

A Short Literature on Linear Programming Problem

^{1,*} Shubham Kumar Tripathi, and ¹R Kumar

¹VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India

Abstract

Researchers and scientists have developed various approaches and methodologies over time to model and analyze different types of linear programming problems, such as assignment problems and parametric programming problems. This paper provides a critical review and classification of existing modelling approaches and solution methods related to linear programming problems. Moreover, the simplex method is discussed in detail through a comprehensive literature review. The paper concludes by presenting an integrated research framework that is directly applicable to the present context, along with suggestions for future research directions.

Keywords: operational research, Linear Programming Problem, simplex Method

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*Corresponding author: shubham.vit22@gmail.com

1. Introduction

During World War II [1939-1944], Watt and Rowe developed Operational Research in response to the need to manage scarce resources [1]. This discipline involves the use of quantitative and qualitative tools, such as mathematical modeling, optimization, simulation, statistics, and computer science, to analyze complex systems and find solutions to problems. Its applications are diverse, ranging from transportation [2], healthcare [3], and supply chain management [4], to risk management [5], cost reduction in manufacturing [6], energy management systems [7], and call center operations [8]. According to Taha [9], Operations Research is "The application of scientific methods, techniques, and tools to problems involving the operations of systems so as to provide those in control of the operation with optimum solutions to the problems." Its primary objectives include improving efficiency, reducing costs, and increasing effectiveness. Table 1 and Fig. 1 below highlight some of the significant contributions to Operational Research.

Table 1. Influence of operational research in the real-life application by the different researchers in different areas.

Authors	Year	Applications	Significance
Thompson and Zawack [10]	1985	Job scheduling problem	The objective of this manuscript is to apply parametric programming techniques to solve the problem of job scheduling.
Yu and Lui [11]	2006	Non-convex spectrum of a multi-carrier system.	The goal is to find the best solution for a multi-carrier system's nonconvex spectrum.
Sulaiman, and Hamadameen [12]	2008	Multi-objective optimization problems	The objective of this paper is to address the problem of solving multi-objective optimization problems.
Mohsenian-Rad et al. [13]	2010	Energy consumption	This study focuses on the problem of scheduling energy consumption in the context of a smart grid.

Vilaplana et al. [14]	2014	Cloud-based service	The purpose of this research is to gain insight into about the service quality in cloud computing.
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A pictorial representation of the Table.1 is shown in Fig.1

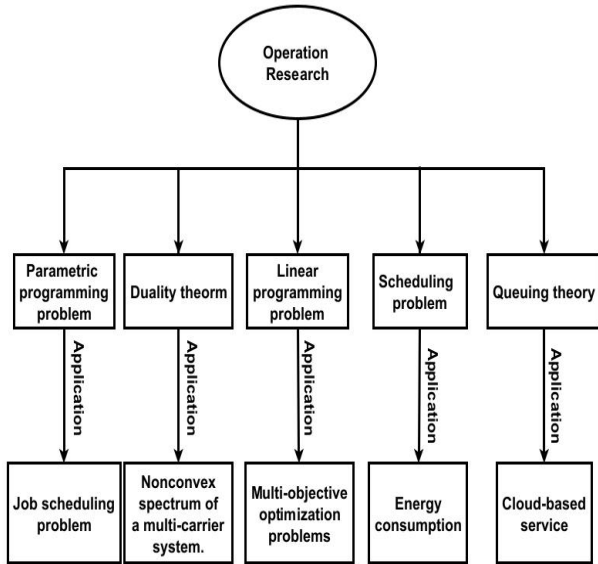


Fig.1 Pictorial representation of Table 1

Overall, Operation research plays an important role in decision-making and problem-solving across many industries and sectors. LPP is a common topic in Operation research which is discussed in detail in the below paragraph.

In this paper, our main aim is to discuss the recent trends and applications of LPP. Linear programming (LP) was invented in the late 1930s by Soviet mathematician Leonid Kantorovich and independently by the American mathematician George Dantzig in 1947 [1]. It has many applications in various fields, including economics [15], finance [16], engineering [17], transportation [18], and so on. According to V.K. Kapoor [19], "Linear Programming is the process of optimizing (maximizing or minimizing) a linear function subject to linear constraints." Additionally, we have also discussed some of the major contributions to the LPP in below Table 2 and Fig. 2.

