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Phase Impairment Estimation for mmWave MIMO Systems with Low Resolution ADC and Imperfect CSI

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Abstract

Multiple-Input Multiple-Output systems operating at millimeter wave band (mmWave MIMO) are a promising technology next generations of mobile networks. In practice, the non-ideal hardware is a challenge for commercially viable mmWave MIMO transceivers and come from non-linearities of the amplifier, phase noise, quantization errors, mutual coupling between antenna ports, and In-phase/Quadrature (I/Q) imbalance. As a result, the received signals are affected by non-ideal transceiver hardware components, thus reduce the performance of such systems, especially phase impairment caused by phase noise and carrier frequency offset (CFO). In this paper, we consider a mmWave MIMO system model that takes into account many practical hardware impairments and imperfect channel state information (CSI). Our main contributions are a problem formulation of phase impairments with imperfect CSI and a low-complexity estimation method to solve the problem. Numerical results are provided to evaluate the performance of the proposed algorithm.

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1. Introduction

Millimeter-wave multiple-input multiple-output (mmWave MIMO) systems can leverage the advantages of multiple-antenna technologies and the large bandwidth of mmWave band to provide high-speed, reliable and low-latency transmissions for next generations of mobile networks. Nevertheless, there are many challenges to realize all the benefits of mmWave MIMO systems in practice. One of such challeges is to reduce the costs related to both hardware components and computational complexity of algorithms. Given the use of a large number of antennas and radio frequency (RF) chains in mmWave MIMO systems, a natural approach is to use inexpensive hardware components at the cost of lower accuracy and hence performance degradation. This approach often causes changing the phase of transmited signal and/or received signal, unbalance of in phase channel and quadrature phase channel, nonlinear power amplifier, and power losses caused by stage of beamforming in analog domain [1], [2], [3]. The impact of non-ideal transceiver hardware can be reduce by calibration and compensation methods. Due to the influence of many factors, the impact of non-ideal hardware cannot be eliminated completely [1], [3], [4], [5], [6]. Phase noise is one of the most important hardware impairment comes from non-ideal oscillators of the transceiver during the up and down conversion between baseband and bandpass signal [7], [1], [8]. Basically, there are two types of using oscillators at transceiver, named common local oscillators (CLO) and separate local oscillators (SLO). In case of SLO, each RF chain is connected to an oscillator. In case of CLO, all RF chains are connected to one oscillator [3]. Phase noise is multiplied by output signal at oscillators. Due to the high oscillation frequency of mmWave transceivers and difficult estimation, phase noise is a significant challenge. In addition, using low

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resolution of ADC/DACs has many challenges such as quantization noise and affecting on the other blocks of receivers [9], [10], [11].

Recently, the effect of non-ideal transceiver hardware for MIMO systems at low frequency has obtained a lot of attentions. In [1], Hani Mehrpouyan *et al.* has analyzed how phase noise effect on the performance of MIMO systems operating at low frequency. In this work, authors proposed a method to simultaneously estimate channel gains and phase noise by using an Extended Kalman Filter (EKF). Hence, bit error ratio (BER) performance can be significantly improved. Koohian *et al.* in [12] studied full duplex MIMO systems at millimeter wave band. Authors proposed a method to simultaneously estimate channel gains and phase noise by using an EKF and figured out the lower bound. Emil *et al.* in [3] has discuss how non-ideal transmitter hardware impact on massive MIMO systems and figured out the achievable data rate. Furthermore, authors also point out that, in case of SLO architecture higher data rates are provided than in the case CLO architecture, however, the trade-off is that power consumption will increase. Prior work has also considered channel gains estimation problem under the effect of phase noise and carrier frequency offset for OFDM systems in [13], and [14]. Myer *et al.* in [8], [15] has investigated the combining of phase noise and channel estimation technique for fully digital beamforming mmWave systems with few bit quantization. The authors in [15] proposed lifting techniques to solve estimation problem for narrowband channel systems while [8] proposed message passing technique for wideband channel estimation. Nuria *et al.* in [16], and [17] has analyzed the combining channel estimation and synchronization for mmWave MIMO hybrid analog and digital beamforming mmWave MIMO systems with assumption that phase noise and quantization noise can be ignored. Corvaja *et al.* in [5] has considered designing analog/digital precoding and combining matrices under the effect of phase noise at transceiver. The authors in [2] has discuss how the impact of non-ideal hardware on mmWave massive MIMO systems with fully array connected architecture. They figured out the achievable data rate and showed that phase noise is the highest degradation to the system performance.

