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Frontier and Semifrontier Sets in Intuitionistic Topological Spaces

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Abstract

The notions of frontier and semifrontier in intuitionistic topology have been studied and several of their properties are proved. Many counter examples have been pointed out for the relevant classifications.

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1. Introduction

Atanassov [1], in 1986, established the fundamentals of intuitionistic fuzzy set as a generalization of fuzzy sets of Zadeh [12] on the degree of membership and non membership. The fundamentals of intuitionistic topological spaces was instigated by Coker[4], in the year 2000. Intuitionistic sets ($IS's$) have been applied in areas of science and technology. Salama [8] has used intuitionistic topology (IT) for studying land cover changes. Considering the inherent nature of Geographic Information Science (GIS) phenomina, it seems more suitable to study the problem of land cover changes using intuitionistic fuzzy topology. For recasting the GIS problem in terms of intuitionistic topology, the study of intuitionistic frontier is necessary.
This paper provides the notion of intuitionistic frontier and its properties in intuitionistic topological spaces $ITS(X)$. By intuitionistic semiopen sets [5], the notion of intuitionistic semifrontier is defined and we characterize intuitionistic semi continuous functions with reference to intuitionistic semi frontier. Counter examples given herein are constructed upon the intuitionistic topological space defined by Coker[4].

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2. Intuitionistic Frontier

Definition : 2.1.[2] Consider a nonempty set as X_1. An IS A , having the form $A =< X_1, A_1, A_2 >$, where A_1 and A_2 are subsets of X_1 satisfying $A_1 \cap A_2 = \underset{\sim}{\phi}$. The set of members of A is A_1, and the set of non members is A_2. The set of all ITS in X_1 is denoted as ITS(X_1).
Definition : 2.2.[4] The nonempty set X_1 and A,B are IS's in the form A=$< X_1, A_1, A_2 >$, B=$< X_1, B_1, B_2 >$ respectively. Then
(a) A⊆B iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
(b) A = B iff A⊆B and B⊆A.
(c) $\overline{A} = < X_1, A_2, A_1 >$.
(d) $A - B = A \cap \overline{B}$.
(e) $\underset{\sim}{\phi} = < X_1, \phi, X_1 >$, $\underset{\sim}{X} = < X_1, X_1, \phi >$.
(f) A∪B = $< X_1, A_1 \cup B_1, A_2 \cap B_2 >$.
(g) A∩B = $< X_1, A_1 \cap B_1, A_2 \cup B_2 >$.
Definition : 2.3.[4] An IT on a nonempty set X_1 is a family τ of IS's in X_1 satisfying the following axioms:
(a) $\underset{\sim}{\phi}, \underset{\sim}{X} \in \tau$
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.
In this case, the pair (X, τ) *is called an intuitionistic topological space* (ITS *for short*) *and any intuitionistic set in* τ *is known as an intuitionistic open set* (IOS *for short*) *in X.*
Definition : 2.4. [4] Let (X, τ) be an intuitionistic topological space ($ITS(X)$) and $A =< X, A_1, A_2 >$ be an IS in X. Then the interior and closure of A are defined