Table 2: Influence of linear programming problem in the real-life application by the different researchers in different areas.

Author s	Year	Method ologies	Applicati ons	Significance
Adlak ha et al. [20]	2006	Heuristi c algorithm	Transpor tation problem	The purpose of the implementation is to provide a solution for more-for-less paradox in transportation problem with mixed restrictions.

Wang et al. [21]	2008	Simple x method	Bilevel programming problem	Their approach to solving the linear-quadratic bilevel programming problem involves the use of the Simplex Method.
Bedekar et al. [22]	2009	Dual simplex method	Overcurrent relays problem	Bedekar et al. have utilized the dual simplex method to obtain coordination of overcurrent relays in the distribution system.
Kim and Dong [23]	2014	Hungarian method	Power allocation problem	The objective of this article is to find the optimal solution for power allocation in a relay-aided device-to-device communication system under a cellular network.
Karimi et al. [24]	2015	Branch and Bound Method	Supply chain schedule problem	Implementation of branch and bound technique to get a batch delivery schedule for the multi-factory supply chain.

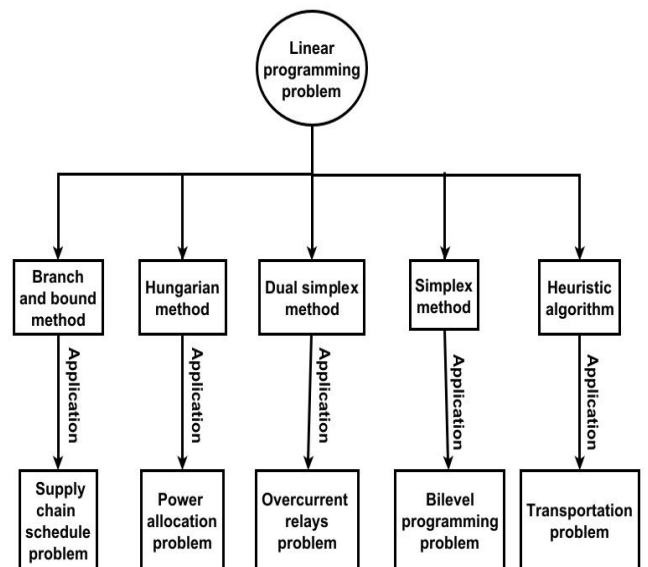


Fig.2 Pictorial representation of Table 2

Overall, Linear programming problem is an important tool in Operation research because it provides a systematic and rigorous approach to solving optimization problems. Additionally, we can say that the Linear programming problem (LPP) is a mathematical model that seeks to optimize (maximize or minimize) a linear objective function subject to a set of linear constraints. The variables in the Linear programming model represent the decision variables, which are the quantities to be determined. After reviewing the introduction, it is evident that there are several gaps in the study of the linear programming problem. Limited research has been conducted on linear programming related issues that arise in various real-world applications. In light of this, our study aims to address these gaps by discussing recent trends and advancements in linear programming problems.

The primary objective of our study is to provide insights into evolutionary methods commonly used in linear programming. We aim to highlight key aspects of linear programming, such as the transportation problem and duality problem, to provide researchers and students with a comprehensive understanding of linear programming models from a single review paper. This summary of linear programming models will assist researchers in discovering and encouraging advancements and developments in the field of linear programming problems. In this review article, we cover a comprehensive survey of important aspects of the simplex linear programming model. We also discuss the existence of simplex linear programming problems and their sub-areas in real-life problems.

The structure of the paper is as follows: Section 2, we provide an overview of the simplex method and discuss related research on its trending extensions. We also summarize the contributions made by various researchers on these models in a tabular form. Section 3 presents a summary of our conclusions regarding the study of the simplex method.