The effect of quantization has considered in many researches. Ribeiro *et al.* in [9] has analyzed fully array connected mmWave MIMO systems with low bit DACs and showed that DACs with the number of bit greater than 3 do not significantly improve spectral efficiency, but also reduce energy efficiency. The authors in [18] has studied partially array connected mmWave MIMO systems with low bit DACs and proposed a method to select the number of RF chains so, energy efficiency increased 30%. Assumed the number of bit resolution ADCs on RF chains are the same, J.Mo *et al.* in [10] figured out the trade off between power consumption and achievable data rate of a hybrid beamforming mmWave MIMO with fully array connected architecture. The results of simulation showed that ADCs at receivers with 4-5 bits quantization is the best for system performance. The authors in [19] has investigated how the quantization noise effect on massive MIMO systems with fully digital beamforming architecture and hybrid analog/digital beamforming. Assumed the number of bit resolution ADCs on RF chains are the different, in [20] proposed the ADC bit-allocation algorithm for mmWave MIMO systems with hybrid beamforming to overcome the effects of quantization noise.

In this paper, we consider a mmWave MIMO system model that takes into account imperfect CSI model, hybrid beamforming, and many hardware impairments, such as quantization noise, phase noise, and carrier frequency offset. Assuming the available knowledge of other modeling parameters, we formulate an estimation problem of phase impairments due to varying phase noise and constant carrier frequency offset. We propose a new low-complexity linear method for phase impairment estimation to solve the formulated problem. Numerical results are provided to evaluate the accuracy of the proposed method. Our major contribution are summarized as follows:

1. We formulate an estimation problem of phase impairments for a quite realistic mmWave MIMO system model. The effect of timming and synchronization error is ignored. Phase noise is assumed fixed in a symbol [21]. Training symbols is assumed beginning each frame.

2. We propose a low complexity method to solve the formulated phase impairment estimation problem. The main idea is using vectorization received signal to estimate the vector of phase rotation factors affecting the received pilot symbols due to phase noise values at the receiver. This paper focused on phase impairment estimation in mmWave MIMO systems. Noted that, system model in this paper is more general than the almost previous studies relevant to systems, because it included phase noise component [3], CFO component [8] and quantization noise component [11] in hybrid beamforming mmWave MIMO systems.

3. Our proposed method is evaluated based on the Mean Square Error (MSE) for various simulation scenarios. Numerical results show that our proposed method works quite well in a wide range of phase noise variances.

The remainder of this paper is organized as follows. Section 2 introduces the system model and the signal model. Section 3 formulates the phase impairment estimation problem and then proposes a low complexity estimation method. Section 4 provides numerical results. Section 5 concludes the paper and suggests some directions for future work.