by
$Icl(A) = \cap\{K : K \text{ is an ICS in X and } A \subseteq K\}$
$Iint(A) = \cup\{G : G \text{ is an IOS in X and } G \subseteq A\}$.
It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A) = A and is an IOS in X iff Iint(A) = A.
Definition : 2.5.[2] (a) Let X be a nonempty set and p $\in$ X, a fixed element in X. Then the IS $\underset{\sim}{p}$ defined by $\underset{\sim}{p} = < X, \{p\}, \{p\}^c >$ is called an intuitionistic point (IP for $short$) in X.
(b) Let $\underset{\sim}{p}$ be an IP in X and A = $< X, A_1, A_2 >$ an IS in X. Then $\underset{\sim}{p}$ is said to be contained in A ($\underset{\sim}{p} \in A$ for short) if and only if p $\in A_1$.
Definition : 2.6.[4] (a) Let (X, τ) and (Y, Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each IS in Φ is an IS in τ.
(b) Let (X, τ) and (Y, Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be open iff the image of each IS in τ is an IS in Φ.
Definition : 2.7.[5] Let (X, τ) be an ITS(X). An intuitionistic set A of X is said to be intuitionistic semiopen if $A \subseteq Icl(Iint(A))$. The collection of all intuitionistic semiopen sets are denoted by ISOS(X). The complement of every intuitionistic semiopen set is intuitionistic semiclosed set and the collection of all intuitionistic sets are denoted by ISCS(X).
Definition : 2.8.[5] Let (X, τ) be an intuitionistic topological space and $A =< X, A_1, A_2 >$ be an IS in X. Then the intuitionistic semiinterior and intuitionistic semiclosure of A are defined by
$Iscl(A) = \cap\{K : K \text{ is an ISCS in X and } A \subseteq K\}$
$Isint(A) = \cup\{G : G \text{ is an ISOS in X and } G \subseteq A\}$.
It can be shown that Iscl(A) is an ISCS and Isint(A) is an ISOS in X and A is an ISCS in X iff Iscl(A) = A and A is an ISOS in X iff Isint(A) = A.
Definition : 2.9.[4] A, B, C and A_i be intuitionistic sets in $X(i \in J)$. Subsequently
(a) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$
(b) $A_i \subseteq B$ for each $i \in J \Rightarrow \cup A_i \subseteq B$
(c) $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \cap A_i$
(d) $(\cup A_i)^c = \cap A_i^c$
(e) $(\cap A_i)^c = \cup A_i^c$
$(f$ $A \subseteq B \Leftrightarrow B^c \subseteq A^c$
(g) $(A^c)^c = A$
$(h$ $(\underset{\sim}{\phi})^c = \underset{\sim}{X}$ and
(i) $(\underset{\sim}{X})^c = \underset{\sim}{\phi}$
Definition : 2.10.[11] Let (X, τ) and (Y, Φ) be two intuitionistic topological spaces and let $f : X \to Y$ be a function. Then f is said to be intuitionistic semi continuous, if the inverse image of every intuitionistic open set of (Y, Φ) is intuitionistic semi open in (X, τ).
Proposition : 2.11.[4] Let X be a nonempty set and let A,B are intuitionistic sets in the form A = $< X, A_1, A_2 >$, B = $< X, B_1, B_2 >$ respectively. Then
(a) Icl$(A \cup B) = Icl(A) \cup Icl(B)$
(b) Iint$(A \cap B) = Iint(A) \cap Iint(B)$
(c) Icl$(A \cap B) \subset Icl(A) \cap Icl(B)$
(d) Icl$(A \cup B) \supset Icl(A) \cup Icl(B)$.