2. Discuss some of the Recent Trends and advancements in Simplex Theory

Hedi Nabli [25](2009) examines variations working models of simplex algorithm and suggests two novel approaches. The first novelty is the introduction of the formal tableau, which makes it simple to derive the dual solution from the most recent primal table without distinguishing between slack and the original variables. Furthermore, a new alternative to the dual-simplex algorithm is derived using the formal tableau. By the approach, the dual is solved using the revised simplex algorithm, and the primal is obtained by use of the formal tableau. To avoid using big-M numbers or arbitrary variables, a Non-Feasible Basis approach (NFB) is offered as an alternative for starting the simplex algorithm. Computational experiments on problems with varying numbers of constraints and decision variables demonstrate the effectiveness of the proposed NFB method, which is computationally more efficient than the two-phase and big-M methods. Then lately, Boonperm and Sinapiromsaran [26] introduced a new approach i.e., splitting constraints into three groups to solve the simplex method.

For the simplex algorithm, Boonperm and Sinapiromsaran [26](2014) suggested a method that forgoes the usage of artificial variables. Their approach entails projecting the issue using objective plane and categorizing the restrictions into three groups according to the fixed variable's coefficient. Initially, the

technique uses restrictions from the positive coefficient group to solve the issue along the objective direction. If the positive coefficient group is nonempty, then loosens restrictions from the negative and zero coefficient groups. They provide an example problem to compare their method to the simplex algorithm. Boonperm and Sinapiromsaran's [26] method solves the problem in two iterations, while the simplex algorithm uses fake variables to solve the problem in two phases. Thus, their method decreases the number of variables and constraints to solve the problem by implementing few parameters than the simplex algorithm. Further, Nabli and Chahdoura [27] developed a new simplex algorithm efficiency approach.

Nabli and Chahdoura (2015) [27] proposed simplex algorithm without the use of artificial variables. This method can detect infeasibility and redundant constraints. To achieve feasibility, it can be combined with the NFB method. However, the basis obtained by this method is generally not feasible, which can be resolved by using the non-feasible basis method (NFB). Additionally, Nabli and Chahdoura [27] (2015) propose a new pivoting rule for NFB that improves its numerical and time complexity. To help decide which algorithm to use between NFB and formal non-feasible basis method (FNFB), we also develop an efficient criterion. Nabli and Chahdoura [27] (2015) also propose a more efficient pivoting rule for NFB. Additionally, they also developed the primal-formal non-feasible basis method using the formal tableau notion. Hence, it is observed that Nabli and Chahdoura [27] (2015) method helps us to reduce the number of iterations as well as time. Finally, Sangngern and Boonperm [28] (2020) proposed another different way to handle the simplex method.

Sangngern and Boonperm [28](2020) developed a new simplex method with the initial basis, that considers the angle between the objective function coefficient vectors and each constraint. This method uses restrictions with modest angles to generate an initial basis close to the ideal answer. The non-feasible basis method avoids artificial variables if the initial basis yields infeasible solutions. By using linear programming problems, computational experiments are randomly generated. Sangngern and Boonperm [28](2020) technique, the Two-phase method, Big-M method and non-feasible basis method solved the problems. Sangngern and Boonperm [28](2020) technique outperformed other issues by demonstrating its potential for tackling real-world optimization problems.

The statement provides an overview of various approaches to handle the simplex algorithm efficiently. The methods comprise by introducing novel ideas like the formal tableau and the non-feasible basis method, grouping constraints into three categories, creating an initial basis by measuring the objective function using coefficient vectors, and implying new pivoting rules. These approaches aim to reduce the number of iterations, time, and the use of artificial variables required to solve linear programming problems. Computational experiments have demonstrated the effectiveness of these approaches and their potential usefulness for solving real-world optimization problems.

Conclusion

Linear programming problems, such as assignment problems and parametric programming problems, have been the subject of extensive research by scientists and researchers. This paper critically reviews and classifies existing modelling approaches

and solution methods related to linear programming problems. The simplex method is discussed in detail through a comprehensive literature review. Additionally, the paper proposes an integrated research framework that is relevant to the present context and offers suggestions for future research directions. We aim to conduct a thorough investigation of the current literature on fuzzy theories, including concepts like triangular, trapezoidal, and others. We believe that relying solely on classical theory is insufficient for effectively handling uncertainty. By focusing on this perspective, our objective is to explore the literature on fuzzy theory in greater depth, enhancing our understanding in this field.

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