Notation: a, $\mathbf{a}$, $\mathbf{A}$ is a scalar, a vector and a matrix, respectively. $\mathbf{A}^T$, $\mathbf{A}^H$, $\|\mathbf{A}\|_F$ represents the transpose, the Hermitian (conjugate transpose) and Frobenius norm of a matrix $\mathbf{A}$, respectively. $\mathrm{tr}(\mathbf{A})$ is the trace of a matrix. $\mathbf{I}_N$ stands for an N-by-N identity matrix. $\mathbf{1}_N$ stand for an N-by-1 vector of all ones. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix. $\mathbf{R}_{\mathbf{ab}}$ is the correlation matrix of vector $\mathbf{a}$ and $\mathbf{b}$ defined as $\mathbb{E}\left[\mathbf{ab}^H\right]$. $\mathbf{A} \otimes \mathbf{B} \in \mathbb{C}^{MP \times NQ}$ is the Kronecker product of $\mathbf{A} \in \mathbb{C}^{M \times N}$ and $\mathbf{B} \in \mathbb{C}^{P \times Q}$. $\mathbf{A} \odot \mathbf{B} = [\mathbf{a}_1 \otimes \mathbf{b}_1 \ \mathbf{a}_2 \otimes \mathbf{b}_2 \ \cdots \ \mathbf{a}_N \otimes \mathbf{b}_N]$ is the Khatri-Rao product of $\mathbf{A} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_N] \in \mathbb{C}^{M \times N}$ and $\mathbf{B} = [\mathbf{b}_1 \ \mathbf{b}_2 \ \cdots \ \mathbf{b}_N] \in \mathbb{C}^{P \times N}$.

2. System model

In this paper we consider a single-user point-to-point mmWave MIMO system where a transmitter with N_t antennas communicates with a receiver with N_r antennas, as illustrated in Fig. 1. Assume a discrete-time block-fading channel model where the channel coefficients remain unchanged within the duration of a radio frame and change randomly and independently frame-by-frame. Let K be the number of symbol periods in each radio frame. Denote k as the symbol index within a frame, where $k \in \mathcal{K} = \{1, 2, \cdots, K\}$. Without loss of generation, we assume that the K_p pilot symbols are located at the beginning of the frame [3]. The remaining K_d symbols, where $K = K_\mathrm{p} + K_\mathrm{d}$, are used for data transmission. Let $\mathcal{K}_\mathrm{p} = \{1, 2, \cdots, K_\mathrm{p}\}$ be the index set of the pilot symbols and $\mathcal{K}_\mathrm{d} = \{K_\mathrm{p} + 1, K_\mathrm{p} + 2, \cdots, K\}$ be the index set of the data symbols.

To focus on phase impairment effects, we assume that the receiver has obtained an estimate $\hat{\mathbf{H}} \in \mathbb{C}^{N_\mathrm{r} \times N_\mathrm{t}}$ of the matrix of channel coefficients $\mathbf{H} \in \mathbb{C}^{N_\mathrm{r} \times N_\mathrm{t}}$. Let $\tilde{\mathbf{H}} \in \mathbb{C}^{N_\mathrm{r} \times N_\mathrm{t}}$ be the channel estimation error matrix. We assume that the entries of both $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. In addition, for tractability, we adopt the following model where $\mathbf{H}$ is decomposed as

$$\mathbf{H} = \sqrt{1 - \alpha^2}\hat{\mathbf{H}} + \alpha\tilde{\mathbf{H}} \tag{1}$$

where $\alpha \in [0, 1]$ represents the level of estimation accuracy. Note that $\alpha = 0$ corresponds to perfect channel estimation, i.e. $\hat{\mathbf{H}} = \mathbf{H}$ and that $\alpha = 1$ corresponds to the case that $\hat{\mathbf{H}}$ is independent of $\mathbf{H}$. For notational convenience, denote $\bar{\alpha} = \sqrt{1 - \alpha^2} \in [0, 1]$.

Since mmWave propagation environment has limited scattering, the mmWave channel can be described by a geometric channel model with L scatterers. Moreover, for ease of illustration, it is assumed that each scatterer contributes a propagation path between the transmitter and the receiver. Let $\phi_{\mathrm{t},\ell} \in [0, 2\pi]$ and $\phi_{\mathrm{r},\ell} \in [0, 2\pi]$ are angles of departure (AoD) and angles of arrival (AoA) corresponding to the ℓ-th path, for $\ell = 1, 2, ..., L$. Denote $\mathbf{a}_\mathrm{t}(\phi_{\mathrm{t},\ell})$ as the antenna array's steering vector at the transmitter and $\mathbf{a}_\mathrm{r}(\phi_{\mathrm{r},\ell})$ as the antenna array's response vector at the receiver for the ℓ-th path. As a result, the channel matrix can be decomposed as

