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Mean time to recruitment for three grades with two sources of depletion

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Abstract

In any organization when the policy decisions related to emoluments, benefits and objectives are announced then the exit of personnel is occurred. When the exit of personnel occurs, time overwhelming and expense happened for the recruitment , so that the recruitment cannot introduced in real time and also frequent recruitment are not encouraged, when the cumulative loss of manpower on critical occasion cross the level called as threshold, then only the recruitment is introduced and made. A mathematical model is developed using univarity policy of recruitment based on shock model approach in this paper.
The inter-policy decision times and the inter-transfer decision times form same renewal process for three grades to obtain the mean variance of the time to recruitment. Mathematical equations for mean time to recruitment are developed using Laplace transform.

Keywords: Three grade system, two sources of depletion, univaiate policy of recruitment, renewal process

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1. Introduction

In any administrative and manufacturing company the frequent exit of personnel is common, in organization whenever the revised policies related to sales target, revision of wages, incentives and benefits are announced departure of employees is possible. The sales turnover of the company is adversely affected when the total strength of marketing personnel is reduced. As several are involved in the recruitment it is common that the organization is disinclined to go for frequent recruitments. The company reaches an uneconomic status which is known as the breakdown point when the threshold level then the recruitment is made at the time. the time to reach the breakdown point is an important factor for the management of the company.

Barthlomew [**1**] and Barthlomew and Forbes [**2**] have developed many models in this system. The problem of time to recruitment in a marketing organization under different conditions have been analyzed by Sathyamoorthi and Elangovan [3] and Sathyamoorthi and Parthasarathy [4]. Two different cases are occurred in any organization for the depletion of manpower (i) policy decisions related to emoluments, benefits and work schedule are revised and (ii) transfer of persons to the sister concern of the same management.

Elangovan et.al [5] have developed a model of time to recruitment for an organization with one grade and variance of the time to recruitment by univariate policy of recruitment is obtained in the presence of these two different sources of depletion when (i) the two sources of depletion and its threshold are independent and are exponential random variables in the exit of manpower in any organization. (ii) The renewal process is same for both inter-policy decision times and inter-transfer decision times. Usha et.al [6] have analyzed a stochastic model for determining the expected time to recruitment by assuming (i) the depletion of manpower at every epoch of decision making and at every transfer they are considered a discrete random variables and (ii) recruitment is made when the total depletion crosses the threshold which is the discrete random variables.

For determining the expected time to recruitment by assuming that inter-arrival times between every epoch of policy decisions are not independent but they are correlated whereas the inter-arrival times between every epoch of transfers are independent and identically distributed random variables have been developed by the same authors [7].

Threshold with two components known as the allowable level of wastage and the available backup resource manpower has been developed by Vijayasankar, et.al.[8].

A single grade system subjected to exit of personnel due to policy decisions related to pay decided by the organization has been constructed by Rojamary and Uma [9] and the mathematical expression for the long-run average cost was derived by assuming geometric process for survival time process and two components threshold. Mean and variance of time to recruitment for three grade manpower system with two sources of depletion has been determined and the inter-policy decision times and also the inter-transfer decision times are the same for renewal process has been presented by Rojamary and Uma [10].

In this paper, it is proposed to obtain mean time to recruitment with two sources of depletion for three grade manpower system. The inter-policy decision times and also inter-transfer decision times are considered the same for the renewal processes.

2. Materials and Methods

Consider a three grade organization with univariate policy of recruitment which takes decisions at random era. At every decision making an era a random number of persons quit the organization. There is an associated loss of manhours to the organization if a person quits. The loss of manhours at any decision forms a sequence of independent and identically distributed random variables. For i=1,2,3… let X_{i1}, X_{i2} and X_{i3} be the continuous random variables representing the amount of depletion of manpower in grades A,B and C respectively caused due to the i[th] policy decision. $g_1(.)$, $g_2(.)$ and $g_3(.)$ are the p.d.f of X_{i1}, X_{i2} and X_{i3} for each i and each form a sequence of i.i.d random variables. $\overline{X}_{m1}=\sum_{i=1}^{m} X_{i1}$, $\overline{X}_{m2}=\sum_{i=1}^{m} X_{i2}$ and $\overline{X}_{m3}=\sum_{i=1}^{m} X_{i3}$. For j=1,2,3… let Y_{j1}, Y_{j2} and Y_{j3} d Y_{j3} for each j and each form a sequence of i.i.d random variables. $\overline{Y}_{n1}=\sum_{j=1}^{n} Y_{j1}$, $\overline{Y}_{n2}=\sum_{j=1}^{n} Y_{j2}$ and $\overline{Y}_{n3}==\sum_{j=1}^{n} Y_{j3}$ be the continuous random variables representing the amount of depletion of manpower in grades A,B and C respectively caused due to the j[th] transfer decision. $h_1(.)$, $h_2(.)$ and $h_3(.)$ are the p.d.f of Y_{j1}, Y_{j2} an. Let Z_1, Z_2 and Z_3 be independent exponentially distributed threshold levels for the depletion of manpower in grades A, B and C with mean $\frac{1}{\theta_1}, \frac{1}{\theta_2}\ and\ \frac{1}{\theta_3}$ respectively and let Z be the threshold level for the depletion of manpower in the organization with probability density function l(.). Let the inter-decision times for the three grades are correlated with distribution F(.) and the inter-transfer times with distribution V(.) and probability density function f(.) and v(.) respectively with mean $\frac{1}{\mu_1}$, $\frac{1}{\mu_2}$ ($\mu_1, \mu_2 > 0$). It is assumed that three sources of depletion are independent. Let T be the random variable denoting the time to recruitment with distribution L(.),mean E(T).