3. Intuitionistic Frontier

Definition: 3.1. Consider (X_1, μ) be an ITS(X_1) and K $\in IS(X_1)$. Then $\underset{\sim}{q} \in IFrP(X_1)$ is called an intuitionistic frontier point $(IFrP)$ of K if $\underset{\sim}{q} \in Icl(K) \cap Icl(K^c)$. The union of all IFrPs of K is termed as an IFrP of K and is represented by $IFr(K)$. It is clear that $IFr(K) = Icl(K) \cap Icl(K^c)$.
Proposition : 3.2. Each IS M in $X_1, M \cup IFr(M) \subset Icl(M)$.
Proof: Let M be an IS in X_1 and Icl$(M) = (\underset{\sim}{X_1} - Icl(X_1 - M))$ and Icl$(M^c) = Icl(X_1 - M)$. Also IFr$(M) = Icl(M) \cap Icl(M^c) = (\underset{\sim}{X_1} - Icl(\underset{\sim}{X_1} - M)) \cap (Icl(\underset{\sim}{X_1} - M)$ $= \underset{\sim}{\phi}$. So M$\cup IFr(M) = M$. Since M$\subset Icl(M)$, M $\cup IFr(M) \subset Icl(M)$.
Remark: 3.3. Equality concept cannot be replaced in Proposition 3.2.
Example:3.4. Consider $X_1 = \{a_1, b_1, c_1\}$ with intuitionistic topology $\mu = \{\underset{\sim}{X_1}, \underset{\sim}{\phi}, < X_1, \{c_1\}, \{b_1\} >, < X_1, \{a_1, c_1\}, \phi >\}$. For $M =< X_1, \{a_1, c_1\}, \phi >$, $Icl(M) = \underset{\sim}{X_1}$ and IFr$(M) =< X_1, \phi, \{a_1, c_1\} >$.
But M$\cup IFr(M) =< X_1, \{a_1, c_1\}, \phi > \neq Icl(M)$.
Theorem:3.5. For an ITS(X_1, μ), the following results hold.
(a) IFr$(M) = IFr(M^c)$.
(b) If M is an ICS then IFr$(M) \subseteq M$.
(c) If M is an IOS then IFr$(M) \subseteq M^c$.
(d) $(IFr(M))^c = Iint(M) \cup Iint(M^c)$.
Proof:(a) $IFr(M) = Icl(M) \cap Icl(M^c) = Icl(M^c) \cap Icl(M)$ $= Icl(M^c) \cap Icl((M^c))^c = IFr(M^c)$.
(b) Considering as M be an ICS in X_1, IFr$(M) = Icl(M) \cap Icl(M^c) \subseteq Icl(M) = M$. Hence IFr$(M) \subseteq M$.
(c) M is an IOS implies M^c is ICS. By (b), $IFr(M^c) \subseteq M^c$, and by $(a)$$IFr(M) \subseteq M^c$.
(d) $(IFr(M))^c = (Icl(M) \cap Icl(M^c))^c = (Icl(M))^c \cap (Icl(M^c))^c = Iint(M^c) \cup Iint(M)$.
Remark:3.6. In general, the converse of (b) and (c) of Theorem 3.5 is not satisfied.
Example:3.7. Let $X_1 = \{a_1, b_1, c_1\}$ with intuitionistic topology $\mu = \{\underset{\sim}{X_1}, \underset{\sim}{\phi}, < X_1, \{a_1, b_1\}, \{c_1\} >, < X_1, \{a_1\}, \phi >,$
$< X_1, \{a_1, c_1\}, \phi >, < X_1, \phi, \{a_1, b_1\} >, < X_1, \{a_1\}, \{b_1\} >,$
$< X_1, \phi, \{c_1\} >, < X_1, \{a_1, b_1\}, \phi >, < X_1, \{a_1\}, \{c_1\} >,$
$< X_1, \{a_1\}, \{b_1, c_1\} >, < X_1, \phi, \phi >, < X_1, \{b_1\}, \phi >,$
$< X_1, \{b_1\}, \{a_1\} >, < X_1, \phi, \{b_1, c_1\} >, < X_1, \{c_1\}, \{b_1\} >,$
$< X_1, \{a_1, c_1\}, \{b_1\} >, < X_1, \{b_1, c_1\}, \{a_1\} >, \ < X_1, \phi, \{a_1\} >,$

$< X_1, \{c_1\}, \phi >< X_1, \{c_1\}, \{a_1, b_1\} >, < X_1, \phi, \{a_1, c_1\} >, < X_1, \phi, \{b_1\} >\}$.$(i)$ Let M=$< X_1, \{b_1\}, \{a_1, c_1\} >$, then IFr$(M)$= $< X_1, \phi, \{a_1, c_1\} >$ which implies IFr$(M)\subseteq$ M but M $\nsubseteq IFr(M)$. (ii) Let N=$< X_1, \{a_1\}, \{c_1\} >$, then IFr$(N)$ = $< X_1, \phi, \{a_1, c_1\} >$ which implies, IFr$(N)\subseteq$ N^c but N$^c \nsubseteq IFr(N)$.

Theorem:3.8. If M and N be IS's in an ITS(X_1), then IFr$(M \cup N) \subseteq IFr(M) \cup IFr(N)$.

