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Improvement of Efficiency of Inverters in Hydro Photovoltaic Power Station with Particle Swarm Optimization

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Abstract

In the sparsely populated areas without electricity, the hydro photovoltaic power station is a feasible solution for electricity supply. The strategy of distributing the power among the inverters is critical to the efficiency of them. The conventional distributing strategies result in low efficiency of the inverters. In order to improve the efficiency, this paper analysed the loss and efficiency characteristics of the inverter and expressed the power distributing problem as an optimal control problem minimizing the total loss for the inverters. The optimal control problem was solved with particle swarm optimization and the efficiency optimum power distribution strategies in three operation scenarios were obtained. The quantitative analysis method was adopted to evaluate the effect of the efficiency optimum power distribution strategies. The total efficiency of the inverters with the optimal strategies and the conventional strategies were calculated respectively. The optimal distribution strategies were compared quantitatively with conventional power distribution strategies on the basis of the efficiency. The results demonstrated the validity of the strategies obtained in this paper in improving the total efficiency of the inverters.

Keywords: efficiency, power distribution, particle swarm optimization

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1. Introduction

Electricity is indispensable for modern society. It facilitates every aspect of daily life of the people. However, it is unavailable to more than 774 million people around the world yet in 2022 according to the data of International Energy Agency [1]. Most of them live in sparsely populated remote areas, where the construction and operation of public power systems are not economically feasible [2, 3]. However, many of these areas have abundant renewable energy resources, such as solar energy and hydro energy. It is a feasible solution for supplying electricity to these areas to construct hybrid power stations consisting of small hydro power stations and photovoltaic (PV) systems [4-6].

The PV systems in the hydro-PV hybrid power station operate in quite different way from those in conventional grid-connected PV systems, where they operate in maximum power point tracking (MPPT) mode to harvest maximum electrical power from the PV arrays and the inverters of the PV systems convert the direct current (DC) power to the alternating current (AC) power and inject all the power into the grid as current sources [7, 8]. Although the output power of the grid-connected PV systems fluctuates with the condition of the weather, the detrimental impact on the grid is negligible because the capacity of the PV systems is very small compared with that of the grid [9]. In the hydro-PV power station, the proportion of PV systems is relatively big and the fluctuation of the solar power cannot be neglected. So the energy storage system (ESS) is indispensable for the stability of the hydro-PV power station [10]. With the introduction of the ESS, the output power of the PV inverters in the power station does not fluctuate with the variation of the solar radiation any more. Its output power is dispatched by the energy management system (EMS) of the PV system to implement active power balance of the power station [11, 12]. The difference between the instantaneous output power of the inverters and the PV array is balanced by charging or discharging the ESS. The EMS plays an important role in

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the hydro-PV power station. It controls the operation of the PV systems such as selection of the operation mode, the strategies for connection or disconnection of the inverters, and the distribution of the output power among the inverters according to the status of the loads and the power sources.

The efficiency of the inverter plays an important role in reducing the total loss of the power station. Many factors, such as the output power, input voltages, and the output voltage, affect the efficiency of the inverter. Among the factors mentioned above, the output power has the most significant influence on the efficiency.

Generally, the sum of the power ratings of all the inverters in the power station is larger than the total power demand on the PV systems. So the EMS has many different strategy choices to distribute the power among the inverters, which results in different efficiency characteristics. A common strategy is to distribute the output power among the inverters in proportion to their power ratings. Another common strategy is to distribute the power among the inverters on average. The third common strategy is to give the priority of power distribution to the inverter with the most energy in the ESS. None of these strategies guarantees optimal overall efficiency because the efficiency characteristic of the inverter is not considered in the power distribution. In order to solve the problem, this paper researches the output power distribution strategy of the inverters to maximize the efficiency of them and is organized as follows. After brief introduction in this section, section 2 discusses the architecture and operation modes of the hydro-PV power station and analyses the efficiency characteristic of the PV inverter. Section 3 proposes the efficiency optimal distribution strategy of the inverters in the power station and solves the problem with particle swarm optimization (PSO) algorithm. Section 4 demonstrates and verifies the validity of the proposed strategy under three practical operation scenarios. Section 5 concludes the whole paper.

