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DETERMINATION OF MANPOWER MODEL FOR A SINGLE GRADE SYSTEM WITH TWO SOURCES OF DEPLETION AND TWO COMPONENTS FOR THRESHOLD

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Abstract

In this paper, an organization with single grade subjected to exodus of personnel due to policy decisions taken by it, is considered. Here the problem of time to recruitment for this organization is analysed when the breakdown threshold, a level of maximum allowable manpower depletion, has two components namely normal threshold of depletion of manpower, threshold of frequent breaks of existing workers and threshold of backup or reservation of manpower sources In any organization when the policy decisions related to emoluments, benefits and objectives are announced then the exit of personnel is occurred. When the exit of personnel occurs, time overwhelming and expense happened for the recruitment, so that the recruitment cannot introduced in real time and also frequent recruitment are not encouraged, when the cumulative loss of manpower on critical occasion cross the level called as threshold, then only the recruitment is introduced and made. A mathematical model is developed using univarity policy of recruitment based on shock model approach in this paper. The inter-policy decision times and the inter-transfer decision times form same renewal process for three grades to obtain the mean variance of the time to recruitment. Mathematical equations for mean time to recruitment are developed using Laplace transform.

Keywords: Three grade system, two sources of depletion, univariate policy of recruitment, renewal process

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1. Introduction

In an organization, the depletion of manpower can occur due to two different cases:
1) whenever the policy decisions regarding pay, perquisites and work schedule are revised and
2) due to transfer of personnel to the other organization of the same management.

In the presence of these ***two different sources*** of depletion, Many models have been discussed using different kinds of wastages and different types of distributions for the threshold. Such models could be seen in [2],[3],[4],[10]and [11]. The problem of time to recruitment is studied by several authors both for single and multi-graded systems for different types of thresholds according as the interdecision times are independent and identically distributed random variables or correlated random variables. In a multi-graded system, transfer of

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personnel from one grade to another may or may not be permitted. Most of these authors have used univariate CUM policy of recruitment by which recruitment is done whenever the cumulative loss of manpower crosses a threshold. In [16] the author has obtained the performance measures namely mean and variance of the time for a two graded system when (i) the loss of manpower and the threshold for the loss of manpower in each grade are exponential random variables (ii) the interdecision times are independent and identically distributed exponential random variables forming the same renewal process for both the grades and (iii) threshold for the organization is the max(min) of the thresholds for the two rades(max(min)model) using the above cited univariate cumulative policy of recruitment. In [1] the authors have studied the maximum model discussed in [16] when both the distributions of the thresholds have SCBZ property. Assuming that the inter decision times are exchangeable and constantly correlated random variables, the performance measures of time to recruitment are obtained in [12] according as the loss of manpower and thresholds are discrete or continuous random variables. In [20] the author has extended the results in [12] for geometric thresholds when the inter-decision times for the two grades form two different renewal processes.. In [29] these performance measures are obtained when the inter-decision times are exchangeable and constantly correlated exponential random variables and the distributions of the thresholds have SCBZ property. In [31] the authors have studied the results in [16] when the threshold for the organization is the sum of the thresholds for the grades. This paper has been extended in [19] when threshold distributions have SCBZ property. In [13] the work in [31] is studied when the loss of manpower and thresholds are geometric random variables according as the inter-decision times for the two grades are correlated random variables or forming two different renewal processes. This author has also obtained the mean time for recruitment for constant combined thresholds using a univariate max policy for recruitment. In [17] the authors have studied the work in [16] when the thresholds for the loss of manpower in the two grades follow an extended exponential distribution with shape parameter 2. In [6], [7] and [8] the authors have considered a new univariate recruitment policy involving two thresholds in which one is optional and the other a mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter-decision times are independent and identically distributed exponential random variables or the inter-decision times are exchangeable and constantly correlated exponential random variables. In [9] the authors have also obtained the mean time to recruitment when the optional and mandatory thresholds are geometric random variables. In [21] the authors have studied the problem of time to recruitment for a two graded manpower system when (i) the loss of manpower in the organization due to i^{th} decision is maximum of the loss of manpower in this decision in grades A and B (ii) the thresholds for the organization is max (min) of the thresholds for the loss of manpower in the two grades under different conditions using a univariate CUM policy of recruitment. They have also studied max policy of recruitment by assuming constant threshold. In [22] the authors have extended the work of [21] when the threshold for the loss of man hours in the organization is the sum of the corresponding thresholds of the two grades according as the thresholds are exponential or extended exponential thresholds. In [23], [24], [25] and [26] the authors have extended the results in [6] for a two grade system according as the thresholds are exponential random variables or extended exponential random variables or SCBZ property possessing random variables or geometric random variables. In [27] the authors have extended the results in [6] for a two grade system according as the optional thresholds are exponential random variable and the distributions of the mandatory thresholds have SCBZ property. For a single graded manpower system, in [19] the authors have obtained the mean and variance of time to recruitment when (i) the loss of manpower form a sequence of independent and identically distributed Poisson random variables (ii) the threshold for the loss of manpower follow geometric distribution and the number of policy decisions announced by the organization is governed by a renewal process with independent and identically distributed exponential inter-decision times. In all the earlier research work the monotonicity of inter-decision times which do exists in reality, is not taken into account. In [15] the above limitation is removed and the authors have obtained the mean time to recruitment for a single grade manpower system by assuming that (i) the inter decision times form a geometric process in which the monotonicity is inbuilt in the process itself (ii) the loss of manpower is a sequence of independent and identically distributed exponential random variables and (iii) the distribution of the threshold for the loss of manpower in the organization is exponential. In [5] the authors have studied the results of [15] for a two grade system when the threshold for the loss of manpower in the two grades are exponential thresholds or SCBZ property possessing thresholds or extended exponential thresholds or geometric thresholds. They have also studied this work in [28] by considering optional and mandatory thresholds for the loss of manpower in the two grades. In [14] the authors have extended the work of [24],[27] when the loss of manpower for the organization is the maximum of the loss of manpower in the two grades by assuming exponential, extended exponential and SCBZ property possessing thresholds for the loss of manpower. In all the above cited works, authors have assumed the loss of manpower is a sequence of independent and identically distributed random variables. Uma et.al [29],[30] suggested two models in which threshold distributions following exponential and having SCBZ property to overcome the problems with the constant threshold models. The total of the maximum allowable attrition and the maximum available backup resource are considered as threshold in that paper. The backup resource is nothing but the manpower inventory on hand which can be used whenever there is a necessity. The long -run average cost per unit time for a single grade system under univariate policy of recruitment was developed by Rojamary and Uma [18].