Proof:If $IFr(M \cup N) = Icl(M \cup N) \cap Icl(M \cup N)^c$
$\subseteq (Icl(M) \cup Icl(N)) \cap (Icl(M^c) \cap Icl(N^c))$
$\subseteq ((Icl(M) \cup Icl(N)) \cap (Icl(M^c))) \cap ((Icl(M) \cup Icl(N)) \cap Icl(N^c))$
$\subseteq ((Icl(M) \cap Icl(M^c)) \cup (Icl(N) \cap Icl(M^c))) \cap ((Icl(M) \cap Icl(N^c)) \cup (Icl(M) \cap Icl(N^c)))$
$\subseteq (IFr(M) \cup (Icl(N) \cap Icl(M^c))) \cap ((Icl(M) \cap Icl(N^c)) \cup IFr(N))$
$\subseteq (IFr(M) \cup IFr(N)) \cap ((Icl(N) \cap Icl(M^c)) \cup (Icl(M) \cap Icl(N^c)))$
$\subseteq IFr(M) \cup IFr(N)$.

Converse of Theorem 3.8, does not hold.

Example:3.9. Let $X_2 = \{a_2, b_2, c_2\}$ with intuitionistic topology $\underset{\sim}{\mu} = \{X_2, \underset{\sim}{\phi}, < X_2, \{c_2\}, \{b_2\} >, < X_2, \{a_2, c_2\}, \phi >\}$ and let$A_2 =< X_2, \{a_2, c_2\}, \phi >, B_2 =< X_2, \{a_2, b_2\}, \{c_2\} >$. Then IFr$(A_2) =< X_2, \phi, \{a_2, c_2\} >$, IFr$(B_2)$= $< X_2, X_2, \phi >$, and IFr$(A_2 \cup B_2)$= $< X_2, \phi, X_2 >$, IFr$(A_2) \cup IFr(B_2)$= $< X_2, X_2, \phi >$ which implies IFr$(A_2 \cup B_2) \subseteq$ IFr$(A_2) \cup IFr(B_2)$ but IFr$(A_2) \cup IFr(B_2) \nsubseteq$ IFr$(A_2 \cup B_2)$.

Theorem:3.10. Let A_1 and B_1 be two IS's in an ITS(X_1), $IFr(A_1 \cap B_1) \subseteq (IFr(A_1) \cap Icl(B_1)) \cup (IFr(B_1) \cap Icl(A_1))$.

Proof:Consider $IFr(A_1 \cap B_1) = Icl(A_1 \cap B_1) \cap Icl(A_1 \cap B_1)^c$
$\subseteq ((Icl(A_1) \cap Icl(B_1)) \cap (Icl(A_1^c) \cup Icl(B_1^c)))$
$\subseteq ((Icl(A_1) \cap Icl(B_1)) \cap Icl(A_1^c)) \cup ((Icl(A_1) \cap Icl(B_1)) \cap Icl(B_1^c))$
$\subseteq (IFr(A_1) \cap Icl(B_1)) \cup (IFr(B_1) \cap Icl(A_1))$.

The reverse process of Theorem 3.10. does not satisfied.

Example:3.11. Consider $X_3 = \{11, 22\}$ with intuitionistic topology $\mu = \{\underset{\sim}{X_3}, \underset{\sim}{\phi}, < X_3, \phi, \{22\} >, < X_3, \{11\}, \{22\} >\}$ and let$A_3 =< X_3, \{22\}, \{11\} >, B_3 =< X_3, \{11\}, \{22\} >$. Then $IFr(A_3) =< X_3, \{22\}, \{11\} >$, IFr$(B_3) =< X_3, \{22\}, \{11\} >$ and IFr$(A_3 \cap B_3) = \underset{\sim}{\phi}$ which implies $IFr(A_3) \cap Icl(B_3) =< X, \{22\}, \{11\} >$ and $IFr(B_3) \cap Icl(A_3) = \underset{\sim}{\phi}$ implies IFr$(A_3 \cap B_3) \subseteq (IFr(A_3) \cap Icl(B_3))\cup$(IFr$(B_3) \cap Icl(A_3))$ but $(IFr(A_3)\cap$Icl$(B_3)) \cup (IFr(B_3) \cap Icl(A_3)) \nsubseteq$ IFr$(A_3 \cap B_3)$.