2. Hydro-PV Power station and Inverter Efficiency

2.1. Architecture of the Power station

As shown in Fig.1, the hydro-PV power station consists of the hydro power station, the PV systems, the load and the power station energy management system (EMS). Generally, there are one hydro power station and several PV systems in a typical hydro-PV power station. The PV system consists of the PV array, the DC/DC PV controller, the ESS battery, the PV inverter, and the EMS. The scheduling system of the power station determines the output power of the hydro power station and the PV systems to achieve power balance and other operation objectives. The EMS distributes the output power to each inverter.

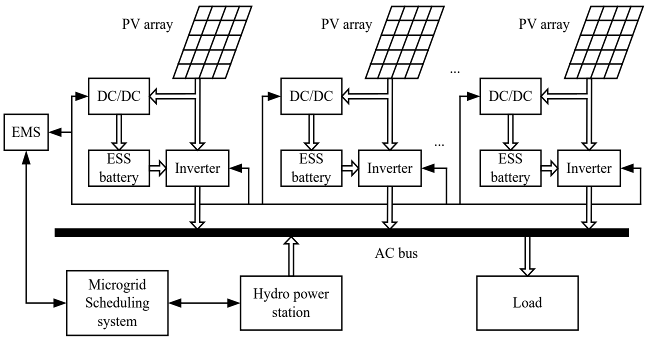

Figure 1. Architecture of hydro-PV power station

2.2. Loss and Efficiency Analysis of Inverter

For simplicity, the efficiency can be regarded as the function of the output power P_o. A typical efficiency curve of the inverter is shown in Fig.2.

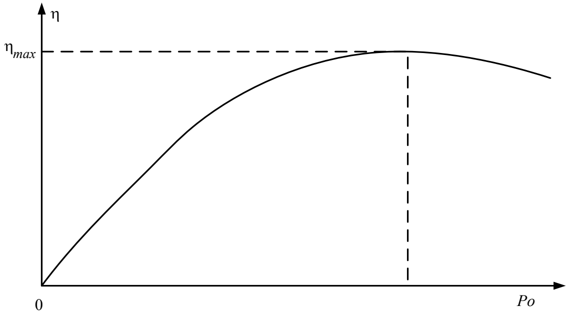

Figure 2. Typical inverter efficiency

The efficiency of the inverter increases gradually from zero at zero output power to the peak, then decreases with the increase of the output power. The reason of such a characteristic is because there are two kinds of loss in the inverter. The first category is the invariable or constant loss, which means that loss in this category keeps invariable or constant when the output power of the inverter varies. The core loss of the magnetic components is the main part of the constant loss. The loss in control circuit, driver circuit, protection circuit, and filter capacitors can be classified into this category too. The second category is the variable loss, which means that the loss in this category varies with the output power of the inverter. Losses in this category mainly include copper loss of the magnetic components, conduction loss and switching loss of the power electronics devices. The overall function relationship between the variable loss and the output power is complex and hard to be express accurately in analytic formulas. But the main part of the variable loss is resistive loss. If the voltage of the power station keeps constant, the output power varies linearly with the output power while the loss is approximately proportional to the square of the output current. So the efficiency curve exhibits the characteristic of increasing first and then decreasing.

The analysis above explains the variation tendency of the efficiency of the inverter. Different inverters have different efficiency curves. Accurate efficiency characteristic can be obtained by testing at different operational points and curve fitting. The general principle of power distribution is to make the inverters operate as close as possible to their maximum efficiency points to achieve optimal efficiency.