$$\hat{\mathbf{H}} = \alpha \sum_{\ell=1}^{L} \beta_\ell \mathbf{a}_\mathrm{t}(\phi_{\mathrm{t},\ell})\mathbf{a}_\mathrm{r}^H(\phi_{\mathrm{r},\ell}) \tag{2}$$

where α is the normalization factor such that $\mathbb{E}\left[\hat{\mathbf{h}}\hat{\mathbf{h}}^H\right] = 1$ and β_ℓ is the complex gain of the ℓ-th propagation path.

Let $N_\mathrm{s}[k]$ be the number of complex-valued modulated symbols to be transmitted at symbol $k \in \mathcal{K}$. We assume that $N_\mathrm{s}[k] = N_\mathrm{p}$ for all $k \in \mathcal{K}_\mathrm{p}$, i.e. symbol k is used for pilot transmission, and that $N_\mathrm{s}[k] = N_\mathrm{d}$ for all $k \in \mathcal{K}_\mathrm{d}$, i.e. symbol k is used for data transmission. Let $\mathbf{s}[k] \in \mathbb{C}^{N_\mathrm{s}[k] \times 1}$ be the symbol vector to be transmitted at symbol $k \in \mathcal{K}$ such that $\mathbb{E}(\mathbf{s}[k]\mathbf{s}^H[k]) = \frac{1}{N_\mathrm{s}[k]}\mathbf{I}_{N_\mathrm{s}[k]}$. We assume that the same set of M_t RF transmit chains are used at the transmitter for both pilot transmission and data transmission. Assume that both pilot transmission and data transmission rely on a hybrid precoding method. The transmission procedure is as follows: First, the symbol vector is precoded at the baseband by a digital precoder. The output of the digital precoder is converted from the digital domain into the analog domain and then up-converted to RF before being transmitted via the antennas into the propagation environment. To focus on the hardware impairment at the receiver, we assume that the digital-to-analog converters at the transmitter have relatively high resolutions so that quantization noise is negligible [10, 11]. We also assume that the phase noise and CFO effects at the transmitter is negligible [8, 15]. These assumptions are reasonable for downlink transmission where the base station is often composed of high-quality electronic components.

The transmitted signal is given by

$$\mathbf{x}[k] \quad = \mathbf{F}_\mathrm{RF}[k]\mathbf{F}_\mathrm{BB}[k]\mathbf{s}[k] \tag{3}$$

where $\mathbf{F}_\mathrm{BB}[k] \in \mathbb{C}^{M_\mathrm{t} \times N_\mathrm{s}[k]}$ is the precoder in the digital domain; $\mathbf{F}_\mathrm{RF}[k] \in \mathbb{C}^{N_\mathrm{t} \times M_\mathrm{t}}$ is the precoder in the analog domain.

The receiver applies a hybrid combining method to the signal received at the antennas. Denote $\mathbf{W}_\mathrm{RF}[k] \in \mathbb{C}^{N_\mathrm{r} \times M_\mathrm{r}}$ and $\mathbf{W}_\mathrm{BB}[k] \in \mathbb{C}^{M_\mathrm{r} \times N_\mathrm{s}[k]}$ as the analog combining matrix and the digital (baseband) combining matrix, respectively, where M_r is the number of RF receive chains at the receiver. For notational convenience, denote $\mathbf{F}[k] = \mathbf{F}_\mathrm{RF}[k]\mathbf{F}_\mathrm{BB}[k] \in \mathbb{C}^{N_\mathrm{t} \times N_\mathrm{s}[k]}$ and