Theorem:3.12. An intuitionistic continuous mapping be $h : (X_1, \mu) \rightarrow (Y_1, \nu)$ then IFr$(h^{-1}(B_1))\subseteq h^{-1}(IFr(B_1))$ in any IS B_1 in Y_1.

Proof: Let h is intuitionistic continuous. B_1 be an IS in Y_1. Then
IFr$(h^{-1}(B_1)) = Icl(h^{-1}(B_1)) \cap Icl(h^{-1}(B_1))^c$
$\subseteq Icl(h^{-1}(Icl(B_1))) \cap Icl(h^{-1}(Icl(B_1)^c))$
$\subseteq h^{-1}(Icl(B_1)) \cap h^{-1}(Icl(B_1)^c) = h^{-1}(Icl(B_1) \cap Icl(B_1^c))$
$\subseteq h^{-1}(IFr(B_1))$.

Lemma : 3.13.LetA$_4$ and B_4 are two intuitionistic sets, $A_4 \subseteq B_4$ and B$_4$is ICS(X), then IFr$(A_4) \subseteq B_4$.

Proof. Since $A_4 \subseteq B_4$ implies $Icl(A_4) \subseteq Icl(B_4)$, which implies $IFr(A_4) = Icl(A_4) \cap Icl(A_4^c) \subseteq Icl(B_4) \cap Icl(B_4^c) \subseteq Icl(B_4) = B_4$.

Theorem:3.14. Consider $h_1 : X_2 \rightarrow Y_2$ be an IO mapping, B_2 be an IS in Y_2. Then $h_1^{-1}(IFr(B_2)) \subseteq$ IFr$(h_1^{-1}(B_2))$.

Proof:Suppose h_1 is an IO function, B_2 is an IS in Y_2. Let $A_2 = Icl(IFr(h_1^{-1}(B_2)))$. Then A_2 is IO, therefore $h_1(A_2)$ is IO in Y_2. This gives $Icl(h_1(A_2)) \in ICS(Y_2)$. This follows B$_2\subseteq Icl(h_1(A_2))$. By Lemma 3.13, $h_1^{-1}(IFr(B_2)) \subseteq h_1^{-1}(Icl(h_1(A_2))) \subseteq Icl(A_2) = Icl(Icl(IFr(h_1^{-1}(B_2)))) = IFr(h_1^{-1}(B_2))$.
Consequently, IFr$(h_1^{-1}(B_2)) \subseteq IFr(h_1^{-1}(B_2))$.

4. Intuitionistic Semi Frontier

Levine [6] generalized the notion of open sets as semiopen sets. The generalized work was helpful to develop a wider framework for the study of continuity and its different variants.

Definition:4.1. Consider (X_4, μ) be an ITS(X_4), $M \in IS(X_4)$. Also $\underset{\sim}{q} \in IFrP(X_4)$ is defined as intuitionistic semifrontier point $(IsFrP)$ of M if $\underset{\sim}{q} \in Iscl(M) \cap Iscl(M^c)$. The union of all the IsFrPs of M is termed as an intuitionistic semifrontier of M. It can be noted as $(IsFr(M))$. Also IsFr(M) = Iscl$(M) \cap Iscl(M^c)$ holds.

Theorem: 4.2. For IS's M and N in an ITS(X_4),
(a) Iscl$(Iscl(M))$=Iscl(M)
(b) Isint$(Isint(M)) = Isint(M)$
(c) Isint$(M)\cup$Isint(N) $\subseteq$Isint$(M \cup N)$
(d) Isint$(M \cap N)$ = Isint$(M) \cap Isint(N)$
(e) Iscl$(M \cup N)$=Iscl$(M) \cup Iscl(N)$
(f) Iscl$(M \cap N) \subseteq Iscl(M) \cap Iscl(N)$.