It is proposed to determine time to recruitment for a single grade manpower system with two sources of depletion and the renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are same. Also it is assumed that threshold for the loss of manpower has two components namely the maximum allowable attrition and the maximum available manpower due to extra time work. The performance measures namely mean time to recruitment and the variances of time v(.) to: recruitment are derived.

2. MODEL DESCRIPTION

Consider an organisation having single grade in which decisions are taken at random epoch. The depletion of man-hours occurs at every decision making epoch and also due to transfer of personnel to the other organisation of the same management. It is assumed that the interarrival times between successive epochs of policy decisions and that of transfers are i.i.d random variables .The renewal processes governing the inter-policy decisions times and that of the inter-transfer decisions times are same . It is assumed that the two sources of depletion are independent. There is a threshold level for the level of wastage and also available for the manhours due to extra time work. If the total loss of manhours crosses the sum of the threshold and the available manhours due to extra time work the break down occurs. The process that generates the loss of manhours and the threshold put together is linearly independent. Recruitment takes place only at decision points and time of recruitment is negligible. The recruitment is made whenever the cumulative loss of manhours exceeds the threshold of the organization.

NOTATIONS

L_{i1} (L_{i2}) : independent and identically distributed continuous random variable denoting the loss of man hours due to i^{th} policy decision (j^{th} transfer), i ,j ≥ 1. Its probability density function is f(.) (k(.)) and cumulative distribution is F(.) (K(.)).

T_1 (T_2) : continuous random variable representing the threshold level of policy decisions (transfer) with probability density function $g_1(.)$ ($g_2(.)$) and cumulative distribution function $G_1(.)$ ($G_2(.)$).

$T = T_1 + T_2$: continuous random variable representing threshold level of the organization

Its probability density function is g(.) and cumulative distribution function is G(.).

R : continuous random variable representing the time to the breakdown of the organization. Its probability density function is l(.) and cumulative distribution function is L(.).

v(.) : probability density function of inter-decision times ,$v_m(.)$ and $v^*(.)$ are its m-fold convolution and Laplace transform respectively.

w(.) : probability density function of inter-trans times with $w_n(.)$ and $w^*(.)$ are its n-fold convolution and Laplace transform respectively.

V(.) : cumulative distribution function of inter- decision times with $V_m(.)$ and $\overline{V}(.)$ are its k-fold convolution and Laplace-Stieltjes transform respectively.

W(.) : cumulative distribution function of inter-transfer times with $W_n(.)$ and $\overline{W}(.)$ are its n-fold convolution and Laplace-Stieltjes transform respectively.

$f_m(.)$: probability density function of $\sum_{i=1}^{m} L_{i1}$, $f_m^*(.)$ is its Laplace transform

$k_n(.)$: probability density function of $\sum_{i=1}^{n} L_{i2}$, $k_n^*(.)$ is its Laplace transform

$K_n(.)$: cumulative distribution function of $\sum_{i=1}^{n} L_{i2}$, $\overline{K}_n(.)$ is its Laplace - Stieltjes transform

λ_1, λ_2 : parameters of the distribution function of thresholds T_1, T_2

μ_1, μ_2 : parameters of the distribution function of inter-decision time times and inter transfer times .

ξ_1, ξ_2 : parameters of the distribution function of the loss of manhours due to decision and due to transfer.

EXPECTED TIME AND VARIANCE OF TIME TO RECRUITMENT

In this section, the analytical expressions for the expected time and the variance of time to recruitment are derived. The recruitment is done whenever the cumulative loss of man-hours exceeds the sum $T_1 + T_2$.

Probability that the manpower system will fail only after time t is P(R > t) = $\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}$ {probability that there are exactly m policy decisions in (0, t] in grade I} {Probability that there are exactly m policy decisions and n transfer of personnel in (0, t] } {Probability that the cumulative damage in the manpower system does not cross its threshold level}.