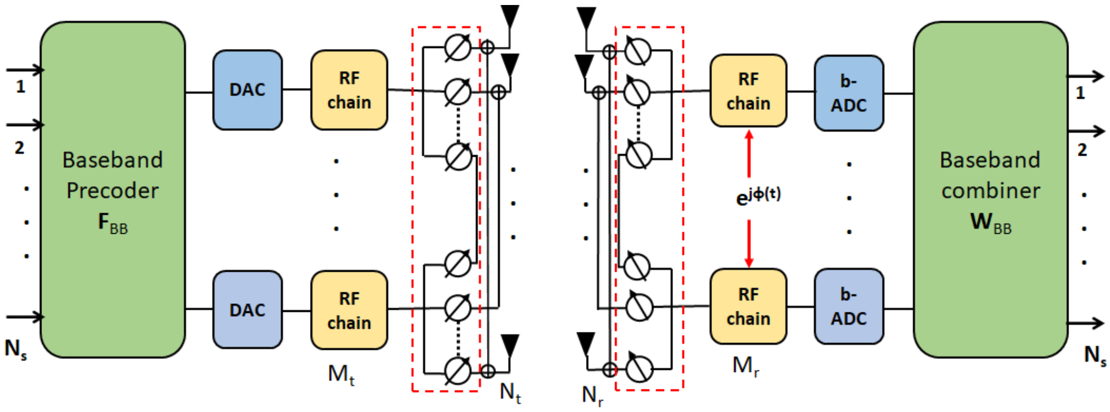

Figure 1. The diagram of the considered mmWave MIMO system model with non-ideal hardware.

$\mathbf{W}[k] = \mathbf{W}_{\mathrm{RF}}[k]\mathbf{W}_{\mathrm{BB}}[k] \in \mathbb{C}^{N_r \times N_s[k]}$ as the equivalent hybrid analog-digital precoding and combining matrix. To focus on phase impairment estimation, it is assumed that $\mathbf{F}_{\mathrm{RF}}[k]$, $\mathbf{F}_{\mathrm{BB}}[k]$, $\mathbf{W}_{\mathrm{RF}}[k]$ and $\mathbf{W}_{\mathrm{BB}}[k]$ have been known, thus $\mathbf{F}[k] = \mathbf{F}$ and $\mathbf{W}[k] = \mathbf{W}$ has also been known, for all $k \in \mathcal{K}$.

We assume that a common CLO is used for all RF receive chains, thus the all the RF chains are affected by the same random phase noise $\theta[k]$, for all $n = 1, 2, \cdots, M_r$. Assuming the discrete-time Wiener phase noise model [3], we have for all $k \in \mathcal{K}_p \setminus \{1\}$

$$\theta[k] = \theta[k-1] + \psi[k] \tag{4}$$

where $\theta[k-1]$ is the phase noise value affecting the RF receive chains in the previous symbol and $\psi[k]$ is an independent phase innovation. For tractable analysis, we assume that the phase innovation values at the RF receive chains are i.i.d. white real Gaussian random variables, i.e. $\psi[k] \sim \mathcal{CN}(0, \sigma_\psi^2)$ for all $k \in \mathcal{K}_p \setminus \{1\}$ and $n = 1, 2, \cdots, M_r$. The phase noise variance may be computed as $\sigma_\psi^2 = 4\pi^2 f_c c_n T_s$ where f_c is the carrier frequency, c_n is a constant that characterizes the quality of the LO and T_s is the symbol time [1–3]. We also assume that the received signal is affected by a constant CFO, which is denoted as Δf. This CFO results from the differences between the local oscillator at the transmitter and that at the receiver.

The received training signal in the analog domain at the output of the RF receive chains, is given by

$$\mathbf{y}_{\mathrm{p,A}}[k] = e^{j\beta[k]}\mathbf{W}_{\mathrm{RF}}^H\mathbf{H}\mathbf{x}_{\mathrm{p}}[k] + \mathbf{W}_{\mathrm{RF}}^H\mathbf{n}_{\mathrm{p}}[k] \tag{5}$$

where $\beta[k] = \theta[k] + 2\pi\Delta f k, \forall k \in \mathcal{K}_p$ represents that phase impairment at training symbol $k \in \mathcal{K}_p$ and $\mathbf{n}_{\mathrm{p}}[k] \sim \mathcal{CN}(\mathbf{0}_{N_r}, \sigma_n^2\mathbf{I}_{N_r})$ is the vector of AWGN.