Proof:(a) M is ISC [5] iff $M = Iscl(M)$. Since Iscl(M) is ISC, Iscl$(Iscl(M))$=Iscl(M).
(b) Since Isint(M) is ISO and M is ISO iff $M = Isint(M)$, therefore $Isint(Isint(M)) = Isint(M)$.
(c) Isint(M) and $Isint(N)$ are both ISO sets and M$\subseteq$ M$\cup$N, N$\subseteq$ M$\cup$N implies Isint$(M) \subseteq$ Isint$(M \cup N)$ and Isint$(N) \subseteq$ Isint$(M \cup N)$. This implies Isint$(M)\cup$Isint(N) $\subseteq$Isint $(M \cup N)$.
(d) As M$\cap$N is an intuitionistic subset of M and M$\cap N$ is an intuitionistic subset of N implies Isint$(M \cap N) \subseteq$ Isint$(M)\cap$Isint(N). Conversely Isint$(M) \subseteq$ M and Isint$(N) \subseteq$ N implies Isint$(M)\cap$ Isint$(N) \subseteq M\cap$N and Isint$(M)\cap$Isint(N) is an ISO set. But Isint $(M \cap N)$ is the biggest ISO set contained in M$\cap$N. Hence Isint$(M)\cap$Isint$(N) \subseteq M\cap$N. This gives the equality.
(e) Since M is an intuitionistic subset of $M \cup N$ and N is an intuitionistic subset of $M \cup N, Iscl(M) \subset$