3. Efficiency Optimization of Inverters

3.1. Optimal Control Model of Efficiency

From the perspective of the EMS, the task of the efficiency optimization of the inverters is to select the strategy which minimizes the total loss of all the inverters in the hydro-PV power station from all the possible strategies that follow the instruction of the EMS and satisfy the constraints imposed by the inverters and the ESS [13]. Assume that there are n inverters to be distributed output power to, with the state of the charge of the ESS batteries as the state variables $x = [x_1 \quad x_2 \quad \dots \quad x_n]^T$ and the output power distribution instructions to the n inverters as the control input variables $u(t) = [u_1(t), \quad u_2(t), \quad \dots, \quad u_n(t)]^T$, the efficiency optimization issue can be regarded as an optimal control problem shown in Eq.1.

$$
\begin{aligned}
&\min J(u(k)) = \sum_{k=0}^{N-1}\sum_{i=1}^{n}\left(1-\eta_i(u_i(k))\cdot u_i(k)\right)\\
&s.t.\\
&\quad x(k+1) = A_d x(k) + B_d(k)u(k)\\
&\quad = x(k) + \begin{bmatrix} -\dfrac{T_s}{\eta_1(u_1(k))} & 0 & \dots & 0 \\ 0 & -\dfrac{T_s}{\eta_2(u_2(k))} & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & -\dfrac{T_s}{\eta_n(u_n(k))} \end{bmatrix} \cdot u(k)\\
&\quad x(0) = x_0\\
&\quad x(N-1) = x_f \in [x_{f,\min}, x_{f,\max}]\\
&\quad x_{f,\min} = 0\\
&\quad x_{f,\max} = x_0\\
&\quad \sum_{k=0}^{N-1} u(k) = P_r\\
&\quad 0 \le x(k) \le x_0\\
&\quad 0 \le u(k) \le P_m
\end{aligned}
\tag{1}
$$

x_0 and x_f are the initial and final values of the state variables, i.e. the remaining energy in the ESS batteries, respectively. $x_{f,\min}$ and $x_{f,\max}$ are allowable minimum and maximum values respectively. P_m is a column vector whose elements are output power ratings of the inverters. P_r is the total output power of the PV systems, which is being distributed among the inverters. The distribution strategy can be divided into $N = t / T_s$ steps, where T_s is the time interval of distribution adjustment.

3.2. Solution for Optimal Efficiency

Traditionally, optimal control issue can be solved with the calculus of variations or dynamic programming [14-16]. But the calculus of variations is difficult to be applied to the researched issue because of its nonlinearity and complexity [17]. Although the dynamic programming can solve the issue, it is computationally intensive and time-consuming.

The intelligent algorithms have an advantage over the traditional methods mentioned above in solving the time-varying, nonlinear, and control constrained input constrained optimal control problems for their simplicity and adaptability. Particle swarm optimization (PSO) is one of the most widely used intelligent algorithms.

The PSO was proposed by Kennedy and Eberhart in 1995 [18]. Since the proposal, it has been used in many different fields in the electric engineering to solve optimization problems. It uses a number of particles simulating the foraging behaviour of the bird swarm to find approximate solutions to the researched problem. Every particle has two attributions, i.e. velocity v and position p. The fitness function in the PSO is used to evaluate the fitness values of all the particles. According to the evaluation results, the best position of individual particle p_{best} and the best position of the whole swarm g_{best} can be obtained by comparing the fitness function values of the particles. The particles move around in the search space and keep adjusting their movement states according to the best position of every individual particle p_{best} and the best position of the whole swarm g_{best}. p_{best} and g_{best} are updated in every iteration. The methods with which particle i adjusts its velocity v and position p in the nth iteration are shown in Eq.2 and Eq.3 respectively. In order to balance the exploration capability and the exploitation capability of the PSO, it is necessary to set a reasonable velocity range by limiting the maximum velocity of the particle.