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))\; P\left[\left(\sum_{i=1}^{m} L_{i,1} + \sum_{i=1}^{n} L_{i,2}\right) < (T_1 + T_2)\right]$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\int_0^{\infty} P\left[(x_1 + x_2) < (T_1 + T_2) \,\middle|\, \sum_{i=1}^{m} L_{i,1} = x_1,\ \sum_{i=1}^{n} L_{i,2} = x_2\right] dG_m(x_1)\, dH_n(x_2) \tag{3.1}$$

Since the threshold levels follow exponential distributions with parameters λ_1 and λ_2, the probability density function of $T_1 + T_2$ is

$$g(y) = \int_0^{y} g_1(x)\, g_2(y - x)\, dx$$

$$= \frac{\theta_1\, \theta_2}{\theta_1 - \theta_2}\left(e^{-\theta_2 y} - e^{-\theta_1 y}\right)$$

The equation (3.1) becomes

$$P(R > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\int_0^{\infty}\int_0^{\infty}\int_{x_1 + x_2}^{\infty} \frac{\theta_1\, \theta_2}{(\theta_1 - \theta_2)}\left(e^{-\theta_2 y} - e^{-\theta_1 y}\right) dy\; dG_m(x_1)\, dH_n(x_2)$$

$$= \frac{1}{(\theta_1 - \theta_2)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\int_0^{\infty} \theta_1\, e^{-\theta_2 x_2}\, f_m^*(\theta_2)\, dH_n(x_2) - \int_0^{\infty} \theta_2\, e^{-\theta_1 x_2}\, f_m^*(\theta_1)\, dH_n(x_2)\right\}$$

$$= \frac{1}{(\theta_1 - \theta_2)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\theta_1\, f_m^*(\theta_2)\, k_n^*(\theta_2) - \theta_2\, f_m^*(\theta_1)\, k_n^*(\theta_1)\right\}$$

$$= \left(\frac{1}{\theta_1 - \theta_2}\right) \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (V_m(t) - V_{m+1}(t))\,(W_n(t) - W_{n+1}(t))$$

$$\left\{\theta_1\, (f^*(\theta_2))^m\, (k^*(\theta_2))^n - \theta_2\, (f^*(\theta_1))^m\, (k^*(\theta_1))^n\right\}$$

$$\left(\frac{1}{\theta_1 - \theta_2}\right) \sum_{m=0}^{\infty} (V_m(t) - V_{m+1}(t))$$

$$(\theta_1\, (f^*(\theta_2))^m - \theta_2\, (f^*(\theta_1))^m)$$

$$- \theta_1\, (f^*(\theta_2))^m \left(\sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_2))^{n-1} - \sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_2))^n\right)$$

$$+ \theta_2\, (f^*(\theta_1))^m \left(\sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_1))^{n-1} - \sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_1))^n\right)$$

$$= \left(\frac{1}{\theta_1 - \theta_2}\right) \sum_{m=0}^{\infty} (V_m(t) - V_{m+1}(t))$$

$$\left\{\theta_1\, (f^*(\theta_2))^m - \theta_2\, (f^*(\theta_1))^m\right.$$

$$- \theta_1\, (f^*(\theta_2))^m\, (1 - k^*(\theta_2)) \sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_2))^{n-1}$$

$$\left. + \theta_2\, (f^*(\theta_1))^m\, (1 - k^*(\theta_1)) \sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_1))^{n-1}\right\}$$

$$= \left(\frac{\theta_1}{\theta_1 - \theta_2}\right)$$

$$\left(1 - (1 - k^*(\theta_2)) \sum_{n=1}^{\infty} W_n(t)\, (k^*(\theta_2))^{n-1}\right)$$

$$\left(1-(1-f^*(\theta_2))\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_2))^{m-1}\right)$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)$$

$$\left(1-(1-k^*(\theta_1))\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_1))^{n-1}\right)$$

$$\left(1-(1-f^*(\theta_1))\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_1))^{m-1}\right)$$

$1-P(R>t)$

$$L(t) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)$$

$$\left\{(1-f^*(\theta_2))\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_2))^{m-1}\right.$$

$$+(1-k^*(\theta_2)\sum_{n=1}^{\infty} W_n(t)\,(f^*(\theta_2))^{n-1}$$

$$\left.-(1-f^*(\theta_2))(1-k^*(\theta_2))\left(\sum_{m=1}^{\infty} V_m(t)(f^*(\theta_2))^{m-1}\right)\left(\sum_{n=1}^{\infty} W_n(t)(f^*(\theta_2))^{n-1}\right)\right\}$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)$$

$$\left\{(1-f^*(\theta_1))\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_1))^{m-1}\right.$$

$$+(1-k^*(\theta_1)\sum_{n=1}^{\infty} W_n(t)\,(f^*(\theta_1))^{n-1}$$

$$-(1-f^*(\theta_1))(1-k^*(\theta_1))\left(\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_1))^{m-1}\right)$$

$$\left.\left(\sum_{n=1}^{\infty} R_n(t)\,(f^*(\theta_1))^{n-1}\right)\right\}$$

(3.2)

$$l(t) = \frac{d}{dt}L(t)$$

$$= \left(\frac{\theta_1}{\theta_1-\theta_2}\right)$$

$$\left\{(1-f^*(\theta_2))\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_2))^{m-1}\right.$$

$$+(1-k^*(\theta_2)\sum_{n=1}^{\infty} w_n(t)\,(k^*(\theta_2))^{n-1}$$

$$-(1-f^*(\theta_2))(1-k^*(\theta_2))$$

$$\left(\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_2))^{n-1}\right)$$

$$\left(\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_2))^{m-1}\right)$$

$$-(1-f^*(\theta_2))(1-k^*(\theta_2))$$

$$\left.\left(\sum_{n=1}^{\infty} w_n(t)\,(k^*(\theta_2))^{n-1}\right)\left(\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_2))^{m-1}\right)\right\}$$

$$-\left(\frac{\theta_1}{\theta_1-\theta_2}\right)$$

$$\left\{(1-f^*(\theta_1))\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_1))^{m-1}\right.$$

$$+(1-k^*(\theta_1))\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_1))^{n-1}$$

$$-(1-f^*(\theta_1))(1-k^*(\theta_1))$$

$$\left(\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_1))^{n-1}\right)$$

$$\left(\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_1))^{m-1}\right)$$

$$-(1-f^*(\theta_1))(1-k^*(\theta_1))$$

$$\left.\left(\sum_{n=1}^{\infty} w_n(t)\,(k^*(\theta_1))^{n-1}\right)\left(\sum_{m=1}^{\infty} R_m(t)\,(f^*(\theta_1))^{m-1}\right)\right\}$$

