outlines the intended goal of the analysis and a set of linear or polynomial constraints that represent the real-time conditions/limitations within which the system must operate.

Once the problem is formulated, the objective function can be solved optimally to satisfy the purpose of the analysis. This solution best satisfies the objective under the prevailing conditions.

In many population-based optimization algorithms random numbers play an important role. In the initialization phase, they are used to randomly initialize the population of candidate solutions within the bounds. These random numbers play a critical role in iteratively updating candidate solutions. During optimization, they help to control the exploration and exploitation of the search space. They also play critical role in maintaining the population diversity, reduce the chances of pre-mature convergence and escape local optima. Random numbers are predominantly used in selection, crossover, and mutation operations in many algorithms. The speed of convergence and solution quality can be influenced by proper random number distributions. Hence, appropriate use of random numbers in algorithms is crucial for effective optimization performance. Along with random numbers, other strategies also must be used to make the updating expressions more effective. Grey Wolf Optimizer (GWO) [1], Teaching-Learning Based Optimization (TLBO) [2], Particle Swarm Optimization (PSO) [3] are a few algorithms that use random numbers effectively in their updating expressions. Baskar [4] analysed seven indigenous expressions in an analysis based on the concept,

$$\text{New Solution, } X_{(i+1)} = \text{Old Solution, } X_i \pm \text{Fraction of } X. \ (X \text{ maybe } X_i \text{ or “Best” or “Worst” solution}) \quad (1)$$

The “fraction of $X$” is generated using random numbers straightaway or by trigonometric functions.

This study considers five important classes of Operations Research (OR) optimization problems. New solutions are generated by random numbers in all these cases:

Travelling Salesman Problem (TSP): The objective is to determine the shortest possible tour for a person to visit a set of cities exactly once and return to the starting point. The total travelling distance or cost is minimized here. In this type of problem, the computation time increases exponentially with the size of the problem.

Permutation Flowshop Scheduling Problem (PFSSP): PFSSP involves scheduling “$n$” jobs on “$m$” machines in the same processing order. The objective is generally to determine the sequence of processing that minimises the overall completion time, termed the makespan.

Mini-Sum Facility Location Problem: The objective is to determine a location so that the total distance or cost between the location and the customers (other data points) it serves is minimized. In some cases, more than one facility also needs to be finalized. Since the sum of distances is minimized, such problems find applications in locating warehouses, service centres and distribution centres and so on.

Mini-Max Facility Location Problem: In this case, the facility is located to minimise the maximum distance between any data point and the optimal facility. The mini-max problem focuses on locating the best possible worst-case service distance, generally the radius.

Constrained Real-World Engineering Problems: For any optimization algorithm, solving constrained real-world engineering problems demonstrates that the proposed algorithm is capable of addressing practical industrial and engineering real-world challenges.

Together, these types of problems represent different applications involving routing, job scheduling, optimal facility location and practical engineering applications. This analysis demonstrates the effectiveness of randomized numbers used in optimization heuristics across diverse optimization conditions.

## 2. Strategies Used for TSP

The general strategy followed in solving this type is; initial two points (initial partial tour) is selected based on different criteria. Thereafter, a random sequence equal to the number of data points is generated with or without using the Levy Flight Strategy (LFT) [5]. Then the points (except the initial two points) are inserted one by one in a place that minimizes the partial cost. This continues until all data points are inserted.

This “insertion strategy” was effectively used by the NEH Algorithm [6] proposed to optimize the makespan in Permutation Flowshop Scheduling Problems (PFSSP) for scheduling “n” jobs in ‘m’ machines to get minimum makespan (total completion time).

The original NEH strategy comprises three stages: (i) arranging the jobs in non-increasing order of their total processing times on all machines, (ii) considering the first two jobs as the initial partial sequence, and (iii) inserting other jobs one by one at a place that minimises the partial makespan.

No random numbers are used by the original NEH algorithm. This work adapts only the insertion technique and uses it with random numbers to generate new TSP solutions.

The steps involved in finding the approximate tour to cover ‘n’ points are:

- Select two points
- Generate a random sequence with natural numbers of size 'n' (use the Levy Flight Strategy if required)
- Insert other points one by one at a place that minimizes the partial cost.

The initial two points may be selected randomly or following any strategy. However, the generated sequence is completely random. We analyse five options with and without LFT, totalling 10 heuristics (Table 1). LFT uses a shape parameter, $\beta = 1.5$ in this work ($\beta$ can assume any value between 0 and 2).

## 3. Benchmarks Used in TSP

We consider a total of 12 instances [7], with nodes ranging from 48 to 783. Table 2 presents the problems, number of nodes, and known optimal values. Since the proposed strategies use the NEH insertion strategy, we first estimate the cost reported by the NEH applied to TSP for each problem. The distance metric (cost) used here is the Euclidean distance. These costs and the relative per cent error with respect to the optimum are presented in the same Table 2.

Table 1. Options and Their Strategies

| Strategy | Initial Partial Sequence (2 points) | Reference Sequence (All points) | Remark |
|---|---|---|---|
| Option-I | Randomly 2 points | Random Sequence | -- |
| Option-I-Levy | Randomly 2 points | Random Sequence*RL | RL= Levy(1,n,1.5) n = number of points Levy = Levy Function |
| Option-II | First and last points of the sequence obtained after sorting in descending order of the sum of distances for each point | Random Sequence | -- |
| Option-II-Levy | First and last points of the sequence obtained after sorting in descending order of the sum of distances for each point | Random Sequence*RL | RL= Levy(1,n,1.5) |
| Option-III | First and last points of the sequence obtained after sorting in descending order by the row with the maximum value in the distance matrix. | Random Sequence | -- |
| Option-III-Levy | First and last points of the sequence, obtained after sorting in descending order by the row with the maximum value in the distance matrix. | Random Sequence*RL | RL= Levy(1,n,1.5) |
| Option-IV | Points having maximum and minimum Distance | Random Sequence | -- |
| Option-IV-Levy | Points having maximum and minimum Distance | Random Sequence*RL | RL= Levy(1,n,1.5) |
| Option-V | First and last points of the sequence obtained after sorting in descending order of the standard deviation of the distance matrix. | Random Sequence | -- |
| Option-V-Levy | First and last points of the sequence obtained after sorting in descending order of the standard deviation of the distance matrix. | Random Sequence*RL | RL= Levy(1,n,1.5) |

Table 2. Benchmark TSP Problems and their Optimal Costs

| Problem No. | Problem | Nodes | Optimum | NEH Cost | % Error |
|---|---|---|---|---|---|
| 1 | ATT48 | 48 | 33522 | 35843.3 | 6.92 |
| 2 | EIL51 | 51 | 426 | 494.0 | 15.96 |
| 3 | BERLIN52 | 52 | 7542 | 8210.7 | 8.87 |
| 4 | ST70 | 70 | 675 | 770.6 | 14.16 |
| 5 | PR76 | 76 | 108159 | 115661.2 | 6.94 |
| 6 | KROA100 | 100 | 21282 | 24597.8 | 15.58 |
| 7 | EIL101 | 101 | 629 | 720.4 | 14.53 |
| 8 | LIN105 | 105 | 14379 | 15648.3 | 8.83 |

| | | | | | |
|---|---|---|---|---|---|
| 9 | CH130 | 130 | 6110 | 7035.9 | 15.15 |
| 10 | CH150 | 150 | 6528 | 7583.4 | 16.17 |
| 11 | PR439 | 439 | 107217 | 139695.5 | 30.29 |
| 12 | RAT783 | 783 | 8806 | 11004.6 | 24.97 |
| Mean Error | -- | -- | -- | -- | 14.86 |

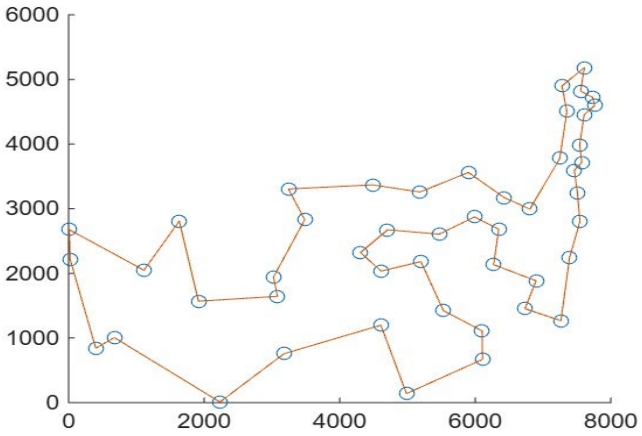
The NEH algorithm itself performs reasonably well for the benchmarks considered. The % Error is minimum (6.92%) for the 48-node ATT48 problem and maximum (30.29%) for the 439-node PR439 instance when compared with the optimal cost. Table 3 shows the simulation results of all 5 options for the 12 TSP problems. We use 1000 independent trials in all cases. That is, it is equivalent to independently running the NEH algorithm 1000 times for each problem. Because the insertion is applied to a random sequence, each trial yields a different solution. From the third to seventh column, the reported numerical values are "minimum", "mean", and "standard deviation" in that order. The integers in brackets indicate the number of trials (out of 1000); the cost is better than the NEH cost. It is about 16.1% to 26.1% for the BERLIN52 problem and 100% for the PR439 and RAT783 problems. That is, for the two highest numbers of nodes considered, the improvement is 100%. For each problem, the first row shows the results for that "Option" without LFT, and the second row shows the results with LFT.