The covariance matrix of $\mathbf{y}_{\mathrm{p,A}}[k]$ is denoted as $\mathbf{R}_{\mathbf{y}_{\mathrm{A}}}[k]$ and is given by

$$\mathbf{R}_{\mathbf{y}_{\mathrm{p,A}}}[k] = \mathbb{E}\left[\mathbf{y}_{\mathrm{p,A}}[k]\mathbf{y}_{\mathrm{p,A}}^H[k]\right] \tag{6}$$

The received signal in the analog domain $\mathbf{y}_{\mathrm{p,A}}[k]$ is converted into the digital domain by a number of ADCs. Let $\mathcal{Q}(.)$ denote the input-output relationship of the ADC signal process. We assume that the ADCs work with the same resolution level of b bits and hence the same distortion factor ξ. The distortion factor of ADC can be computed as

$$\xi = \frac{\pi\sqrt{3}}{2}2^{-2b}. \tag{7}$$

Define $\bar{\xi} = 1 - \xi$, assuming that the additive quantization noise model (AQNM) and the gain of the automatic gain control is set appropriately [10]. The output signal of the ADCs is written as

$$\mathbf{y}_{\mathrm{p,BB}}[k] = \bar{\xi}\mathbf{y}_{\mathrm{p,A}}[k] + \mathbf{e}_{\mathrm{A}}[k] \tag{8}$$

where $\mathbf{e}_{\mathrm{A}}[k] \in \mathbb{C}^{M_r \times 1}$ is the additive quantization noise vector with the probability distribution $\mathcal{CN}(\mathbf{0}, \mathbf{R}_{e_{\mathrm{A}}}[k])$.

The covariance matrix of $\mathbf{e}_{\mathrm{A}}[k]$ is computed as [20]

$$\mathbf{R}_{\mathbf{e}_{\mathrm{A}}}[k] = \mathbb{E}(\mathbf{e}_{\mathrm{A}}[k]\mathbf{e}_{\mathrm{A}}^H[k]) = \xi\bar{\xi}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p,A}}}[k]\right). \tag{9}$$

The post-processed received baseband signal at symbol $k \in \mathcal{K}_p$ is

$$\begin{aligned}\mathbf{y}_{\mathrm{p}}[k] &= \mathbf{W}^H\left(\bar{\xi}e^{j\beta[k]}\mathbf{H}\mathbf{F}\mathbf{s}[k] + \bar{\xi}\mathbf{n}_{\mathrm{p}}[k]\right) + \mathbf{W}_{\mathrm{BB}}^H\mathbf{e}_{\mathrm{A}}[k]\\ &= \underbrace{\bar{\xi}\bar{\alpha}e^{j\beta[k]}\mathbf{W}^H\hat{\mathbf{H}}\mathbf{F}\mathbf{s}[k]}_{\text{desired training signal}}\\ &\quad + \underbrace{\bar{\xi}\alpha e^{j\beta[k]}\mathbf{W}^H\tilde{\mathbf{H}}\mathbf{F}\mathbf{s}[k] + \bar{\xi}\mathbf{W}^H\mathbf{n}_{\mathrm{p}}[k] + \mathbf{W}_{\mathrm{BB}}^H\mathbf{e}_{\mathrm{A}}[k]}_{\text{effective noise}}.\end{aligned} \tag{10}$$

The receiver has to estimate the phase impairment values, which include Δf and $\theta[k], \forall k \in \mathcal{K}_p$, based on the post-processed received signal $\mathbf{y}_{\mathrm{p}}[k], \forall k \in \mathcal{K}_p$.