(3.3)

As the inter-decision times and inter transfer times follow exponential distributions with parameters σ1 and σ2respectively, we have

$$v_m(t) = \frac{\mu_1^m\, e^{-\sigma_1 t}\, t^{m-1}}{(m-1)!} \text{ and}$$

$$w_n(t) = \frac{\mu_2^n\, e^{-\sigma_2 t}\, t^{n-1}}{(n-1)!}$$

Now, $\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_i))^{m-1} = \sum_{m=1}^{\infty} \frac{\mu_1^m\, e^{-\sigma_1 t}\, t^{m-1}}{(m-1)!}\,(f^*(\theta_i))^{m-1}$

$$= \mu_1\, e^{-\sigma_1 t\,(1-f^*(\theta_i))}, \quad i = 1, 2. \tag{3.4}$$

and $\sum_{n=1}^{\infty} r_n(t)\,(k^*(\theta_i))^{n-1} = \sum_{m=1}^{\infty} \frac{\mu_2^n\, e^{-\sigma_2 t}\, t^{n-1}}{(i-1)!}\,(k^*(\theta_i))^{n-1}$

$$= \mu_2\, e^{-\sigma_2 t\,(1-k^*(\theta_i))},\ i = 1, 2 \tag{3.5}$$

Using the equations (3.4) and (3.5), the equation (3.3) becomes

$$l(t) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)\Big\{(1-f^*(\theta_2))\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_2))^{m-1}$$

$$-(1-f^*(\theta_2))(1-k^*(\theta_2))\,\mu_1\, e^{-\sigma_1 t\,(1-f^*(\theta_2))}\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_2))^{n-1}$$

$$+(1-k^*(\theta_2))\sum_{n=1}^{\infty} w_n(t)\,(k^*(\theta_2))^{n-1}$$

$$-(1-f^*(\theta_2))(1-k^*(\theta_2))\,\mu_2\, e^{-\sigma_2 t\,(1-k^*(\theta_2))}\sum_{n=1}^{\infty} V_n(t)\,(f^*(\theta_2))^{n-1}\Big\}$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)\Big\{(1-f^*(\theta_1))\sum_{m=1}^{\infty} v_m(t)\,(f^*(\theta_1))^{m-1}$$

$$-(1-f^*(\theta_1))(1-k^*(\theta_1))\,\sigma_1 e^{-\sigma_1 t(1-f^*(\theta_1))}\sum_{n=1}^{\infty} W_n(t)\,(k^*(\theta_1))^{n-1}$$

$$+(1-k^*(\theta_1))\sum_{n=1}^{\infty} w_n(t)\,(k^*(\theta_1))^{n-1}$$

$$-(1-f^*(\theta_1))(1-k^*(\theta_1))\,\theta_2\, e^{-\sigma_2 t\,(1-k^*(\theta_1))}\left(\sum_{m=1}^{\infty} V_m(t)\,(f^*(\theta_1))^{m-1}\right)\Big\}$$

Taking Laplace transform on both sides and using the notations $a=(1-f^*(\theta_1))\,\mu_1$, $b=(1-f^*(\theta_2))\,\mu_1$, $c=(1-k^*(\theta_1))\,\mu_2$ and $d=(1-k^*(\theta_2))\,\mu_2$ we have,

$$l^*(s) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)\left\{\left((1-f^*(\theta_2))\sum_{m=1}^{\infty}(f^*(\theta_2))^{m-1}\, v_m^*(s)\right)\right.$$

$$-(1-f^*(\theta_2))(1-h^*(\theta_2))\sum_{n=1}^{\infty}\mu_1\,(k^*(\theta_2))^{n-1}\,\overline{W}_n(b+s)$$

$$+(1-k^*(\theta_2))\left(\sum_{n=1}^{\infty}(k^*(\theta_2))^{n-1}\, w_n^*(s)\right)$$

$$\left.-(1-f^*(\theta_2))(1-k^*(\theta_2))\left(\sum_{m=1}^{\infty}\mu_2\,(f^*(\theta_2))^{m-1}\,\overline{V}_m(d+s)\right)\right\}$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)$$

$$\left\{(1-\mathrm{f}^*(\theta_1))\left(\sum_{\mathrm{m=1}}^{\infty}(\mathrm{f}^*(\theta_1))^{m-1}\,\mathrm{v}_{\mathrm{m}}^*(s)\right)\right.$$

$$-(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))$$

$$\sum_{\mathrm{n=1}}^{\infty}\mu_1\,(\mathrm{k}^*(\theta_1))^{n-1}\,\overline{\mathrm{W}}_{\mathrm{n}}(a+s)$$

$$+(1-\mathrm{k}^*(\theta_1))\sum_{\mathrm{n=1}}^{\infty}(\mathrm{k}^*(\theta_1))^{n-1}\,\mathrm{w}_{\mathrm{n}}^*(s)$$

$$-(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))$$

$$\left.\left(\sum_{\mathrm{m=1}}^{\infty}\mu_2\,(\mathrm{f}^*(\theta_1))^{m-1}\,\overline{\mathrm{V}}_{\mathrm{m}}(c+s)\right)\right\} \qquad (3.6)$$