Table 3. Results of Option-I to Option-V Heuristics and (minimum/ mean/ SD): 1000 Trials

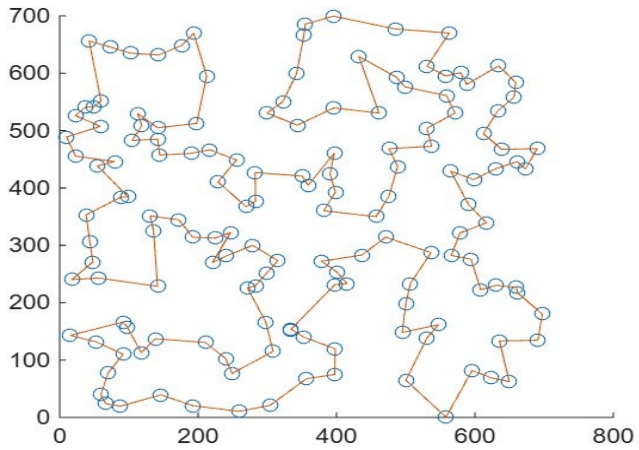
| No. | Problem | Option-I (Better than NEH) | Option-II (Better than NEH) | Option-III (Better than NEH) | Option-IV (Better than NEH) | Option-V (Better than NEH) |
|---|---|---|---|---|---|---|
| 1 | ATT48 | 33619.92, 35659.06, 760 (622) | 33701.26, 35673.88, 726 (611) | 33555.28, 35711.44, 769 (602) | 33755.35, 35527.83, 737 (683) | 33600.56, 35738.73, 776 (574) |
| | Levy1 | 33718.66, 35660.57, 735 (618) | 33680.14, 35716.48, 759 (600) | 33607.68, 35728.23, 761 (595) | 33750.23, 35512.17, 738 (695) | 33607.68, 35756.85, 773 (587) |
| 2 | EIL51 | 432.23, 461.49, 10.24 (997) | 430.24, 460.77, 10.26 (999) | 433.10, 459.94, 10.12 (999) | 433.60, 459.99, 10.19 (1000) | 430.60, 460.17, 10.25 (996) |
| | Levy2 | 432.81, 460.96, 10.40 (1000) | 434.94, 460.68, 10.56 (996) | 431.58, 459.54, 10.24 (997) | 428.98, 460.49, 10.30 (999) | 431.17, 460.15, 10.35 (999) |
| 3 | BERLIN52 | 7544.37, 8408.75, 265 (221) | 7572.85, 8369.35, 246 (247) | 7544.37, 8383.78, 251 (221) | 7544.37, 8405.54, 263 (225) | 7699.79, 8432.51, 224 (163) |
| | Levy3 | 7544.37, 8411.86, 269 (216) | 7544.37, 8372.84, 260 (261) | 7544.37, 8378.95, 232 (222) | 7544.37, 8416.72, 268 (220) | 7725.29, 8424.29, 221 (161) |
| 4 | ST70 | 683.03, 725.17, 17.42 (989) | 684.46, 731.94, 19.22 (968) | 683.64, 731.24, 18.54 (984) | 683.46, 724.51, 17.08 (988) | 682.56, 723.85, 17.20 (992) |
| | Levy4 | 681.96, 724.84, 17.57 (989) | 684.82, 731.42, 19.02 (976) | 684.96, 730.10, 17.85 (981) | 684.71, 724.46, 16.46 (992) | 677.44, 724.27, 17.97 (983) |
| 5 | PR76 | 109261.75, 115810.72, 3080 (513) | 109051.02, 115748.67, 3188 (514) | 108833.49, 116777.88, 3382 (404) | 108925.16, 114960.70, 3125 (629) | 108323.18, 116586.11, 2941 (414) |
| | Levy5 | 108724.24, 115814.45, 3094 (526) | 108233.99, 115859.70, 3241 (520) | 108895.39, 116724.02, 3399 (429) | 108338.50, 115092.74, 3091 (620) | 108445.50, 116654.70, 2995 (402) |
| 6 | KROA100 | 21357.16, 23018.50, 685 (989) | 21377.99, 23168.45, 700 (974) | 21306.84, 23054.19, 699 (985) | 21439.56, 23221.60, 718 (963) | 21408.27, 23061.85, 645 (990) |
| | Levy6 | 21389.31, 23038.07, 701 (988) | 21461.57, 23214.93, 742 (969) | 21485.97, 23071.41, 666 (987) | 21421.62, 23183.46, 736 (967) | 21463.67, 23097.93, 656 (980) |
| 7 | EIL101 | 661.40, 699.07, 11.91 (960) | 663.91, 699.48, 11.67 (956) | 661.59, 698.58, 11.67 (971) | 660.51, 699.13, 12.31 (952) | 660.91, 698.57, 12.60 (955) |
| | Levy7 | 662.56, 699.40, 11.80 (951) | 660.46, 699.39, 11.43 (953) | 663.38, 698.32, 11.66 (966) | 659.94, 699.31, 12.34 (941) | 657.63, 699.10, 11.93 (960) |
| 8 | LIN105 | 14697.51, 15841.62, 430 (338) | 14552.61, 15791.40, 398 (374) | 14456.23, 15784.58, 421 (377) | 14630.54, 15839.84, 422 (335) | 14711.76, 15908.28, 403 (263) |
| | Levy8 | 14686.59, 15878.75, 414 (296) | 14508.02, 15810.83, 426 (382) | 14582.36, 15771.91, 407 (390) | 14495.76, 15851.26, 417 (319) | 14511.20, 15919.42, 405 (250) |
| 9 | CH130 | 6254.70, 6660.33, 140.9 (994) | 6222.44, 6670.23, 141.7 (992) | 6227.79, 6637.37, 139.8 (996) | 6232.20, 6685.91, 143.0 (995) | 6231.16, 6673.64, 138.6 (996) |

| | | | | | | |
|---|---|---|---|---|---|---|
| | Levy9 | 6190.00, 6648.61, 136.2 (997) | 6302.45, 6659.63, 137.3 (993) | 6217.60, 6643.80, 139.5 (995) | 6313.75, 6695.79, 149.7 (985) | 6280.00, 6669.54, 141.1 (989) |
| 10 | CH150 | 6811.40, 7260.68, 158.9 (977) | 6784.75, 7244.64, 149.4 (984) | 6763.90, 7239.43, 158.2 (985) | 6780.50, 7247.29, 163.7 (981) | 6714.73, 7257.16, 163.1 (974) |
| | Levy10 | 6785.62, 7260.96, 159.3 (975) | 6694.62, 7249.18, 157.3 (980) | 6739.30, 7247.16, 160.6 (976) | 6753.66, 7247.03, 156.2 (980) | 6782.64, 7250.78, 157.8 (978) |
| 11 | PR439 | 114133.06, 121506.93, 2271 (1000) | 115315.79, 120764.70, 1999 (1000) | 114662.42, 120826.03, 2049 (1000) | 113742.66, 121332.06, 2262 (1000) | 115090.07, 121771.51, 2373 (1000) |
| | Levy11 | 114816.78, 121340.56, 2254 (1000) | 115082.79, 120868.34, 2136 (1000) | 114306.29, 121034.71, 2143 (1000) | 114614.72, 121396.56, 2158 (1000) | 114228.94, 121656.56, 2324 (1000) |
| 12 | RAT783 | 9741.29, 9994.33, 82.24 (1000) | 9770.18, 9997.56, 84.05 (1000) | 9730.95, 9991.69, 81.91 (1000) | 9758.98, 9992.28, 80.06 (1000) | 9715.38, 9994.50, 82.96 (1000) |
| | Levy12 | 9722.28, 10000.93, 83.64 (1000) | 9702.20, 9995.60, 84.59 (1000) | 9716.10, 9990.26, 85.71 (1000) | 9759.72, 9985.07, 82.14 (1000) | 9700.38, 9996.04, 84.29 (1000) |
| Better Results in | | 15 | 23 | 28 | 28 | 26 |