$$
\begin{aligned}
v_i(n+1) = \omega(n)\cdot v_i(n) &+ c_1\cdot r_1\cdot(p_{best,i}(n) - p_i(n))\\
&+ c_2\cdot r_2\cdot(g_{best,i}(n) - p_i(n))
\end{aligned}
\tag{2}
$$

$$
p_i(n+1) = p_i(n) + v_i(n+1) \tag{3}
$$

ω is the inertial weight, which affects the performance of the PSO significantly. Rational values of ω benefit the balance between the exploration and exploitation of the PSO. There are many different inertial weight strategies such as constant inertial weight, linear decreasing inertia weight, and dynamic inertial weight. Linear decreasing inertia weight strategy is adopted for its good global optimization ability in the initial stage and local optimization ability in the final stage. With this strategy, the inertial weight decreases linearly with the increase of the number of the iteration. The inertial weight in the nth iteration is calculated according to Eq. 4.

$$\omega(n) = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{N-1}(n-1) \tag{4}$$

$\omega_{\max}$ is the maximum of the inertial weight, which is adopted in the first iteration. $\omega_{\min}$ is the minimum of the inertial weight, which is adopted in the Nth iteration.

c_1 and c_2 are acceleration factors related to p_{best} and g_{best} respectively. The two factors allow the particles approach the optimal position by using individual experience and exchanging information among the particles.

r_1 and r_2 are two random numbers between 0 and 1.

The flow chart of PSO is shown in Fig. 3. In the beginning, the parameters of the inverters, power instructions, and the state of the ESS are input; the population number and parameters of the PSO are configured; and the position and velocity of the particles are initialized. The position is an $n \times m$ matrix, where n is the number of the inverter taking part in the output power dispatching and m is the number of the step during the output power dispatching. The elements of the matrix are output power of the inverters. The elements in line 1 to line n-1 are output power of inverter 1 to inverter n-1 in every dispatching step respectively. They are initialized with random numbers between zero and the power ratings of the inverters. The elements in line n are the output power of inverter n in every dispatching step and are determined by subtracting the sum of output power of inverter 1 to n-1 from the power output instruction rather than generated with random numbers to meet the total power output requirement.

Then the fitness of every particle is calculated. In the issue to be solved, the fitness is defined as the total loss of the inverters corresponding to the power dispatching strategy, which is the position of the particle. It is calculated by accumulating the loss of the inverters in each step during the whole dispatching process.

If the dispatching strategy results in appropriate output power of the last inverter, for example, less than zero or greater than the power rating, or results in appropriate remaining energy of the ESS out of the range from $x_{f,\min}$ to $x_{f,\max}$, it will be considered as an invalid one and the fitness of it will be added a penalty which is much larger than normal loss. If the dispatching strategy results in appropriate output power of the last inverter, for example, less than zero or greater than the power rating, or results in appropriate remaining energy of the ESS out of the range from $x_{f,\min}$ to $x_{f,\max}$, it will be considered as an invalid one and the fitness of it will be added a penalty which is much larger than normal loss.

After calculating the fitness, the PSO algorithm update the optimal position and fitness of each particle and whole swarm and check it the convergence condition is satisfied. The convergence condition is set as the distance between the best position of the whole swarm g_{best} and the best position of all the individual particle p_{best} is less than a tiny positive number. The distance is set as the 1-norm of the difference matrix of the best global position matrix and the best local position matrix.

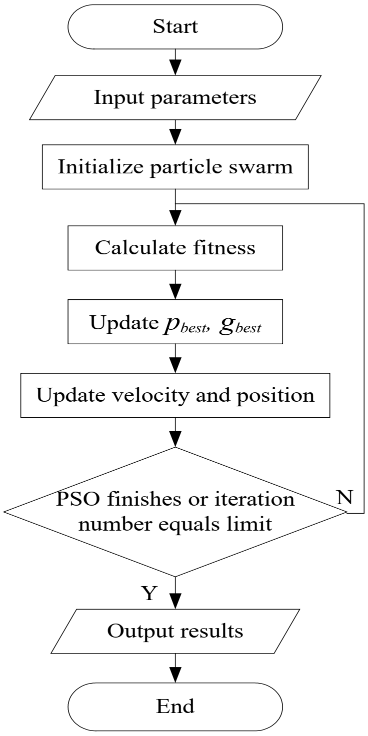

Figure 3. Flow chart of PSO

If the PSO has finished converging or the iteration number equals the limit, the program breaks the loop. Otherwise, it continues iterating until the terminating condition is met. At last, the optimized results or failure information is displayed.