Results

The probability distribution of T is given by

$P(T > t) =${Probability that there are exactly m decisions in all the three grades and n transfer decisions and total loss of manpower does not cross the threshold Z}

$$P[T > t] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[F_m(t) - F_{m+1}(t)][V_n(t) - V_{n+1}(t)]P[\overline{X}_{m1} + \overline{X}_{m2} + \overline{X}_{m3} + \overline{Y}_{n1} + \overline{Y}_{n2} + \overline{Y}_{n3} < Z] \tag{1}$$

By the total law of probability

$$P[\overline{X}_{m1} + \overline{X}_{m2} + \overline{X}_{m3} + \overline{Y}_{n1} + \overline{Y}_{n2} + \overline{Y}_{n3} < Z] = \int_0^{\infty} P[\overline{X}_{m1} + \overline{X}_{m2} + \overline{X}_{m3} + \overline{Y}_{n1} + \overline{Y}_{n2} + \overline{Y}_{n3} < Z]k(z)\,dz \tag{2}$$

The explicit expressions for E (T) is obtained by using the equations (1) and (2)

Consider the threshold is the minimum of thresholds of all the three grades.

That is Z=min (Z_1 , Z_2 , Z_3)

$P(T > t) =${Probability that there are exactly m decisions in all the three grades and n transfer decisions and the total loss of manpower does not cross the threshold Z }

$$P(T>t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[F_m(t)-F_{m+1}(t)\right]\left[V_n(t)-V_{n+1}(t)\right]P\left[\overline{X_1}+\overline{X_2}+\overline{X_3}+\overline{Y_1}+\overline{Y_2}+\overline{Y_3}<Z\right]$$

$$P\left[\overline{X_1}+\overline{X_2}+\overline{X_3}+\overline{Y_1}+\overline{Y_2}+\overline{Y_3}<Z\right]=\int_0^{\infty}R_{\overline{x_1}+\overline{x_2}+\overline{x_3}+\overline{y_1}+\overline{y_2}+\overline{y_3}}(Z).\theta.e^{-\theta z}dz$$

Where $\theta=\theta_1+\theta_2+\theta_3$

$$=\theta\int_0^{\infty}R_{\overline{x_1}+\overline{x_2}+\overline{x_3}+\overline{y_1}+\overline{y_2}+\overline{y_3}}(Z).e^{-\theta z}dz$$

$$=\theta.R^*_{\overline{x_1}+\overline{x_2}+\overline{x_3}+\overline{y_1}+\overline{y_2}+\overline{y_3}}(\theta)$$

$$F_m^*(s)=\frac{1}{s}f_m^*(s)$$

$$P\left[\overline{X_1}+\overline{X_2}+\overline{X_3}+\overline{Y_1}+\overline{Y_2}+\overline{Y_3}<Z\right]=R^*_{\overline{x_1}+\overline{x_2}+\overline{x_3}+\overline{y_1}+\overline{y_2}+\overline{y_3}}(\theta)$$

$$=r_{\overline{x_1}}^*(\theta)r_{\overline{x_2}}^*(\theta)r_{\overline{x_3}}^*(\theta)r_{\overline{y_1}}^*(\theta)r_{\overline{y_2}}^*(\theta)r_{\overline{y_3}}^*(\theta)$$

$$=\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^m\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^n$$

$$P(T>t)=\sum_{m=0}^{\infty}\left[F_m(t)-F_{m+1}(t)\right]\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^m\sum_{n=0}^{\infty}\left[V_n(t)-V_{n+1}(t)\right]\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^n$$

$$L(t)=1-P(T>t)$$

$$=\left\{1-\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[\sum_{m=0}^{\infty}\left[F_m(t)-F_{m+1}(t)\right]\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\right]\right\}$$

$$\left\{1-\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[\sum_{m=0}^{\infty}\left[F_m(t)-F_{m+1}(t)\right]\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\right]\right\}$$

$$=\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[\sum_{m=0}^{\infty}\left[F_m(t)\right]\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\sum_{n=0}^{\infty}\left[V_n(t)-V_{n+1}(t)\right]\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^n\right]$$

$$l(t)=\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\sum_{m=1}^{\infty}f_m(t)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}$$
$$+\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\sum_{n=1}^{\infty}v_n(t)\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^{n-1}$$
$$-\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\sum_{m=1}^{\infty}f_m(t)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\sum_{n=1}^{\infty}V_n(t)\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^{n-1}$$
$$-\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\sum_{m=1}^{\infty}F_m(t)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\sum_{n=1}^{\infty}v_n(t)\left[h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^{n-1}$$