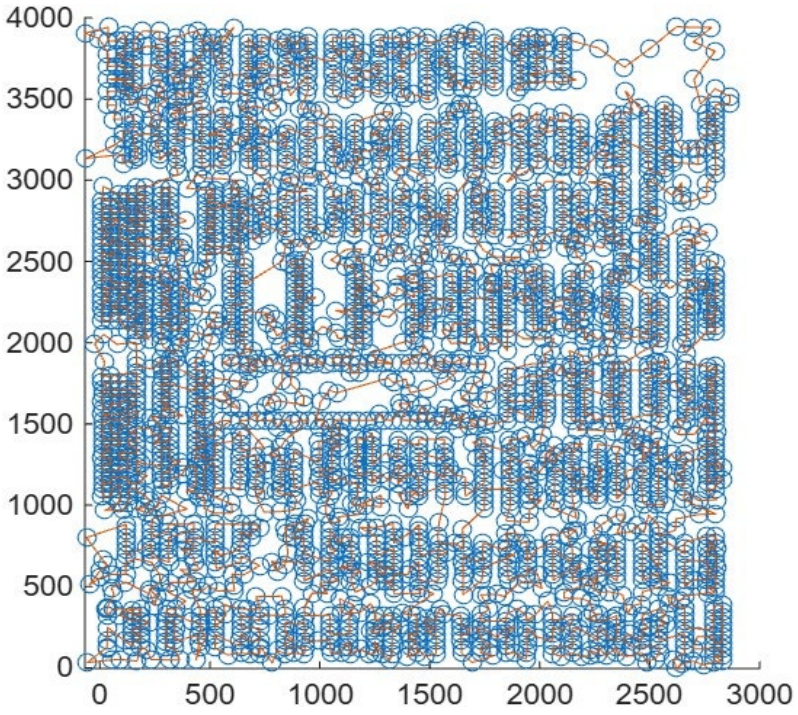
If we count better results across all four metrics (minimum, mean, standard deviation, and better than NEH cost), Option-III and Option-IV report better results in 28 cases each, followed by Option-V in 26 cases. Option-I is the poorest performer among all. Fig. 1 shows the tours obtained during one trial for ATT48, CH150, and PCB3038 instances.




(a). ATT48




(b). CH150




(c). PCB3038

**Figure 1.** Tours Obtained During Trials

Tables 4 to 8 show the minimum cost obtained, % Improvement over NEH Cost for the minimum cost, % Error for the minimum cost with respect to the optimal cost, the mean cost reported for 1000 independent trials, and the sum of ranks.

The minimum % improvement over NEH cost is 5.53%, and the maximum is 18.58% (Table 5). The average % improvement is 10.37 for the better performer, Option-III.

When considering % error with respect to the optimal costs, the minimum error is 0.03% for the BERLIN52 instance, and the maximum error is 10.95% for the RAT783 instance. The mean %error is 2.75% for Option-III (Table 6).

Table 8 summarises the ranks and shows that Option-III is the better performer across the 12 instances considered. It reports an overall sum of ranks of 118, followed by Option-IV with 137. Option-I is the poorest performer overall.

We then run simulations for 10000 and 20000 trials. For the better-performing Option-III, Table 9 presents the minimum cost and the number of times the trials report better results than the NEH cost (in brackets) for 10 problem instances. The mean %Error is 1.328% for 10000 trials and 1.125% for 20000 trials. For Option-III, the corresponding result is 1.508% for 1000 trials. That is, the results improve marginally over 1000 trials (Table 10).

From Tables 7 and 10, the Lévy variants perform similarly to the non-Lévy variants on this set of problems.

Table 11 shows a few results available in the literature [8], which are compared with the results of Option-III. Out of 7 instances, Option-III (Levy version) yields better results in 5 cases, followed by Option-III in 3 cases. ACO could report the better result in only one case, CH150.

In another analysis [9], the results of a few popular algorithms are considered (Table 12), including GA, SA, TS, and SCO. Here, Option-III also performs better than the other algorithms.

"Option-III" and its Levy version, "Option-III-Levy", are run for 1000 trials, whereas both the "Best Tour" and "Random Tour" Levy variants of TP-AB are run for 500 iterations each to keep the number of function evaluations the same. Since the TP-AB algorithm [10] has two function evaluations per iteration, we halve the total iterations for a fair assessment.

Table 13 shows the simulation results for the metrics: minimum, mean, and standard deviation. Option-III (Levy) yields better results in 17 cases, followed by Option-III with 12.

Finally, the "minimum" results of 7 instances for which the results of one of the recent algorithms, "Half Max Insertion Heuristic (HMIH)" proposed by Asani et. al. [11], are presented in Table 14.

Table 4. Results of Option-I to Option-V Heuristics (1000 Trials): Minimum Cost

| Problem | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|
| ATT48 | 33619.92 | 33701.26 | 33555.28 | 33755.35 | 33600.56 |
| Rank | 3 | 4 | 1 | 5 | 2 |
| Levy1 | 33718.66 | 33680.14 | 33607.68 | 33750.23 | 33607.68 |
| Rank | 4 | 3 | 1 | 5 | 1 |
| EIL51 | 432.23 | 430.24 | 433.1 | 433.6 | 430.6 |
| Rank | 3 | 1 | 4 | 5 | 2 |
| Levy2 | 432.81 | 434.94 | 431.58 | 428.98 | 431.17 |
| Rank | 4 | 5 | 3 | 1 | 2 |
| BERLIN52 | 7544.37 | 7572.85 | 7544.37 | 7544.37 | 7699.79 |
| Rank | 1 | 4 | 1 | 1 | 5 |
| Levy3 | 7544.37 | 7544.37 | 7544.37 | 7544.37 | 7725.29 |
| Rank | 1 | 1 | 1 | 1 | 5 |
| ST70 | 683.03 | 684.46 | 683.64 | 683.46 | 682.56 |
| Rank | 2 | 5 | 4 | 3 | 1 |
| Levy4 | 681.96 | 684.82 | 684.96 | 684.71 | 677.44 |
| Rank | 2 | 4 | 5 | 3 | 1 |
| PR76 | 109261.75 | 109051.02 | 108833.49 | 108925.16 | 108323.18 |
| Rank | 5 | 4 | 2 | 3 | 1 |
| Levy5 | 108724.24 | 108233.99 | 108895.39 | 108338.5 | 108445.5 |
| Rank | 4 | 1 | 5 | 2 | 3 |
| KROA100 | 21357.16 | 21377.99 | 21306.84 | 21439.56 | 21408.27 |
| Rank | 2 | 3 | 1 | 5 | 4 |
| Levy6 | 21389.31 | 21461.57 | 21485.97 | 21421.62 | 21463.67 |
| Rank | 1 | 3 | 5 | 2 | 4 |
| EIL101 | 661.4 | 663.91 | 661.59 | 660.51 | 660.91 |
| Rank | 3 | 5 | 4 | 1 | 2 |
| Levy7 | 662.56 | 660.46 | 663.38 | 659.94 | 657.63 |
| Rank | 4 | 3 | 5 | 2 | 1 |
| LIN105 | 14697.51 | 14552.61 | 14456.23 | 14630.54 | 14711.76 |
| Rank | 4 | 2 | 1 | 3 | 5 |
| Levy8 | 14686.59 | 14508.02 | 14582.36 | 14495.76 | 14511.2 |
| Rank | 5 | 2 | 4 | 1 | 3 |
| CH130 | 6254.7 | 6222.44 | 6227.79 | 6232.2 | 6231.16 |
| Rank | 5 | 1 | 2 | 4 | 3 |
| Levy9 | 6190 | 6302.45 | 6217.6 | 6313.75 | 6280 |
| Rank | 1 | 4 | 2 | 5 | 3 |
| CH150 | 6811.4 | 6784.75 | 6763.9 | 6780.5 | 6714.73 |
| Rank | 5 | 4 | 2 | 3 | 1 |
| Levy10 | 6785.62 | 6694.62 | 6739.3 | 6753.66 | 6782.64 |
| Rank | 5 | 1 | 2 | 3 | 4 |
| PR439 | 114133.06 | 115315.79 | 114662.42 | 113742.66 | 115090 |
| Rank | 2 | 5 | 3 | 1 | 4 |
| Levy11 | 114816.78 | 115082.79 | 114306.29 | 114614.72 | 114228.94 |
| Rank | 4 | 5 | 2 | 3 | 1 |
| RAT783 | 9741.29 | 9770.18 | 9730.95 | 9758.98 | 9715.38 |
| Rank | 3 | 5 | 2 | 4 | 1 |
| Levy12 | 9722.28 | 9702.2 | 9716.1 | 9759.72 | 9700.38 |
| Rank | 4 | 2 | 3 | 5 | 1 |