Using the relation between the Laplace transform of the density function and Laplace transform of distribution function, we have

$$\mathrm{l}^*(s) = \left(\frac{\theta_1}{\theta_1-\theta_2}\right)$$

$$\left\{(1-\mathrm{f}^*(\theta_2))\sum_{m=1}^{\infty}(\mathrm{f}^*(\theta_2))^{m-1}\,\mathrm{v}_{\mathrm{m}}^*(s)\right.$$

$$-(1-\mathrm{f}^*(\theta_2))\,(1-\mathrm{k}^*(\theta_2))$$

$$\sum_{\mathrm{n=1}}^{\infty}\mu_1\,(\mathrm{k}^*(\theta_2))^{n-1}\,\frac{\mathrm{w}_{\mathrm{n}}^*(s+b)}{s+b}$$

$$+(1-\mathrm{k}^*(\theta_2))\sum_{\mathrm{n=1}}^{\infty}(\mathrm{k}^*(\theta_2))^{n-1}\,\mathrm{w}_{\mathrm{n}}^*(s)$$

$$-(1-\mathrm{f}^*(\theta_2))\,(1-\mathrm{k}^*(\theta_2))$$

$$\left.\sum_{\mathrm{m=1}}^{\infty}\mu_2\,(\mathrm{f}^*(\theta_2))^{m-1}\,\frac{\mathrm{v}_{\mathrm{m}}^*(s+d)}{s+d}\right\}$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)$$

$$\left\{(1-\mathrm{f}^*(\theta_1))\sum_{\mathrm{m=1}}^{\infty}(\mathrm{f}^*(\theta_1))^{m-1}\,\mathrm{v}_{\mathrm{m}}^*(s)\right.$$

$$-(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))$$

$$\sum_{\mathrm{n=1}}^{\infty}\mu_1\,(\mathrm{k}^*(\theta_1))^{n-1}\,\frac{\mathrm{w}_{\mathrm{n}}^*(s+a)}{s+a}$$

$$+(1-\mathrm{k}^*(\theta_1))\sum_{\mathrm{n=1}}^{\infty}(\mathrm{k}^*(\theta_1))^{n-1}\,\mathrm{w}_{\mathrm{n}}^*(s)$$

$$-(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))$$

$$\left.\sum_{\mathrm{m=1}}^{\infty}\sigma_2\,(\mathrm{f}^*(\theta_1))^{m-1}\,\frac{\mathrm{v}_{\mathrm{m}}^*(s+c)}{s+c}\right\}$$

$$=\left(\frac{\theta_1}{\theta_1-\theta_2}\right)\left[\frac{(1-\mathrm{f}^*(\theta_2))\,\mathrm{v}^*(s)}{1-\mathrm{f}^*(\theta_2)\,\mathrm{v}^*(s)}\right.$$

$$-\frac{\mu_1\,(1-\mathrm{f}^*(\theta_2))\,(1-\mathrm{k}^*(\theta_2))\,\mathrm{w}^*(s+b)}{(s+b)\,(1-\mathrm{k}^*(\theta_2)\,\mathrm{w}^*(s+b))}$$

$$+\frac{(1-\mathrm{k}^*(\theta_2))\,\mathrm{w}^*(s)}{1-\mathrm{k}^*(\theta_2)\,\mathrm{w}^*(s)}$$

$$\left.-\frac{\mu_2\,(1-\mathrm{f}^*(\theta_2))\,(1-\mathrm{k}^*(\theta_2))\,\mathrm{v}^*(s+d)}{(s+d)\,(1-\mathrm{f}^*(\theta_2)\,\mathrm{v}^*(s+d))}\right]$$

$$-\left(\frac{\theta_2}{\theta_1-\theta_2}\right)\left[\frac{(1-\mathrm{f}^*(\theta_1))\,\mathrm{v}^*(s)}{1-\mathrm{f}^*(\theta_1)\,\mathrm{v}^*(s)}\right.$$

$$-\frac{\mu_1\,(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))\,\mathrm{v}^*(s+a)}{(s+a)\,(1-\mathrm{k}^*(\theta_1)\,\mathrm{w}^*(s+a))}$$

$$+\frac{(1-\mathrm{k}^*(\theta_1))\,\mathrm{w}^*(s)}{1-\mathrm{k}^*(\theta_1)\,\mathrm{w}^*(s)}$$

$$\left.-\frac{\mu_2\,(1-\mathrm{f}^*(\theta_1))\,(1-\mathrm{k}^*(\theta_1))\,\mathrm{v}^*(s+c)}{(s+c)\,(1-\mathrm{f}^*(\theta_1)\,\mathrm{v}^*(s+c))}\right] \text{ ----- (3.7)}$$

Since f(.) and k(.) follow exponential distributions with parameters ξ_1 and ξ_2 respectively, we have

$$\mathrm{f}^*(\theta_i) = \frac{\beta_1}{\beta_1+\theta_{\mathrm{i}}} \quad \text{and} \quad \mathrm{k}^*(\theta_i) = \frac{\beta_2}{\beta_2+\theta_{\mathrm{i}}},$$

i = 1, 2 ------------ (3.8)

Taking

$A_1(s) = (\mu_1 + s)\,(\alpha_1 + {}_2) - \mu_1\,\xi_1$;

$A_2(s)=(\mu_2 + s)\,(\xi_2 + \lambda_2) - \lambda_2\,\xi_2$

$A_3(s) = (\mu_1 + s)\,(\xi_1 + \lambda_1) - \mu_1\,\xi_1$;

$A_4(s)=(\sigma_2 + s)\,(\xi_2 + \lambda_1) - \mu_2\,\xi_2$

$B_1(s) = \lambda_2\,\mu_1 + s\,(\xi_1 + \lambda_2)$;

$B_2(s)=\lambda_2\, \mu_2 + s\,(\xi_2 + \lambda_2)$

$B_3(s) = \lambda_1\, \mu_1 + s\,(\xi_1 + \lambda_1)$;