4. Simulation Results

In order to verify the validity of the proposed optimal control model and its particle swarm optimization solution of the optimal efficiency of the inverters in the hydro-PV hybrid power station, a simulation program is implemented according to the method in Section 3. The researched power station has 3 PV inverters, whose power ratings are 50kW, 100kW, and 120kW respectively. The efficiencies of every inverter at different output power points are shown in Table 1. The output power in the table is the per unit value with the base values are the power ratings of the corresponding inverters respectively.

Table 1. Efficiency of the inverters

Output power(p.u.)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Inverter 1 (%)	0	75.2	86.6	91.3	92.3	92.9	93.2	93.9	95.2	94.3	93.4
Inverter	0	78.	87.	92.	93.	94.	95.	95.	96.	96.	95.

2 (%)		0	4	5	7	1	0	8	1	3	3
Inverter 3 (%)	0	79.2	88.5	94.0	95.1	96.1	96.9	97.6	98.1	97.8	96.9

The efficiencies in Table 1 only have values at some discrete points. The continuous efficiency function is obtained with linear interpolation between two adjacent power points whose efficiencies are known.

The output power instruction to the PV systems is [46.7, 77.3, 114.6, 203.1, 245.5, 236.4, 221.0, 193.7, 161.2, 69.1] kW. The output power to be distributed is adjusted every half hour and lasts for 5 hours.

The initial electrical energy of the ESS of the inverters is 200kWh, 350kWh, and 400kWh respectively. In the first scenario, there is no special requirement on the remaining energy of the ESS in the end of the operation as long as it is greater than zero. The simulation results of the efficiency optimum distribution strategy and the remaining energy in the ESS are shown in Table 2.

Table 2. Power distribution and remaining energy with optimal efficiency strategy in scenario 1

Inverter	Output power(kW)										Remaining energy(kWh)
1	10.0	11.3	15.0	40.0	40.0	40.0	40.0	15.0	40.0	10.0	60.5
2	10.0	30.0	30.0	67.1	90.0	90.0	85.0	82.7	30.0	30.0	63.0
3	26.7	36.0	69.6	96.0	115.5	106.4	96.0	96.0	91.2	29.1	6.9

The key parameters of the PSO during the simulation are that population size is 100; learning factor c_1 is 2.0; learning factor c_2 is 2.0; max inertia weight is 1.0; and min inertia weight is 0.5. The total loss of the inverters with the efficiency optimal dispatching strategy is 35.3kWh and the overall efficiency of the inverters is 95.69%.

If the output power was dispatched to the inverters in proportion to the power ratings of them, the dispatching strategy and remaining energy in ESS are shown in Table 3. The total loss of the inverters with the proportionally dispatched strategy is 37.9kWh and the overall efficiency of the inverters is 95.39%, which is 0.3% lower than that with the optimal efficiency dispatching strategy.

Table 3. Power distribution and remaining energy with proportionally dispatched strategy

Inverter	Output power(kW)										Remaining energy(kWh)
1	8.6	14.3	21.2	37.6	45.5	43.8	40.9	35.9	29.9	12.8	44.5
2	17.3	28.6	42.5	75.2	91.0	87.6	81.9	71.7	59.7	25.6	44.0
3	20.8	34.4	50.9	90.3	109.1	105.0	98.2	86.1	71.6	30.7	39.3

It is to be observed that the third inverter was prioritized with the efficiency optimal distribution strategy in Table 2, which results in low remaining energy in the ESS of it. In some cases, there exists constraint on the minimum remaining energy in the ESS.

In the second scenario, this issue is addressed and minimal remaining energy of ESS of the inverters is 30kWh. The efficiency optimal distribution strategy with PSO is shown in Table 4.