Table 5. % Improvement Over NEH Cost (1000 Trials): Minimum Values

| Problem | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|
| ATT48 | 6.20 | 5.98 | 6.38 | 5.83 | 6.26 |
| Levy1 | 5.93 | 6.04 | 6.24 | 5.84 | 6.24 |
| EIL51 | 12.50 | 12.91 | 12.33 | 12.23 | 12.83 |
| Levy2 | 12.39 | 11.96 | 12.64 | 13.16 | 12.72 |
| BERLIN52 | 8.12 | 7.77 | 8.12 | 8.12 | 6.22 |
| Levy3 | 8.12 | 8.12 | 8.12 | 8.12 | 5.91 |
| ST70 | 11.36 | 11.18 | 11.28 | 11.31 | 11.42 |
| Levy4 | 11.50 | 11.13 | 11.11 | 11.15 | 12.09 |
| PR76 | 5.53 | 5.72 | 5.90 | 5.82 | 6.34 |
| Levy5 | 6.00 | 6.42 | 5.85 | 6.33 | 6.24 |
| KROA100 | 13.17 | 13.09 | 13.38 | 12.84 | 12.97 |

| | | | | | |
|---|---|---|---|---|---|
| Levy6 | 13.04 | 12.75 | 12.65 | 12.91 | 12.74 |
| EIL101 | 8.19 | 7.84 | 8.16 | 8.31 | 8.26 |
| Levy7 | 8.03 | 8.32 | 7.92 | 8.39 | 8.71 |
| LIN105 | 6.08 | 7.00 | 7.62 | 6.50 | 5.98 |
| Levy8 | 6.15 | 7.29 | 6.81 | 7.37 | 7.27 |
| CH130 | 11.10 | 11.56 | 11.49 | 11.42 | 11.44 |
| Levy9 | 12.02 | 10.42 | 11.63 | 10.26 | 10.74 |
| CH150 | 10.18 | 10.53 | 10.81 | 10.59 | 11.45 |
| Levy10 | 10.52 | 11.72 | 11.13 | 10.94 | 10.56 |
| PR439 | 18.30 | 17.45 | 17.92 | 18.58 | 17.61 |
| Levy11 | 17.81 | 17.62 | 18.17 | 17.95 | 18.23 |
| RAT783 | 11.48 | 11.22 | 11.57 | 11.32 | 11.72 |
| Levy12 | 11.65 | 11.84 | 11.71 | 11.31 | 11.85 |
| Average % Imp. | 10.22 | 10.24 | **10.37** | 10.27 | 10.24 |

Table 6. % Error with Respect to Optimal (1000 Trials): Minimum Values

| Problem | Opt. | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|---|
| ATT48 | 33522 | 0.29 | 0.53 | 0.10 | 0.70 | 0.23 |
| Levy1 | 33522 | 0.59 | 0.47 | 0.26 | 0.68 | 0.26 |
| EIL51 | 426 | 1.46 | 1.00 | 1.67 | 1.78 | 1.08 |
| Levy2 | 426 | 1.60 | 2.10 | 1.31 | 0.70 | 1.21 |
| BERLIN52 | 7542 | 0.03 | 0.41 | 0.03 | 0.03 | 2.09 |
| Levy3 | 7542 | 0.03 | 0.03 | 0.03 | 0.03 | 2.43 |
| ST70 | 675 | 1.19 | 1.40 | 1.28 | 1.25 | 1.12 |
| Levy4 | 675 | 1.03 | 1.45 | 1.48 | 1.44 | 0.36 |
| PR76 | 108159 | 1.02 | 0.82 | 0.62 | 0.71 | 0.15 |
| Levy5 | 108159 | 0.52 | 0.07 | 0.68 | 0.17 | 0.26 |
| KROA100 | 21282 | 0.35 | 0.45 | 0.12 | 0.74 | 0.59 |
| Levy6 | 21282 | 0.50 | 0.84 | 0.96 | 0.66 | 0.85 |
| EIL101 | 629 | 5.15 | 5.55 | 5.18 | 5.01 | 5.07 |
| Levy7 | 629 | 5.34 | 5.00 | 5.47 | 4.92 | 4.55 |
| LIN105 | 14379 | 2.22 | 1.21 | 0.54 | 1.75 | 2.31 |
| Levy8 | 14379 | 2.14 | 0.90 | 1.41 | 0.81 | 0.92 |
| CH130 | 6110 | 2.37 | 1.84 | 1.93 | 2.00 | 1.98 |
| Levy9 | 6110 | 1.31 | 3.15 | 1.76 | 3.33 | 2.78 |
| CH150 | 6528 | 4.34 | 3.93 | 3.61 | 3.87 | 2.86 |
| Levy10 | 6528 | 3.95 | 2.55 | 3.24 | 3.46 | 3.90 |
| PR439 | 107217 | 6.45 | 7.55 | 6.94 | 6.09 | 7.34 |
| Levy11 | 107217 | 7.09 | 7.34 | 6.61 | 6.90 | 6.54 |
| RAT783 | 8806 | 10.62 | 10.95 | 10.50 | 10.82 | 10.33 |
| Levy12 | 8806 | 10.41 | 10.18 | 10.33 | 10.83 | 10.16 |
| Average % Error | --- | 2.92 | 2.91 | 2.75 | 2.86 | 2.89 |

Table 7. Results of Option-I to Option-V Heuristics (1000 Trials): Mean Values

| Problem | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|
| ATT48 | 35659.06 | 35673.88 | 35711.44 | 35527.83 | 35738.73 |
| Rank | 2 | 3 | 4 | 1 | 5 |
| Levy1 | 35660.57 | 35716.48 | 35728.23 | 35512.17 | 35756.85 |
| Rank | 2 | 3 | 4 | 1 | 5 |
| EIL51 | 461.49 | 460.77 | 459.94 | 459.99 | 460.17 |
| Rank | 5 | 4 | 1 | 2 | 3 |
| Levy2 | 460.96 | 460.68 | 459.54 | 460.49 | 460.15 |
| Rank | 5 | 4 | 1 | 3 | 2 |
| BERLIN52 | 8408.75 | 8369.35 | 8383.78 | 8405.54 | 8432.51 |
| Rank | 4 | 1 | 2 | 3 | 5 |
| Levy3 | 8411.86 | 8372.84 | 8378.95 | 8416.72 | 8424.29 |
| Rank | 3 | 1 | 2 | 4 | 5 |
| ST70 | 725.17 | 731.94 | 731.24 | 724.51 | 723.85 |
| Rank | 3 | 5 | 4 | 2 | 1 |
| Levy4 | 724.84 | 731.42 | 730.1 | 724.46 | 724.27 |

| | | | | | |
|---|---|---|---|---|---|
| Rank | 3 | 5 | 4 | 2 | 1 |
| PR76 | 115810.72 | 115748.67 | 116777.88 | 114960.7 | 116586.11 |
| Rank | 3 | 2 | 5 | 1 | 4 |
| Levy5 | 115814.45 | 115859.7 | 116724.02 | 115092.74 | 116654.7 |
| Rank | 2 | 3 | 5 | 1 | 4 |
| KROA100 | 23018.5 | 23168.45 | 23054.19 | 23221.6 | 23061.85 |
| Rank | 1 | 4 | 2 | 5 | 3 |
| Levy6 | 23038.07 | 23214.93 | 23071.41 | 23183.46 | 23097.93 |
| Rank | 1 | 5 | 2 | 4 | 3 |
| EIL101 | 699.07 | 699.48 | 698.58 | 699.13 | 698.57 |
| Rank | 3 | 5 | 2 | 4 | 1 |
| Levy7 | 699.4 | 699.39 | 698.32 | 699.31 | 699.1 |
| Rank | 5 | 4 | 1 | 3 | 2 |
| LIN105 | 15841.62 | 15791.4 | 15784.58 | 15839.84 | 15908.28 |
| Rank | 4 | 2 | 1 | 3 | 5 |
| Levy8 | 15878.75 | 15810.83 | 15771.91 | 15851.26 | 15919.42 |
| Rank | 4 | 2 | 1 | 3 | 5 |
| CH130 | 6660.33 | 6670.23 | 6637.37 | 6685.91 | 6673.64 |
| Rank | 2 | 3 | 1 | 5 | 4 |
| Levy9 | 6648.61 | 6659.63 | 6643.8 | 6695.79 | 6669.54 |
| Rank | 2 | 3 | 1 | 5 | 4 |
| CH150 | 7260.68 | 7244.64 | 7239.43 | 7247.29 | 7257.16 |
| Rank | 5 | 2 | 1 | 3 | 4 |
| Levy10 | 7260.96 | 7249.18 | 7247.16 | 7247.03 | 7250.78 |
| Rank | 5 | 3 | 2 | 1 | 4 |
| PR439 | 121506.93 | 120764.7 | 120826.03 | 121332.06 | 121771.51 |
| Rank | 4 | 1 | 2 | 3 | 5 |
| Levy11 | 121340.56 | 120868.34 | 121034.71 | 121396.56 | 121656.56 |
| Rank | 3 | 1 | 2 | 4 | 5 |
| RAT783 | 9994.33 | 9997.56 | 9991.69 | 9992.28 | 9994.5 |
| Rank | 3 | 5 | 1 | 2 | 4 |
| Levy12 | 10000.93 | 9995.6 | 9990.26 | 9985.07 | 9996.04 |
| Rank | 5 | 3 | 2 | 1 | 4 |
| Sum: Without LFT | 39 | 37 | 26 | 34 | 44 |
| Sum: With LFT | 40 | 37 | 27 | 32 | 44 |