Now,

$$v_n(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}\quad\text{and}$$

$$V_n(t)=\int_0^t\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}dt$$

$$l(t)=\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]e^{-\lambda t\left(\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)}\sum_{m=1}^{\infty}f_m(t)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}$$
$$+\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\lambda e^{-\lambda t\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}$$
$$-\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\sum_{m=1}^{\infty}F_m(t)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\lambda e^{-\lambda t\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}$$

Taking Laplace transform,

$$l^*(s)=\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\sum_{m=1}^{\infty}f_m^*\left(s+\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}$$

$$+\frac{\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}{s+\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}-\frac{\lambda\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}{s+\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}$$

$$\left\{f_m^*\left(s+\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\right\}$$

$$\frac{-d}{ds}l^*(s)\Big|_{s=0}=\frac{1}{\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}-\sum_{m=1}^{\infty}f_m^*\left(\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}$$

$$\left\{\frac{\lambda\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}{\lambda^2\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]^2}\right\}$$

$$E(T)=\frac{1}{\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}-\frac{\left[1-g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]}{\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]}$$
$$\left\{\sum_{m=1}^{\infty}f_m^*\left(\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)\left[g_1^*(\theta)g_2^*(\theta)g_3^*(\theta)\right]^{m-1}\right\}$$

$$f_m^*\left(\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)=\frac{(1-R)\left[1+b\lambda\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right]^{1-m}}{\left(1-R+\left[1-h_1^*(\theta)h_2^*(\theta)h_3^*(\theta)\right]\right)\left[b\lambda-b\lambda R+mRb\lambda\right]}$$

G(.) and h(.) are exponentially distributed with parameters α and μ

$$g_1^*(\theta)=\frac{\alpha_1}{\alpha_1+\theta},\ g_2^*(\theta)=\frac{\alpha_2}{\alpha_2+\theta}\ \text{and}\ g_3^*(\theta)=\frac{\alpha_3}{\alpha_3+\theta}$$

$$h_1^*(\theta)=\frac{\mu_1}{\mu_1+\theta},\ h_2^*(\theta)=\frac{\mu_2}{\mu_2+\theta},\ \text{and}\ h_3^*(\theta)=\frac{\mu_3}{\mu_3+\theta},$$

$$E(T)=\frac{(\mu_1+\theta)(\mu_2+\theta)(\mu_3+\theta)}{\lambda\left[\theta^3+\theta^2(\mu_1+\mu_2+\mu_3)+\theta(\mu_1\mu_2+\mu_2\mu_3+\mu_3\mu_1)\right]}$$

$$\left\{1-\left[\begin{array}{l}\left\{\dfrac{(1-R)(\theta^3+\theta^2(\alpha_1+\alpha_2+\alpha_3)+\theta(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)}{b\lambda(1+R)\left[(\alpha_1+\theta)(\alpha_2+\theta)(\alpha_3+\theta)\right]^2}\right\} \\ \left\{\dfrac{\alpha_1\alpha_2\alpha_3}{\left[(\mu_1+\theta)(\mu_2+\theta)(\mu_3+\theta)+b\lambda\left[\theta^3+\theta^2(\mu_1+\mu_2+\mu_3)+\theta(\mu_1\mu_2+\mu_2\mu_3+\mu_3\mu_1)\right]\right]}\right\} \\ \left\{\dfrac{\left[(\mu_1+\theta)(\mu_2+\theta)(\mu_3+\theta)\right]^2}{\left[(1-R)(\mu_1+\theta)(\mu_2+\theta)(\mu_3+\theta)\right]+\theta^3+\theta^2(\mu_1+\mu_2+\mu_3)+\theta(\mu_1\mu_2+\mu_2\mu_3+\mu_3\mu_1)}\right\}\end{array}\right]\right\} \quad (4)$$

3. Conclusion

Expected time to recruitment can be obtained from equation (4). The results of any research work place a vital role for real applications. These results are derived on the basis of real factors and it is of great need in case of stochastic models. The applications of stochastic models are very essential and also very much useful in human activities**.** When an unbalanced condition arises between the demand of manpower and supply, it is very important to identify in the areas of human activity. The transformation of real time situation into mathematical model and the identification on the demand for manpower are to be analyzed for the development of human resource management.

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