$B_4(s)=\lambda_1\, \mu_2 + s\,(\xi_2 + \lambda_1)$

$C_1(s) = (\mu_2 + s)\,(\xi_1 + \lambda_2)\,(\xi_2 + \lambda_2) + \mu_1\, \lambda_2(\xi_2 + \lambda_2) - \mu_2\, \xi_2\,(\xi_1 + \lambda_2)$

$C_2(s) = (\mu_1 + s)\,(\xi_1 + \lambda_2)\,(\xi_2 + \lambda_2) + \mu_2\, \lambda_2(\xi_1 + \lambda_2) - \mu_1\, \xi_1\,(\xi_2 + \lambda_2)$

$C_3(s) = (\mu_2 + s)\,(\xi_1 + \lambda_1)\,(\xi_2 + \lambda_1) + \mu_1\, \lambda_1\,(\xi_2 + \lambda_1) - \mu_2\, \xi_2\,(\xi_1 + \lambda_1)$

$C_4(s) = (\mu_1 + s)\,(\xi_1 + \lambda_1)\,(\xi_2 + \lambda_1) + \mu_2\, \lambda_1(\xi_1 + \lambda_1) - \mu_1\, \xi_1\,(\xi_2 + \lambda_1)$

and using (3.8) the equation (3.7) becomes

$$
\begin{aligned}
I^*(s) = {} & \left(\frac{\theta_1}{\theta_1-\theta_2}\right) \left\{ \frac{\theta_2\,\mu_1}{A_1(s)} + \frac{\theta_2\,\mu_2}{A_2(s)} - \theta_2^2\,\mu_1\,\mu_2 \left[\frac{\xi_1+\theta_2}{B_1(s)\,C_1(s)} + \frac{\xi_2+\theta_2}{B_2(s)\,C_2(s)} \right] \right\} \\
& - \left(\frac{\theta_2}{\theta_1-\theta_2}\right) \left\{ \frac{\theta_1\,\mu_1}{A_3(s)} + \frac{\theta_1\,\mu_2}{A_4(s)} - \theta_1^2\,\mu_1\,\mu_2 \left[\frac{\xi_1+\theta_1}{B_3(s)\,C_3(s)} + \frac{\xi_2+\theta_1}{B_4(s)\,C_4(s)} \right] \right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dI^*(s)}{ds} = {} & \left(\frac{\theta_1}{\theta_1-\theta_2}\right) \left\{ \frac{-\theta_2\,\mu_1\,(\xi_1+\theta_2)}{[A_1(s)]^2} - \frac{\theta_2\,\mu_2\,(\xi_2+\theta_2)}{[A_2(s)]^2} \right. \\
& + \theta_2^2\,\mu_1\,\mu_2 \left[\frac{(\xi_1+\theta_2)[B_1(s)\,(\xi_1+\theta_2)\,(\xi_2+\theta_2) + C_1(s)\,(\xi_1+\theta_2)]}{[B_1(s)\,C_1(s)]^2} \right. \\
& \left.\left. + \frac{(\xi_2+\theta_2)[B_2(s)\,(\xi_1+\theta_2)\,(\xi_2+\theta_2) + C_2(s)\,(\xi_2+\theta_2)]}{[B_2(s)\;C_2(s)]^2} \right] \right\} \\
& - \left(\frac{\theta_2}{\theta_1-\theta_2}\right) \left\{ \frac{-\theta_1\,\mu_1\,(\xi_1+\theta_1)}{[A_3(s)]^2} - \frac{\theta_1\,\mu_2\,(\xi_2+\theta_1)}{[A_4(s)]^2} \right. \\
& + \theta_1^2\,\mu_1\,\mu_2 \left[\frac{(\xi_1+\theta_1)[B_3(s)\,(\xi_1+\theta_1)\,(\xi_2+\theta_1) + C_3(s)\,(\xi+\theta_1)]}{[B_3(s)\,C_3(s)]^2} \right. \\
& \left.\left. + \frac{(\beta_2+\theta_1)[B_4(s)\,(\beta_1+\theta_1)\,(\beta_2+\theta_1) + C_4(s)\,(\beta_2+\theta_1)]}{[B_4(s)\;C_4(s)]^2} \right] \right\}
\end{aligned}
$$

The mean time to recruitment is

$$
\begin{aligned}
E(R) = {} & \left(\frac{\theta_1}{\theta_1-\theta_2}\right) \left\{ \frac{\xi_1+\theta_2}{\mu_1\,\theta_2} + \frac{\xi_2+\theta_2}{\mu_2\,\theta_2} \right. \\
& - \frac{\mu_2\,(\xi_1+\theta_2)^2}{\mu_1\,\theta_2} \left[\frac{2\,\mu_1\,(\xi_2+\theta_2) + \mu_2\,(\xi_1+\theta_2)}{[\mu_2\,(\xi_1+\theta_2) + \mu_1\,(\xi_2+\theta_2)]^2} \right] \\
& \left. - \frac{\mu_1\,(\xi_2+\theta_2)^2}{\mu_2\,\theta_2} \left[\frac{2\,\mu_2\,(\xi_1+\theta_2) + \mu_1\,(\xi_2+\theta_2)}{[\mu_1\,(\xi_2+\theta_2) + \mu_2\,(\xi_1+\theta_2)]^2} \right] \right\} \\
& - \left(\frac{\theta_2}{\theta_1-\theta_2}\right) \left\{ \frac{\xi_1+\theta_1}{\mu_1\,\theta_1} + \frac{\xi_2+\theta_1}{\mu_2\,\theta_1} \right. \\
& - \frac{\mu_2\,(\beta_1+\theta_1)^2}{\mu_1\,\theta_1} \left[\frac{2\,\mu_1\,(\xi_2+\theta_1) + \mu_2\,(\xi_1+\theta_1)}{[\mu_2\,(\xi_1+\theta_1) + \mu_1\,(\xi_2+\theta_1)]^2} \right] \\
& \left. - \frac{\mu_1\,(\xi_2+\theta_1)^2}{\mu_2\,\theta_1} \left[\frac{2\,\mu_2\,(\xi_1+\theta_1) + \mu_1\,(\xi_2+\theta_1)}{[\mu_1\,(\xi_2+\theta_1) + \mu_2\,(\xi_1+\theta_1)]^2} \right] \right\}
\end{aligned}
$$