$Iscl(M \cup N)$ and $Iscl(N) \subset Iscl(M \cup N)$ because M$\subset N \Rightarrow Iscl(M) \subset Iscl(N)$. Hence Iscl$(M) \cup Iscl(N) \subset Iscl(M \cup N) \longrightarrow (1)$.
Since $Iscl(M), Iscl(N)$ are ISC sets, $Iscl(M) \cup Iscl(N)$ is also intuitionistic closed. Also $M \subset Iscl(M)$ and $N \subset Iscl(N)$ implies that $M \cup N \subset Iscl(M) \cup Iscl(N)$. Since Iscl$(M \cup N)$ is the smallest ISC set containing $M \cup N$, $Iscl(M \cup N) \subset Iscl(M) \cup Iscl(N) \longrightarrow (2)$. From (1) and (2), $Iscl(M \cup N) = Iscl(M) \cup Iscl(N)$.
(f) Hence M$\cap$N $\subseteq M$ and M$\cap$N $\subseteq N$ implies Iscl$(M \cap N) \subseteq$ Iscl(M) also Iscl$(M \cap N) \subseteq$ Iscl(N) implies Iscl$(M \cap N) \subseteq Iscl(M) \cap Iscl(N)$.
Theorem:4.3. Let M be an intuitionistic set in an ITS(X_4), the following statement holds.
(a) IsFr(M)= IsFr(M^c).
(b) If M_1 is ISC, then IsFr$(M_1) \subseteq M_1$.
(c) Suppose M_2 is ISO, then IsFr$(M_2) \subseteq M_2^c$.
(d) Let$M \subseteq$N and N$\in ISC(X)$(resp. N$\in ISO(X)$) then IsFr$(M) \subseteq N$(resp. IsFr$(M) \subseteq N^c$) where ISC(X)(resp. ISO(X)) denotes the ISC(ISO resp.) sets in X.
(e) $(IsFr(M_3))^c = Isint(M_3) \cup Isint(M_3^c)$.
(f) IsFr$(M_4) \subseteq IFr(M_4)$.
(g) Iscl$(IsFr(M)) \subseteq IFr(M)$.
Proof:(a) Let $\underset{\sim}{q} \in IsFr(M) \Leftrightarrow$ every intuitionistic neighbourhood (Inhd shortly)[2] of $\underset{\sim}{q}$intersects both M and M^c $\Leftrightarrow$ every Inhd of $\underset{\sim}{q}$ intersects both $(M^c)^c$ and M^c, because $(M^c)^c = M \Leftrightarrow \underset{\sim}{q} \in IsFr(M^c)$.
Proof of $(b), (c), (d)$ and (e) are analogous of Theorem 3.8.
(f) Since Iscl$(M_4) \subseteq Icl(M_4)$ and Iscl(M_4^c) $\subseteq$Icl(M_4^c), then it gives IsFr(M_4)=Iscl$(M_4)\cap$ $Iscl(M_4^c) \subseteq$ Icl$(M_4)\cap$Icl(M_4^c) = IFr(M_4).
(g)Iscl$(IsFr(M)) = Iscl(Iscl(M)(M^c)) \subseteq$ Iscl$(Iscl(M))\cap$Iscl$(Iscl(M^c))$=Iscl(M)$\cap Iscl(M^c) = IsFr(M)\subseteq IFr(M)$.
Example:4.4. Let$X_4 = \{a_4, b_4, c_4\}$ with $\mu = \{\underset{\sim}{X}, \underset{\sim}{\phi}, < X_4, \phi, \{b_4, c_4\} >, < X_4, \phi, \{c_4\} >, < X_4, \{a_4, b_4\}, \{c_4\} >, < X_4, \{a_4, b_4\}, \phi >\}$,
(i) ISC sets are $\{\underset{\sim}{X_4}, \underset{\sim}{\phi}, < X_4, \phi, \phi >, < X_4, \phi, \{a_4\} >, < X_4, \phi, \{c_4\} >, < X_4, \phi, \{a_4, b_4\} >, < X_4, \{b_4\}, \{c_4\} >, < X_4, \{c_4\}, \phi >, < X_4, \{c_4\}, \{b_4\} >, < X_4, \{b_4, c_4\}, \phi >\}$ and $M_1 =< X_4, \phi, \{c_4\} >, IsFr(< X_4, \phi, \{c_4\} >) = < X_4, \phi, \{b_4, c_4\} >$. This implies IsFr$(M_1) \subseteq M_1$ but$M_1 \nsubseteq IsFr(M_1)$.
(ii) ISO sets are $\{\underset{\sim}{X_4}, \underset{\sim}{\phi}, < X_4, \phi, \phi >, < X_4, \phi, \{b_4\} >, < X_4, \phi, \{c_4\} >, < X_4, \phi, \{b_4, c_4\} >, < X_4, \{c_4\}, \{b_4\} >, < X_4, \{c_4\}, \phi >, < X_4, \{a_4, b_4\}, \{c_4\} >, < X_4, \{a_4, b_4\}, \phi >\}$, IsFr$< X_4, \phi, \{b_4, c_4\} > = < X_4, \phi, \{b_4, c_4\} >$ implies IsFr$(< X_4, \phi, \{b_4, c_4\} >) \subseteq M_2^c$ but $M_2^c \nsubseteq IsFr(M_2)$.