Table 8. Sum of Ranks for All 12 Instances (1000 Trials)

| Parameter | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|
| Sum of Ranks (Minimum) | 77 | 77 | 65 | 71 | 60 |
| Sum of Ranks (Mean) | 79 | 74 | 53 | 66 | 88 |
| Total | 156 | 151 | 118 | 137 | 148 |

Table 9. Option III: Minimum Cost (10000 and 20000 Trials)

| | | | | IT 10000 | | IT 20000 | |
|---|---|---|---|---|---|---|---|
| Problem No. | Problem | Optimum | NEH Cost | Option III (Better than NEH) | With Levy (Better than NEH) | Option III (Better than NEH) | With Levy (Better than NEH) |
| 1 | ATT48 | 33522 | 35843.3 | 33588.34 (5946) | 33588.34 (5899) | 33588.34 (11839) | 33607.68 (11620) |
| 2 | EIL51 | 426 | 494.0 | 431.58 (9978) | 428.98 (9982) | 430.24 (19969) | 428.98 (19958) |
| 3 | BERLIN52 | 7542 | 8210.7 | 7544.36 (2238) | 7544.36 (2359) | 7544.36 (4736) | 7544.36 (4586) |
| 4 | ST70 | 675 | 770.6 | 682.64 (9784) | 680.75 (9789) | 680.09 (19592) | 681.96 (19606) |
| 5 | PR76 | 108159 | 115661.2 | 108266.23 (3980) | 108559.70 (4114) | 108595.84 (8238) | 108298.90 (8076) |
| 6 | KROA100 | 21282 | 24597.8 | 21377.26 (9806) | 21357.16 (9816) | 21335.33 (19580) | 21327.85 (19600) |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | EIL101 | 629 | 720.4 | 659.90 (9664) | 658.91 (9621) | 659.72 (19264) | 653.60 (19250) |
| 8 | LIN105 | 14379 | 15648.3 | 14485.76 (3767) | 14527.13 (3880) | 14501.87 (7449) | 14505.50 (7383) |
| 9 | CH130 | 6110 | 7035.9 | 6221.09 (9953) | 6220.59 (9953) | 6190.38 (19904) | 6199.81 (19914) |
| 10 | CH150 | 6528 | 7583.4 | 6721.91 (9854) | 6745.94 (9838) | 6710.70 (19704) | 6699.30 (19711) |

Table 10. Option-III, % Error

| S.No. | Problem | IT 10000 | IT 10000 (with Levy) | IT 20000 | IT 20000 (with Levy) |
|---|---|---|---|---|---|
| 1 | ATT48 | 0.197509 | 0.1979 | 0.1979 | 0.255593 |
| 2 | EIL51 | 1.292924 | 0.699531 | 0.995305 | 0.699531 |
| 3 | BERLIN52 | 0.031282 | 0.031291 | 0.031291 | 0.031291 |
| 4 | ST70 | 1.119184 | 0.851852 | 0.754074 | 1.031111 |
| 5 | PR76 | 0.099043 | 0.370473 | 0.403887 | 0.129347 |
| 6 | KROA100 | 0.445614 | 0.353162 | 0.250587 | 0.21544 |
| 7 | EIL101 | 4.682528 | 4.755167 | 4.883943 | 3.91097 |
| 8 | LIN105 | 0.737 | 1.030183 | 0.85451 | 0.879755 |
| 9 | CH130 | 1.7857 | 1.809984 | 1.315548 | 1.469885 |
| 10 | CH150 | 2.884746 | 3.338542 | 2.798713 | 2.624081 |
| Mean | --- | 1.327553 | 1.343808 | 1.248576 | 1.1247 |

Table 11. Percentage Error Relative to Optimal Solutions for Representative TSP Instances

| Instance | GA (%) | PSO (%) | ACO (%) | Option-III (%) | Option-III-Levy (%) |
|---|---|---|---|---|---|
| ATT48 | 1.43 | 0.86 | 0.58 | 0.1979 | 0.255593 |
| EIL51 | 2.82 | 2.11 | 1.41 | 0.995305 | 0.699531 |
| BERLIN52 | 0.78 | 0.61 | 0.37 | 0.031291 | 0.031291 |
| ST70 | 4.00 | 2.96 | 2.07 | 0.754074 | 1.031111 |
| PR76 | 1.15 | 0.87 | 0.69 | 0.403887 | 0.129347 |
| KROA100 | 1.07 | 0.65 | 0.51 | 0.250587 | 0.21544 |
| CH150 | 1.29 | 0.80 | 0.64 | 2.798713 | 2.624081 |

Table 12. Comparison of %Error with Existing Algorithms

| Instance | TPO | NN | GA | SA | TS | ACO | Option-III | Option-III (Levy) |
|---|---|---|---|---|---|---|---|---|
| EIL51 | 2.64 | 18.56 | 6.60 | 3.08 | 3.08 | 9.73 | 0.995305 | 0.699531 |
| BERLIN52 | 2.17 | 8.50 | 5.36 | 5.55 | 2.63 | 5.04 | 0.031291 | 0.031291 |
| ST70 | 3.28 | 12.82 | 3.81 | 3.16 | 2.26 | 12.08 | 0.754074 | 1.031111 |
| PR76 | 5.26 | 21.05 | 13.70 | 4.48 | 1.64 | 9.78 | 0.403887 | 0.129347 |
| CH150 | 6.35 | 8.42 | 7.30 | 8.18 | 5.12 | 12.60 | 2.798713 | 2.624081 |
| EIL101 | 7.31 | 17.01 | 9.04 | 6.86 | 6.14 | 19.70 | 4.883943 | 3.91097 |
| KROA100 | 5.55 | 16.05 | 6.79 | 4.68 | 5.82 | 7.80 | 0.250587 | 0.21544 |
| CH130 | 6.63 | 17.82 | 8.20 | 7.34 | 9.94 | 13.16 | 1.315548 | 1.469885 |

Table 13. Comparative Results of Option-III and TP-AB Algorithm (minimum/ mean/ SD)

| No. | Problem | Option-III (1000 Trials) | Option_III_Levy (1000 Trials) | TP-AB: Levy (BT)-500 Iterations | TP-AB: Levy (RT)-500 Iterations |
|---|---|---|---|---|---|
| 1 | ATT48 | 33555.28, 35711.44, 769 | 33607.68, 35728.23, 761 | 33772.1 35714.2 748 | 34043.6 35674.7 712 |
| 2 | EIL51 | 433.10, 459.94, | 431.58, 459.54, | 434.3 460.6 | 434.4 461.8 |