---------- (3.9)

$$
\frac{d^2 I^*(s)}{ds^2} = \left(\frac{\theta_1}{\theta_1-\theta_2}\right) \left\{ \frac{2\,\theta_2\,\mu_1\,(\xi_1+\theta_2)}{(A_1(s))^3} + \frac{2\,\theta_2\,\mu_2\,(\xi_2+\theta_2)}{(A_2(s))^3} \right.
$$

$$- \theta_2^2 \mu_1 \mu_2 \left[\frac{2(\xi_1+\theta_2)^3}{(B_1(s)C_1(s))^3} [B_1(s)C_1(s)(\xi_2+\theta_2) - (B_1(s)(\xi_2+\theta_2)+C_1(s))^2] \right.$$

$$\left. + \frac{2(\xi_2+\theta_2)^3}{(B_2(s)C_2(s))^3} [B_2(s)C_2(s)(\xi_1+\theta_2) - (B_2(s)(\xi_1+\theta_2)+C_2(s))^2] \right]\Bigg\}$$

$$- \left(\frac{\theta_2}{\theta_1-\theta_2} \right)$$

$$\left\{ \frac{2\theta_1 \mu_1 (\xi_1+\theta_1)}{(A_3(s))^3} + \frac{2\theta_1 \mu_2 (\xi_2+\theta_1)}{(A_4(s))^3} \right.$$

$$+ \theta_1^2 \mu_1 \mu_2 \left[\frac{2(\xi_1+\theta_1)^3}{(B_3(s)C_3(s))^3} [B_3(s)C_3(s)(\xi_2+\theta_1) - (B_3(s)(\xi_2+\theta_1)+C_3(s))^2] \right.$$

$$\left. + \frac{2(\xi_2+\theta_1)^3}{(B_4(s)C_4(s))^3} [B_4(s)C_4(s)(\xi_1+\theta_1) - (B_4(s)(\xi_1+\theta_1)+C_4(s))^2] \right]\Bigg\}$$

Let $C_{1i} = (C_i(s))_{s=0}$, i = 1, 2

Then,

$$E(R^2) = \left(\frac{\theta_1}{\theta_1-\theta_2} \right) \left\{ \frac{2(\xi_1+\theta_2)^2}{(\mu_1 \theta_2)^2} + \frac{2(\xi_2+\theta_2)^2}{(\mu_2 \theta_2)^2} \right.$$

$$- \theta_2^2 \mu_1 \mu_2 \left[\frac{2(\xi_1+\theta_2)^3}{(C_{11} \theta_2 \mu_1)^3} (C_{11}\lambda_2\sigma_1 (\xi_2+\lambda_2) + (\lambda_2\sigma_1)^2 (\xi_2+\lambda_2)^2 + (C_{11})^2) \right.$$

$$\left. + \frac{2(\xi_2+\theta_2)^3}{(C_{12} \theta_2 \mu_2)^3} (C_{12} \theta_2 \mu_2 (\xi_1+\theta_2) + (\theta_2 \mu_2)^2 (\xi_1+\theta_2)^2 + (C_{12})^2) \right]\Bigg\}$$

$$- \left(\frac{\theta_2}{\theta_1-\theta_2} \right)$$

$$\left\{ \frac{2(\alpha_1+\theta_1)^2}{(\mu_1 \theta_1)^2} + \frac{2(\alpha_2+\theta_1)^2}{(\mu_2 \theta_1)^2} \right.$$

$$- \theta_1^2 \mu_1 \mu_2 \left[\frac{2(\xi_1+\theta_1)^3}{(C_{13} \theta_1 \mu_1)^3} (C_{13}\lambda_1\sigma_1 (\xi_2+\lambda_1) + (\lambda_1\mu_1)^2(\xi_2+\lambda_1)^2 + (C_{13})^2) \right.$$

$$\left. + \frac{2(\xi_2+\theta_1)^3}{(C_{14} \theta_1 \mu_2)^3} (C_{14} \theta_1 \mu_2 (\xi_1+\theta_1) + (\theta_1 \mu_2)^2 (\xi_1+\theta_1)^2 + (C_{14})^2) \right]\Bigg\}$$

-------------- (3.10)

Using the equations (3.9) and (3.10) the variance of time to recruitment can be calculated.

with indent.

References

[1] Akilandeswari, M and A.srinivasan. 2007. Mean time to recruitment for a two graded manpower system when threshold distribution has SCBZ property. Acta Ciencia Indica , 33M(3):1113-1118.

[2] Barthlomew, D.J.1971. The statistical approach to manpower planning. Statistician,20:3-26

[3] Barthlomew, D.J. 1973, stochastic models for social processes, 2nd Ed., John wiley & sons, Newyork.

[4] Barthlomew, D.J. and A.F.Forbes. 1979, statistical techniques for manpower planning, John Wiley & sons, chichester.