(iii) From Example 3.11., Iscl$(IsFr(M)) =< X_3, \phi, \{11\} >$ implies Iscl$(IsFr(M)) \subseteq IFr(M)$ but IFr$(M) \nsubseteq Iscl(IsFr(M))$.
(iv) From Example 3.11, let $M =< X_3, \{11\}, \phi >$, IsFr$(M) =< X_3, \phi, \{11\} >$, IFr$(M) = < X_3, \{22\}, \{11\} >$ implies IsFr$(M) \subseteq IFr(M)$ but IFr$(M) \nsubseteq IsFr(M)$.
Theorem: 4.5. Let M be an IS in an $ITS(X_4)$. Then
(a) IsFr(M_4) = Iscl$(M_4) - Isint(M_4)$.
(b) IsFr$(Isint(M_4)) \subseteq IsFr(M_4)$.
(c) IsFr$(Iscl(M_4)) \subseteq IsFr(M_4)$.
(d) Isint$(M_4) \subseteq M_4 - IsFr(M_4)$.
Proof: (a) Let IsFr(M_4) = Iscl$(M_4) \cap Iscl(M_4^c) = Iscl(M_4) \cap Iscl(\underset{\sim}{X}_4 - M_4) = Iscl(M_4) \cap (Isint(M_4))^c = Iscl(M_4) - Isint(M_4)$.
(b) $IsFr(Isint(M_4)) = Iscl(Isint(M_4)) \cap Iscl(Isint(M_4)^c) \subseteq Iscl(M_4) \cap Iscl(M_4^c) \subseteq IsFr(M_4)$.
(c) IsFr$(Iscl(M_4)) = Iscl(Iscl(M_4)) \cap Iscl(Iscl(M_4)^c) \subseteq Iscl(M_4) \cap Iscl(M_4^c) \subseteq IsFr(M_4)$, by Theorem 4.2.
(d) Let $M_4 - IsFr(M_4) = M_4 - (Iscl(M_4) - Isint(M_4)) \supseteq Isint(M_4)$.
Example:4.6.$i)$ In Example 3.11, let $M_4 =< X_3, \phi, \{1\} >$, $Isint(M_4) = \underset{\sim}{\phi}, IsFr(Isint(M_4)) = \underset{\sim}{\phi}$ and $IsFr(M_4) = < X_3, \phi, \{11\} >$. Thus $IsFr(M_4) \nsubseteq$ IsFr$(Isint(M_4))$.
ii) Let $X_5 = \{a_5, b_5, c_5\}, \mu = \{\underset{\sim}{X_5}, \underset{\sim}{\phi}, < X_5, \phi, \{c_5\} >, < X_5, \{c_5\}, \phi >, < X_5, \{a_5, b_5\}, \{c_5\} >, < X_5, \phi, \{b_5, c_5\} >\}$.
Let $M_4 =< X_5, \{c_5\}, \{b_5\} >$, then $IsFr(Iscl(M_4)) = < X_5, \phi, \{b_5, c_5\} >, IsFr(M_4) =< X_5, \{c_5\}, \{b_5\} >$. This implies $IsFr(M_4) \nsubseteq IsFr(Iscl(M_4))$.
iii) Let $X_4 = \{12, 22\}$with intuitionistic topology $\mu = \{\underset{\sim}{X_4}, \underset{\sim}{\phi}, < X_4, \{12\}, \{22\} >, < X_4, \phi, \{22\} >\}$.
Let $M_4 =< X_4, \phi, \{12\} >$, then $Isint(M_4) = \underset{\sim}{\phi}$. $M_4 - IsFr(M_4) =< X_4, \phi, \{12\} >$. This implies $M_4 - IsFr(M_4) \nsubseteq Isint(M_4)$.
Theorem: 4.7. Let M and N be IS's in an ITS(X_6). Then IsFr$(M \cup N) \subseteq IsFr(M) \cup IsFr(N)$.
Proof:Similar to Theorem 3.8. and converse of Theorem 4.7. need not be true.
Example:4.8. In an ITS$(X_6) = \{a_6, b_6, c_6\}$ with IT $\mu = \{\underset{\sim}{X_6}, \underset{\sim}{\phi}, < X_6, \{a_6\}, \{b_6\} >, < X_6, \{a_6, c_6\}, \phi >\}$. Let M=$< X_6, \{a_6\}, \{b_6\} >$, N=$< X_6, \{a_6, c_6\}, \phi >$. Then IsFr$(M \cup N) =< X_6, \phi, \{a_6, c_6\} >$ and IsFr$(M)\cup$ IsFr$(N) =< X_6, \{b_6\}, \{a_6\} >$. This implies IsFr$(M \cup N) \subseteq$ IsFr$(M) \cup IsFr(N)$ but IsFr$(M) \cup IsFr(N) \nsubseteq IsFr(M \cup N)$.

5. Conclusions

In this paper, the development of intuitionistic frontier and its various properties in intuitionistic topological spaces are studied. Also notions of semifrontier in intuitionistic topology have been studied and several of their properties are proved.

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