| | | | | | |
|---|---|---|---|---|---|
| | | 10.12 | 10.24 | 10 | 10 |
| 3 | BERLIN52 | 7544.37, | 7544.37, | 7549.9 | 7718.7 |
| | | 8383.78, | 8378.95, | 8395.2 | 8418.8 |
| | | 251 | 232 | 267 | 271 |
| 4 | ST70 | 683.64, | 684.96, | 685.9 | 685.4 |
| | | 731.24, | 730.10, | 725.1 | 724.0 |
| | | 18.54 | 17.85 | 18 | 18 |
| 5 | PR76 | 108833.49, | 108895.39, | 109170.3 | 108963.9 |
| | | 116777.88, | 116724.02, | 117028.7 | 116496.3 |
| | | 3382 | 3399 | 3617 | 3298 |
| 6 | KROA100 | 21306.84, | 21485.97, | 21543.8 | 21712.5 |
| | | 23054.19, | 23071.41, | 23033.8 | 23120.2 |
| | | 699 | 666 | 703 | 709 |
| 7 | EIL101 | 661.59, | 663.38, | 665.7 | 667.4 |
| | | 698.58, | 698.32, | 698.3 | 699.2 |
| | | 11.67 | 11.66 | 12 | 12 |
| 8 | LIN105 | 14456.23, | 14582.36, | 14845.3 | 14785.0 |
| | | 15784.58, | 15771.91, | 16018.5 | 15941.7 |
| | | 421 | 407 | 518 | 454 |
| 9 | CH130 | 6227.79, | 6217.60, | 6319.8 | 6288.1 |
| | | 6637.37, | 6643.80, | 6650.9 | 6662.8 |
| | | 139.8 | 139.5 | 139 | 139 |
| 10 | CH150 | 6763.90, | 6739.30, | 6820.4 | 6807.3 |
| | | 7239.43, | 7247.16, | 7260.8 | 7261.9 |
| | | 158.2 | 160.6 | 155 | 170 |
| 11 | PR439 | 114662.42, | 114306.29, | 118645.6 | 118003.1 |
| | | 120826.03, | 121034.71, | 122885.9 | 121958.5 |
| | | 2049 | 2143 | 2697 | 2625 |
| 12 | RAT783 | 9730.95, | 9716.10, | 9923.3 | 9937.4 |
| | | 9991.69, | 9990.26, | 10027.5 | 10051.4 |
| | | 81.91 | 85.71 | 54 | 80 |
| | Best In | 12 | 17 | 6 | 5 |

Table 14. Results of 10 Instances (1000 Trials): Minimum Costs

| No. | Problem | HMIH | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|---|---|
| 1 | ATT48 | 35657 | 33619.92 | 33701.26 | 33555.28 | 33755.35 | 33600.56 |
| | Levy1 | | 33718.66 | 33680.14 | 33607.68 | 33750.23 | 33607.68 |
| 2 | EIL51 | 471 | 432.23 | 430.24 | 433.1 | 433.6 | 430.6 |
| | Levy2 | | 432.81 | 434.94 | 431.58 | 428.98 | 431.17 |
| 3 | EIL101 | 690 | 661.4 | 663.91 | 661.59 | 660.51 | 660.91 |
| | Levy3 | | 662.56 | 660.46 | 663.38 | 659.94 | 657.63 |
| 4 | CH130 | 6650 | 6254.7 | 6222.44 | 6227.79 | 6232.2 | 6231.16 |
| | Levy4 | | 6190 | 6302.45 | 6217.6 | 6313.75 | 6280 |
| 5 | CH150 | 7211 | 6811.4 | 6784.75 | 6763.9 | 6780.5 | 6714.73 |
| | Levy5 | | 6785.62 | 6694.62 | 6739.3 | 6753.66 | 6782.64 |
| 6 | PR439 | 124322 | 114133.06 | 115315.79 | 114662.42 | 113742.66 | 115090 |
| | Levy6 | | 114816.78 | 115082.79 | 114306.29 | 114614.72 | 114228.94 |
| 7 | RAT783 | 10434 | 9741.29 | 9770.18 | 9730.95 | 9758.98 | 9715.38 |
| | Levy7 | | 9722.28 | 9702.2 | 9716.1 | 9759.72 | 9700.38 |

For the 7 problems considered, Option-II and Option-IV report better results in 2 cases each. Other options are better in one problem each. All five Options perform better than the HMIH algorithm. Finally, the CPU time in seconds is computed for 10 problems with a maximum of 150 nodes. 1000 trials are carried out for both the general and Levy options, and the simulations are kept under the same computing environment. The Levy version takes slightly more time than the other, as it generates random numbers using LFT (Table 15). The CPU time increases with varying rates (Fig. 2).

Table 15. CPU Time in Seconds for 10 TSP

| No. | Problem | Option-I | Option-II | Option-III | Option-IV | Option-V |
|---|---|---|---|---|---|---|
| 1 | ATT48 | 3.593 | 3.645 | 3.659 | 3.626 | 3.692 |
| | Levy1 | 3.802 | 3.684 | 3.775 | 3.816 | 3.732 |
| 2 | EIL51 | 4.202 | 4.286 | 4.186 | 4.169 | 4.134 |
| | Levy2 | 4.309 | 4.312 | 4.211 | 4.207 | 4.250 |
| 3 | BERLIN52 | 4.404 | 4.444 | 4.348 | 4.423 | 4.301 |
| | Levy3 | 4.409 | 4.536 | 4.466 | 4.444 | 4.353 |
| 4 | ST70 | 8.300 | 8.282 | 8.228 | 8.289 | 8.392 |
| | Levy4 | 8.364 | 8.464 | 8.315 | 8.371 | 8.418 |
| 5 | PR76 | 9.789 | 9.723 | 9.686 | 9.715 | 9.726 |
| | Levy5 | 9.805 | 9.813 | 9.747 | 9.842 | 9.823 |
| 6 | KROA100 | 18.475 | 18.191 | 18.128 | 18.147 | 18.077 |
| | Levy6 | 18.628 | 18.265 | 18.275 | 18.199 | 18.119 |
| 7 | EIL101 | 18.758 | 18.708 | 18.467 | 18.536 | 18.554 |
| | Levy7 | 18.977 | 18.870 | 18.812 | 18.653 | 18.590 |
| 8 | LIN105 | 20.544 | 20.319 | 20.326 | 20.151 | 20.164 |
| | Levy8 | 20.760 | 20.564 | 20.460 | 20.324 | 20.399 |
| 9 | CH130 | 33.867 | 33.326 | 34.129 | 33.318 | 33.250 |
| | Levy9 | 34.073 | 33.749 | 34.538 | 33.400 | 33.558 |
| 10 | CH150 | 47.479 | 47.453 | 47.508 | 47.547 | 47.021 |
| | Levy10 | 48.086 | 48.010 | 47.898 | 47.773 | 48.631 |

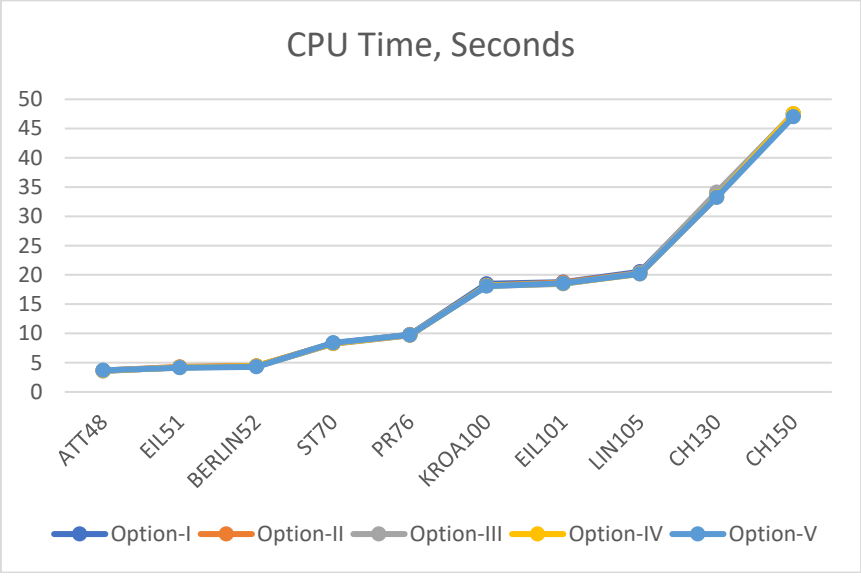



**Figure 2**. CPU Time in Seconds for 10 TSP

## 4. Permutation Flowshop Scheduling Problems (PFSSP)

Researchers have proposed several heuristics and metaheuristics for solving PFSSP, and they are available in the literature. The NEH algorithm is widely accepted as one of the best heuristics for minimising makespan in PFSSP. The NEH algorithm's insertion technique is a powerful strategy. The results outperform several heuristics, even when two jobs are randomly selected as the initial partial sequence [12].

Taillard benchmark instances [13] contain 120 flowshop instances divided into 12 groups. The number of jobs (n) is [20, 50, 100, 200, 500], and the number of machines (m) is [5, 10, 20].

This analysis considers the first 90 problems, and Table 16 presents the results along with those of NEH. In this simulation, iterations refer to independent trials.

The initial partial sequence of two jobs is randomly selected. One random sequence is generated using the expression "random series*LFT", and the jobs, except those two jobs, are inserted one by one at a place that minimises the partial makespan.