3. Linear CLO-based phase impairment estimation

3.1. Problem formulation

In this section, we assume that K_p pilot symbols are located at the beginning of each frame. To focus on phase impairment estimation, we assume that fixed precoders and combiners are used in the pilot transmission stage, i.e. $\mathbf{F}_\mathrm{BB}[k] = \mathbf{F}_\mathrm{BB}$, $\mathbf{F}_\mathrm{RF}[k] = \mathbf{F}_\mathrm{RF}$, $\mathbf{W}_\mathrm{BB}[k] = \mathbf{W}_\mathrm{BB}$, and $\mathbf{W}_\mathrm{RF}[k] = \mathbf{W}_\mathrm{RF}$ for all $k \in \mathcal{K}_\mathrm{p}$. As a result, we have $\mathbf{F}_\mathrm{p} = \mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}$ and $\mathbf{W}_\mathrm{p} = \mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}$. The covariance matrix of the transmitted signal is computed as

$$\begin{aligned}\mathbf{R}_\mathbf{x}[k] &= \mathbb{E}\left(\mathbf{x}[k]\mathbf{x}^H[k]\right)\\ &= \frac{1}{N_\mathrm{p}}\mathbf{F}_\mathrm{p}\mathbf{F}_\mathrm{p}^H. \end{aligned} \tag{11}$$

The covariance matrix of $\mathbf{y}_\mathrm{A}[k]$ is denoted as $\mathbf{R}_{\mathbf{y}_\mathrm{A}}[k]$ and is given by

$$\begin{aligned}\mathbf{R}_{\mathbf{y}_\mathrm{A}}[k] =&\mathbb{E}\left[\mathbf{y}_\mathrm{A}[k]\mathbf{y}_\mathrm{A}^H[k]\right]\\ =&\frac{1}{N_\mathrm{p}}\mathbf{W}_\mathrm{RF}^H\mathbf{H}\mathbf{F}_\mathrm{p}\mathbf{F}_\mathrm{p}^H\mathbf{H}^H\mathbf{W}_\mathrm{RF} + \sigma_\mathrm{n}^2\mathbf{W}_\mathrm{RF}^H\mathbf{W}_\mathrm{RF}. \end{aligned} \tag{12}$$

The covariance matrix of $\mathbf{e}_\mathrm{A}[k]$ is computed as [20]

$$\begin{aligned}\mathbf{R}_{\mathbf{e}_\mathrm{A}}[k] =&\mathbb{E}(\mathbf{e}_\mathrm{A}[k]\mathbf{e}_\mathrm{A}^H[k])\\ =&\xi\bar{\xi}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_\mathrm{A}}[k]\right). \end{aligned} \tag{13}$$

Define $\mathbf{z}_\mathrm{p} = [e^{j\beta[1]},\ e^{j\beta[2]},\ \cdots,\ e^{j\beta[K_\mathrm{p}]}]^T \in \mathbb{C}^{K_\mathrm{p}\times 1}$ as the vector of phase rotation factors affecting the received pilot symbols due to phase noise values and CFO at the receiver. Note that the amplitude of each entry of $\mathbf{z}_\mathrm{p}$ is equal to one $|e^{j\beta[k]}| = 1, \forall k \in \mathcal{K}_\mathrm{p}$. We formulation a problem for linear estimating the phase impairment vector $\mathbf{z}_\mathrm{p}$ given the knowledge of the received training symbols $\mathbf{y}_\mathrm{p}[k]$ for $k \in \mathcal{K}_\mathrm{p}$, channel matrix $\mathbf{H}$, precoding matrices, combining matrices and statistical information of phase impairment as follows