[5] Dhiya, S. A.srinivasan and V.Vasudevan. 2011. Stochastic models for the time to recruitment in a two grade manpower system using same geometric process for inter-decision times. Proceedings of the International Conference on Computational and Mathematical Modeling, Narosa Publishing house Pvt. Ltd., 276-283.

[6] Esther clara J.B and A.srinivasan ,2008. Expected time for recruitment in a single graded manpower system with two thresholds. Proceedings of the national conference on Recent Development of applications of Probability Theory, random process and random variables in computer science,98-102.

[7] Esther clara J.B and A.srinivasan. 2009. A Stochastic model for the expected time for recruitment in a single graded manpower system with two thresholds having SCBZ property. Proceedings of the International Conference on Mathematical methods and Computation, Narosa Publishing house Pvt. Ltd.,274-280

[8] Esther clara J.B and A.srinivasan. 2010. A Stochastic model for the expected time to recruitment in a single graded manpower system with two thresholds having SCBZ property. Proceedings of the International Conference on Mathematical methods and Computation, Narosa Publishing house Pvt. Ltd.,274-280

[9] Esther clara J.B and A.srinivasan. 2010. A Stochastic model for the expected time to recruitment in a single graded manpower system with two discrete thresholds. Antarctica J.Math. 7(3) :261-271

[10] Grinold, R.C. and K.T. Marshal. 1977. Manpower planning Models, North Holland, New York.

[11] Karlin, S and H.M.Taylor. 1975. A first course in stochastic processes, Academic Press, New York, San Francisco, London.
[12] Kasturi .K .2008. Mean time for recruitment and cost analysis on some univariate policies of recruitment in manpower models, Ph.D. thesis, Barathidasasn University.
[13] Mercy Alice, B.2009. Some Stochastic models on the variance of the time to recruitment in a two graded manpower system associated with a univariate policy of recruitment involving combined thresholds. M.Phil., Dissertation, Bharathidasasn University.
[14] Mariappan ,P, A.Srinivasan and G.Iswarya.2011. Mean and variance of the time to recruitment in a two graded Manpower system with two thresholds for the organization. Recent Research in Science and Technology. 3(10):45-54.
[15] Muthaiyan,A. and R.Sathiyamoorthy 2010. A stochastic model using geometric process for inters arrival time between wastages. Acta Ciencia Indica. 46M (4):479-486.
[16] Parthasarathy, S. 2003. On some stochastic models for manpower planning using SCBZ property, Ph.D thesis at department of statistics, Annamalai University.
[17] Parthasarathy, S., M.k.Ravichandran and R.Vinoth. 2010 An application of stochastic models- Grading system in manpower planning. International Business Research 3(2):79-86.
[18] Rojamary.T and Uma.K.P," *A stochastic model for a single grade system with backup resource of manpower"*Malaya Journal of Matematik.
[19] Sathiyamoorthy, R and R.Elangovan. 1998. Shock model approach to determine the expected time for recruitment. Journal of Decision nad Mathematical Sciences. 3(1-3):67-68
[20] Sendhamizh Selvi,S. 2009. A study on expected time to recruitment in manpower models for a multi graded system associated with an univariate policy of recruitment, Ph.D., Thesis, Bharathidasan University.
[21] Srinivasan, A., P.Mariappan and S.Divya.2011. Stochastic models on time to recruitment in a two graded manpower system using different policies of recruitment involving combined thresholds. Recent Research in Science and Technology. 3(4):162-168.
[22] Srinivasan,A and S.Mohanalakshmi. 2011. Stochastic models on time to recruitment in a two graded manpower system using different policies of recruitment involving combined thresholds. Recent Research in Science and Technology. 3(10):55-88
[23] Srinivasan, A and V.Vasudevan .2011. Variance of the time to recruitment in an organization with two grades. Recent Research in Science and Technology. 3(1):128-131.
[24] Srinivasan ,A and V.Vasudevan. 2011. A Stochastic models for the expected time to recruitment in a two graded manpower system. Antarctica J.Math. 8(3):241-248.
[25] Srinivasan ,A and V.Vasudevan. 2011. Amanpower model for a two grade system with univariate policy of recruitment. International Review of Pure and Applied Mathematics. 7(1):79-88.
[26] Srinivasan ,A and V.Vasudevan. 2011. A Stochastic model for the expected time to recruitment in a two graded manpower system with two discrete thresholds. International Journal of Applied Mathematical Analysis and Applications. 6(1-2): 119-126.
[27] Srinivasan ,A and V.Vasudevan. 2011. Expected time to recruitment in an organization with two grades using a univariate recruitment policy involving two thresholds. Recent Research in Science and Technology. 3(10):59-62.
[28] Srinivasan ,A and V.Vasudevan. 2012. Stochastic models for the time to recruitment in a two graded manpower system with two types of thresholds using same geometric process for inter-decision times, Proceeding of Heber International coference on Applications of Mathematics and Statistics, 667-677.
[29] Srinivasan.A, Uma K,P.,Udayachandrika,K. and Saavithri,V., *Expected time to recruitment for an univariate policy when threshold distribution has SCBZ Property,* Proceedings of the third national Conference On Mathematical and Computational Models, Narosa Publishing Book House, New Delhi,(2005) 242-246
[30] Srinivasan.A, Uma K,P.,Udayachandrika,K. and Saavithri,V., *Cost analysis on a bivariate policy of recruitment in man power planning,* Acta Ciencia Indica, Vol. XXXIV M, No. 2, (2008). pp. 753-757
[31] Sureshkumar, R., G.Gopal and P.Sathiyamoorthy. 2006. Stochastic models for the expected time to recruitment in in an organization with two grades. International Journal of Management and systems. 22(2) : 147-164.