That is, unlike NEH, except for the insertion strategy, random numbers are used to generate new solutions.

$$\text{Average Percent Relative Deviation (APRD)} = ((\text{Reported Makespan-Known Upper Bound})*100)/(\text{Known Upper Bound}) \quad (2)$$

Table 16. Best Results for the First 90 Taillard Problems

| | | NEH | RITA | This Work, Option-III | |
|---|---|---|---|---|---|
| Size, m*n | Iterations/ Trials | APRD | APRD (Best) | APRD (Best) | Better Results than NEH for each Set of 10 Instances, % |
| 5*20 | 2000 | 3.30 | 0.144 | 0.185 | 32.26 |
| 10*20 | 4000 | 4.60 | 0.817 | 0.788 | 32.28 |
| 20*20 | 8000 | 3.73 | 0.532 | 0.425 | 30.42 |
| 5*50 | 5000 | 0.73 | 0.089 | 0.044 | 8.17 |
| 10*50 | 10000 | 5.07 | 2.568 | 2.293 | 27.94 |
| 20*50 | 5000 | 6.65 | 3.900 | 3.784 | 34.63 |
| 5*100 | 4000 | 0.53 | 0.144 | 0.056 | 13.17 |
| 10*100 | 2000 | 2.21 | 1.370 | 0.985 | 18.51 |
| 20*100 | 1000 | 5.34 | 4.167 | 4.033 | 20.81 |
| Mean | -- | 3.573 | 1.526 | 1.399 | 24.24 |

Except for the 5*20 dataset, the proposed approach yields better results in 8 out of 9 sets. For the 5*20 instance set, RITA [14] performs better. Overall, Option-III improves the APRD by 60.82% when compared with that of NEH, but at the cost of execution time.

The mean APRD for the best values reported in this simulation is 1.4% against 1.53% of RITA, an improvement of 8.5%. For any set of instances, Table 16 also shows how often the present strategy reports better results than NEH. The improvement ranges from 8.17% to 34.63%. Overall, for the 90 problems, better results are reported than the NEH makespan in 24.24% of cases.

## 5. Optimal Centres and TSP for 20 Cities of Slovakia

Geodetic coordinates (latitudes and longitudes) are used to estimate the Great Circle Distance (GCD) and the distance is the cost metric here to solve mini-sum, mini-max and TSP problem involving Slovakia.

For handling mini-sum and mini-max types of problems, a simple algorithm that generates a set of random numbers is formulated as follows:

$$\text{New Solution} = \text{Old Solution} + R*0.10*(-1+ 2*\text{Random_Numbers}) \quad (3)$$

Where, R = 1 - (Current_Iteration_Number/Maximum Number of Iterations) and its value reduces from 1 to 0 as the iteration progresses.

The strategy is, the geodetic coordinates are converted to GCD and this distance metric is optimized. Initial solution is taken as the arithmetic mean of the geodetic coordinates of the data points termed as the "Mass Centre".

To validate the new algorithm, the 10 points Baskar dataset [15] available in the literature is used. The new random-based algorithm is run for 1000 iterations and only one trial is conducted. The best results are compared with the known results and are presented in Table 17.

Table 17. Results on Baskar Dataset

| Problem | Algorithm | Facility | Total Distance, km (GCD) | Radius, km (GCD) |
|---|---|---|---|---|
| Mini-Sum | This Work | [1.1745, 1.0884] | 67835.9531 | -- |
| | FP-AB | [1.1745, 1.0884] | 67835.9531 | -- |
| | TP-AB | [1.1745, 1.0884] | 67835.9531 | -- |
| Mini-Max | This Work | [0.1327, 0.9765] | 78838.9049 | 12455.3517 |
| | FP-AB | [0.1327, 0.9765] | 78838.9137 | 12455.3558 |
| | TP-AB | [0.1327, 0.9765] | 78838.9045 | 12455.3516 |

For the mini-sum problem, the total distance from the optimal facility reported in this work is 67835.9531 km (GCD) which matches with the cost reported by the Four-Point Direction Search Algorithm (FP-AB) and the two-phase TP-AB Algorithm.

In case of the mini-max problem, the optimal radius reported in this work is 12455.3517 km (GCD) which is less than the cost reported by FP-AB algorithm but, slightly higher than that of TP-AB Algorithm.

To demonstrate its efficacy in solving real-world location science problems, the country of Slovakia is considered. Slovakia is a European Country that has about 197 km at its widest vertical span, East-to-West and about 429 km at its longest horizontal span. Total Area is approximately 49,035 square kilometres [16]. The country is divided into 8 regions which are further divided in to 79 districts.

The objective is to locate a central facility that has the minimum sum of distances from it to cover all 79 district headquarters by road.

For the Travelling Salesman Problem (TSP), 20 major cities are considered (Table 18) and their coordinates are collected. The codes of Option-III are slightly modified to solve TSP.

Table 18. Top 20 Major Cities of Slovakia and their Geodetic Coordinates

| S. No. | City | Latitude (rad) | Longitude (rad) |
|---|---|---|---|
| 1 | Bratislava | 0.840279 | 0.302442 |
| 2 | Košice | 0.849073 | 0.376991 |
| 3 | Prešov | 0.861422 | 0.370359 |
| 4 | Žilina | 0.860025 | 0.321141 |
| 5 | Nitra | 0.842897 | 0.319395 |
| 6 | Banská Bystrica | 0.849771 | 0.333881 |
| 7 | Trnava | 0.845864 | 0.306147 |
| 8 | Martin | 0.860549 | 0.324188 |
| 9 | Trenčín | 0.852041 | 0.315730 |
| 10 | Poprad | 0.864566 | 0.357967 |
| 11 | Prievidza | 0.846911 | 0.319744 |
| 12 | Zvolen | 0.846562 | 0.333183 |
| 13 | Považská Bystrica | 0.856929 | 0.317824 |
| 14 | Michalovce | 0.850644 | 0.385210 |
| 15 | Spišská Nová Ves | 0.861247 | 0.362854 |
| 16 | Komárno | 0.831136 | 0.317126 |
| 17 | Levice | 0.838533 | 0.319395 |
| 18 | Humenné | 0.859327 | 0.380322 |
| 19 | Nové Zámky | 0.835740 | 0.312588 |
| 20 | Bardejov | 0.866836 | 0.369486 |

Table 19. Centres of Slovakia and Optimal Tour for 20 Major Cities (Mass Center: [0.8502, 0.3367] radians): Option-III

| Problem | Iterations, Trials | Location | Total Distance, km (GCD) | Radius, km (GCD) |
|---|---|---|---|---|
| Mini-Sum [79 district head-quarters] | 20000 Iterations, 30 Trials | Banská Bystrica, Slovakia | 8488.3244 | -- |
| Coordinates | [0.8504, 0.3331] radians | [48.7228, 19.0850] degrees | -- | -- |
| Mini-Max [79 district head-quarters] | 20000 Iterations, 30 Trials | Hriňová, Slovakia | 8927.7570 | 191.3575 |
| Coordinates | [0.8482, 0.3422] radians | [48.5986, 19.6084] degrees | -- | -- |
| TSP [20 major cities] | 1000 Iterations, Single Trial | -- | 2758.5388 | -- |
| Best Tour | [14-2-18-3-20-15-10-12-6-8-4-13-9-11-5-17-7-1-19-16-14] | | | |

The results are presented in Table 19.

It is generally accepted that the geographical center of Slovakia is the peak of Hrb in the Poľana mountain range [17]. The location is at Poľana mountains, near the village of Strelníky (Central Slovakia) with coordinates: 48.7305939° N, 19.4570817° E.

In this analysis, two facilities are located; the Mini-Sum Center and the Mini-Max Center which are close to the geographical centre. Distance between the geographical centre and Mini-Sum Center = 27.309 km (GCD) whereas, the distance between the geographical centre and Mini-Max Center = 18.409 km (GCD).

For the TSP to cover 20 major cities of Slovakia, total distance to be covered is 2758.5388 km (GCD) and the best tour is [14-2-18-3-20-15-10-12-6-8-4-13-9-11-5-17-7-1-19-16-14], the numbers indicate the serial number of the city in Table 19.

The best tour is represented as an image as shown in Fig. 3. The latitudes are plotted in X-axis and the longitude in Y-axis.