The PSO parameters in the simulation are the same with those in scenario 1. The total loss of the inverters with the optimal dispatching strategy is 35.7kWh and the overall efficiency of the inverters is 95.64%, which is 0.25% higher than that with the proportionally dispatched strategy in Table 3, which is one possible solution to the issue with the minimum remaining energy constraint.

Table 4. Output power and remaining energy with optimal efficiency strategy in scenario 2

Inverter	Output power(kW)										Remaining energy(kWh)
1	14.7	11.3	40.0	40.0	41.0	40.0	40.0	20.2	15.0	5.0	57.4
2	20.0	30.0	38.6	70.0	90.0	90.0	85.0	77.5	55.4	28.1	42.6
3	12.0	36.0	36.0	93.1	114.5	106.4	96.0	96.0	90.8	36.0	30.0

The third scenario is that there is the upper limit constraint on the remaining energy of the ESS. This scenario applies to the situation when the ESS is required to keep large capacity for storing energy from the PV array in the next step. In the simulation, the upper limit constraint on the remaining energy of the ESS of the first is 15kWh and there are no upper limit constraints on remaining energy of the ESS of the other two inverters. With the same PSO parameters used in prior two scenarios, the resulted optimal efficiency dispatching is shown in Table 5. The total loss of the inverters with the optimal dispatching strategy is 36.9kWh and the overall efficiency of the inverters is 95.51%.

Table 5. Output power and remaining energy with optimal efficiency strategy in scenario 3

Inverter	Output power(kW)										Remaining energy(kWh)
1	24.7	25.3	40.0	31.6	42.6	40.0	40.0	40.0	40.0	35.1	10.0
2	10.0	10.0	62.6	75.5	90.0	90.0	85.0	57.7	30.0	10.0	74.3
3	12.0	42.0	12.0	96.0	112.9	106.4	96.0	96.0	91.2	24.0	43.4

In order to satisfy the constraint on the remaining energy of the ESS of the first inverter, a simple strategy is to prioritize the first inverter during dispatching output power. The output power dispatched to the first inverter equals its power rating as long as the power rating is not greater than the power reference command and the remaining energy of its ESS is greater than the maximum constraint. The other two inverters share other load and the output power is dispatched between them in proportion to their power ratings. The output power and remaining energy with this strategy are shown in Table 6.

Table 6. Output power and remaining energy with conventional dispatching strategy in scenario 3

Inverter	Output power(kW)										Remaining energy(kWh)
1	46.7	50.0	50.0	50.0	50.0	50.0	50.0	0.0	0.0	0.0	14.5
2	0.0	13.65	32.3	76.55	97.75	93.2	85.5	96.85	80.6	34.55	29.0
3	0.0	13.65	32.3	76.55	97.75	93.2	85.5	96.85	80.6	34.55	80.2

The total loss of the inverters with this dispatching strategy is 36.9kWh. So the overall efficiency of the inverters is 95.51%. Comparing the two sets of data of the optimal efficiency dispatching strategy and conventional dispatching strategy in scenario 3, it is can be drawn that the optimal efficiency dispatching strategy has 5.1kWh lower loss and 0.59% higher efficiency than the conventional dispatching strategy.

5. Conclusion

This paper researched the efficiency improvement of the inverters in the hydro-PV power station. The concept, advantages were introduced and operation modes of the hydro-PV power station was discussed. Then the efficiency characteristic of the PV inverter was analysed. The maximum efficiency condition of the inverter was obtained on the basis of the loss analysis. The general principle of optimal efficiency operation of the inverters was proposed. Based on the optimal control theory, the optimal efficiency output power dispatching of the inverters is expressed as a time-varying, non linear, and input-constrained optimal control problem. The particle swarm optimization was adopted to solve the optimal efficiency output power dispatching problem of output power of the inverters. Simulations were conducted to verify validity of the proposed optimal control model and its PSO solution of it in three different scenarios. The simulated results showed that the proposed model of optimal efficiency of the inverter output power dispatching and its PSO solution can improve the overall efficiency of the PV inverters by 0.25-0.59% compared with that of the conventional dispatching strategy.

Acknowledgements

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