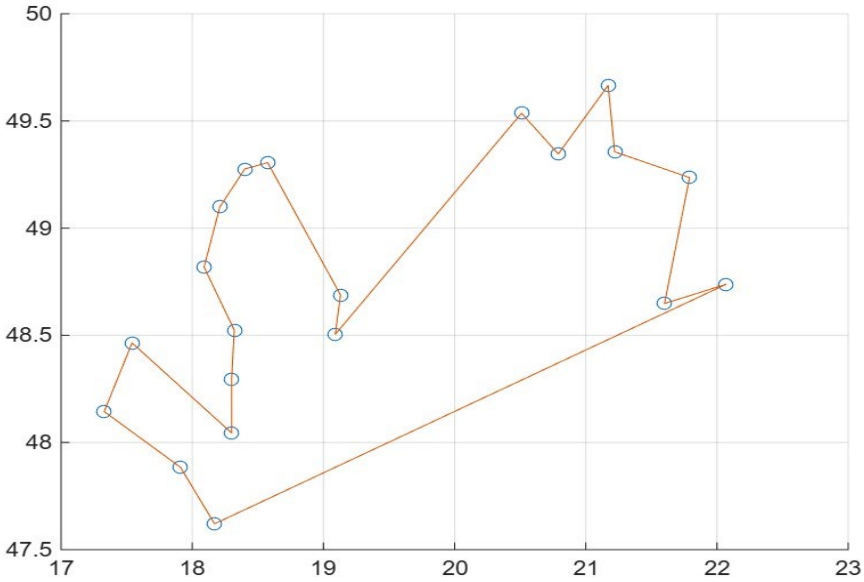



**Figure 3.** Best Tour to Cover 20 Major Cities of Slovakia

## 6. Ten Constrained Real-World Engineering Problems

Altay et al. [18] analysed a few constrained real-world engineering problems and compared the results with 17 popular algorithms namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA.

To assess the impact of random numbers, new solutions are generated using the expression,

$$\text{New Solution} = \text{Old Solution} + R*(-1+2*\text{Random_Numbers}).*\text{abs}(RL) \quad (4)$$

Where, R = 1 - (Current Iteration Number/Maximum Number of Iterations). "RL" is a set of random numbers generated using LFT and the absolute values are multiplied with another set of random numbers. The random algorithm is run for 30 trials of 30000 function evaluations each in line with the 17 algorithms and the minimum cost is evaluated.

Table 20. Performance of Random Algorithm in Solving Constrained Engineering Problems

| S.No. | Problem | Minimum Cost | Rank |
|---|---|---|---|
| 1 | Himmelblau's | -30665.5329654 | 7/17 |
| 2 | Spring | 0.0127066 | 10/17 |
| 3 | Speed Reducer | 2995.2167116 | 7/17 |
| 4 | Pressure Vessel | 6059.7185970 | 9/17 |
| 5 | Welded Beam | 1.7289701 | 15/17 |
| 6 | 3-Bar Truss | 263.8958605 | 6/13 |
| 7 | Cantilever Beam | 1.3399630 | 4/17 |
| 8 | Tubular Column | 26.4864228 | 7/17 |
| 9 | Piston Lever | 8.4155275 | 7/17 |
| 10 | Corrugated Bulkhead | 6.8437809 | 6/17 |
| | | Mean Rank | 7.8 |

The results are presented in Table 20 which shows that the mean rank of the simple random algorithm is 7.8 out of 17 algorithms considered. This shows that if random numbers are properly embedded in the updating expressions, the performance of the algorithm will be comparable with other better performing algorithms.

## 7. Conclusions and Future Work

This paper investigated five random-based insertion heuristics, designated Option-I to Option-V, that predominantly use random numbers, with and without the incorporation of the Lévy strategy. Because these heuristics are highly stochastic, we run multiple trials to obtain improved solutions. The performance of the proposed options is evaluated using the minimum, mean, and standard deviation of results across trials.

The initial analysis covers 12 Travelling Salesman Problem (TSP) instances, with problem sizes up to 783 nodes. Among the five options, Option-III showed the strongest overall performance among the proposed heuristics, achieving better results in 28 cases across the considered statistical measures and a sum of ranks of 118. The results from Option-III are then compared with those of several established algorithms reported in the literature. The comparison shows that Option-III yields better results than several of the algorithms considered.

To further evaluate the applicability of Option-III, we solve 90 Taillard Flowshop Scheduling Problems and compare the results with those obtained using the NEH heuristic and the RITA algorithm. The results indicate an

improvement of approximately 60% over NEH. However, this improvement comes with increased computational time as the number of trials increases. Compared with the stochastic RITA algorithm, Option-III improves by approximately 8.5%.

We demonstrate the applicability of the proposed approach using three real-world location-science problems associated with Slovakia: mini-sum, mini-max, and TSP. These experiments further exhibit the potential of the proposed random-based insertion heuristics, particularly Option-III, across different classes of combinatorial optimization problems.

Finally, 10 constrained real-world engineering problems are solved using simple random algorithm and compared with 17 popular algorithms. The results are encouraging.

Future research will focus on investigating additional variants of optimization problems, exploring further combinations of randomization and Lévy-based strategies, and examining approaches for improving computational efficiency while retaining solution quality.

## References


[1] Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. Advances in engineering software. 2014 Mar 1;69:46-61.

[2] Rao RV, Savsani VJ, Vakharia DP. Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. Computer-aided design. 2011 Mar 1;43(3):303-15.

[3] Kennedy J, Eberhart R. Particle swarm optimization. InProceedings of ICNN'95-international conference on neural networks 1995 Nov 27 (Vol. 4, pp. 1942-1948). ieee.

[4] Baskar A. Simple population-based algorithms for solving optimization problems. RAIRO-Operations Research. 2024 Jan;58(1):253-80.

[5] Yang XS, Deb S, He X. Eagle strategy with flower algorithm. In2013 international conference on advances in computing, communications and informatics (ICACCI) 2013 Aug 22 (pp. 1213-1217). IEEE.

[6] Nawaz M, Enscore Jr EE, Ham I. A heuristic algorithm for the m-machine, n-job flow-shop sequencing problem. Omega. 1983 Jan 1;11(1):91-5.

[7] Symmetric traveling salesman problems, https://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/tsp/tspindex.html [Accessed on 01-08-2026]

[8] Chandomí-Castellanos E, Escobar-Gómez EN, Torres JA, Medina Santiago A, López Zapata BY, Arizaga Silva JA, Roberto-Bermúdez J, Vázquez-Delgado HD. A Hybrid Heuristic Algorithm for the Traveling Salesman Problem with Structured Initialization in Global–Local Search. Algorithms. 2026 Apr 22;19(5):324.

[9] Uddin F, Riaz N, Manan A, Mahmood I, Song OY, Malik AJ, Abbasi AA. An improvement to the 2-opt heuristic algorithm for approximation of optimal TSP tour. Applied Sciences. 2023 Jun 20;13(12):7339.

[10] Baskar A, Xavior MA, Jeyapandiarajan P, Batako A, Burduk A. A novel two-phase trigonometric algorithm for solving global optimization problems. Annals of Operations Research. 2026 Sep;364(2):1387-437.

[11] Asani EO, Okeyinka AE, Ajagbe SA, Adebiyi AA, Ogundokun RO, Adekunle TS, Mudali P, Adigun MO. A novel insertion solution for the travelling salesman problem. Computers, Materials & Continua. 2024 Apr 25;79(1):1581-97.

[12] Baskar A, Xavior MA. Analysis of job insertion technique for different initial sequences in permutation flow shop scheduling problems. International Journal of Enterprise Network Management. 2015;6(3):153-74.

[13] Taillard E. Benchmarks for basic scheduling problems. european journal of operational research. 1993 Jan 22;64(2):278-85.

[14] Amaladosan B, Michael AX, Burduk A, Butdee S, Machado J. A Randomized Insertion-Based Trigonometric Algorithm (RITA) for Permutation Flowshop Scheduling Problems. InInternational Conference on Intelligent Systems in Production Engineering and Maintenance 2025 Jun 24 (pp. 33-47). Cham: Springer Nature Switzerland.

[15] Baskar A, Anthony Xavior M. Solving Facility Location and Supply Chain Management Problems Using Modified Population-Based TP-AB Algorithm: Supply Chain Management Problems. EAI Endorsed Transactions on Digital Transformation of Industrial Processes, 1 (1).(2025) [Internet].

[16] Travel Guide Slovakia, https://www.natucate.com/en/destinations/travel-guide-slovakia [Accessed on 04-09-2026]

[17] Geographical Center of Slovakia, https://mapy.com/en/?source=base&id=1914065 [Accessed on 04-09-2026].

[18] Altay EV, Altay O, Özçevik Y. A comparative study of metaheuristic optimization algorithms for solving real-world engineering design problems. CMES-Computer Modeling in Engineering and Sciences. 2023 Dec 30;139(1):1039-94.