$$\begin{aligned}\mathbb{E}(\mathbf{z}_\mathrm{p}) &= \left[\mathbb{E}(e^{j\beta[1]}),\ \mathbb{E}(e^{j\beta[2]}),\ \cdots,\ \mathbb{E}(e^{j\beta[K_\mathrm{p}]})\right]^T\\ &= \left[e^{\hat{\sigma_\psi}},\ e^{\hat{\sigma_\psi}\cdot 2},\ \cdots,\ e^{\hat{\sigma_\psi}\cdot K_\mathrm{p}}\right]^T \end{aligned} \tag{14}$$

$$\begin{aligned}\mathbf{R}_{z_\mathrm{p}} &= \mathbb{E}(\mathbf{z}_\mathrm{p}\mathbf{z}_\mathrm{p}^*)\\ &= \begin{bmatrix} 1 & e^{\hat{\sigma_\psi}} & \cdots & e^{\hat{\sigma_\psi}(K_\mathrm{p}-1)}\\ e^{\hat{\sigma_\psi}} & 1 & \cdots & e^{\hat{\sigma_\psi}(K_\mathrm{p}-2)}\\ \vdots & \vdots & \ddots & \vdots\\ e^{\hat{\sigma_\psi}(K_\mathrm{p}-1)} & e^{\hat{\sigma_\psi}(K_\mathrm{p}-2)} & \cdots & 1 \end{bmatrix}. \end{aligned} \tag{15}$$

where $\hat{\sigma_\psi} = -\frac{\sigma_\psi^2}{2} + 2\pi\Delta f$.

Define $\mathbf{Y}_\mathrm{p} = \left[\mathbf{y}_\mathrm{p}[1],\ \mathbf{y}_\mathrm{p}[2],\ \cdots,\ \mathbf{y}_\mathrm{p}[K_\mathrm{p}]\right] \in \mathbb{C}^{N_\mathrm{p}\times K_\mathrm{p}}$ as the processed pilot signal matrix in one frame. We can rewrite $\mathbf{Y}_\mathrm{p}$ as follows

$$\begin{aligned}\mathbf{Y}_\mathrm{p} =&\bar{\xi}\bar{\alpha}\mathbf{W}_\mathrm{p}^H\hat{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\mathrm{diag}(\mathbf{z}_\mathrm{p}) + \bar{\xi}\alpha\mathbf{W}_\mathrm{p}^H\tilde{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\mathrm{diag}(\mathbf{z}_\mathrm{p})\\ &+\bar{\xi}\mathbf{W}_\mathrm{p}^H\mathbf{N}_\mathrm{p} + \mathbf{W}_\mathrm{BB}^H\mathbf{E}_\mathrm{A}. \end{aligned} \tag{16}$$

Where: $\mathbf{S}_\mathrm{p} = \left[\mathbf{s}_\mathrm{p}[1],\ \mathbf{s}_\mathrm{p}[2],\ \cdots,\ \mathbf{s}_\mathrm{p}[K_\mathrm{p}]\right] \in \mathbb{C}^{N_\mathrm{p}\times K_\mathrm{p}}$ as the matrix of transmitted pilot symbols and $\mathbf{s}_\mathrm{p} = \mathrm{vec}(\mathbf{S}_\mathrm{p}) \in \mathbb{C}^{N_\mathrm{p}K_\mathrm{p}\times 1}$ as the vectorized version of $\mathbf{S}_\mathrm{p}$; $\mathbf{E}_\mathrm{A} = \left[\mathbf{e}_\mathrm{A}[1],\ \mathbf{e}_\mathrm{A}[2],\ \cdots,\ \mathbf{e}_\mathrm{A}[K_\mathrm{p}]\right] \in \mathbb{C}^{M_\mathrm{r}\times K_\mathrm{p}}$ as the additive quantization noise matrix due to the ADCs during the pilot transmission; $\mathbf{N}_\mathrm{p} = \left[\mathbf{n}[1],\ \mathbf{n}[2],\ \cdots,\ \mathbf{n}[K_\mathrm{p}]\right] \in \mathbb{C}^{N_\mathrm{r}\times K_\mathrm{p}}$ as the AWGN matrix at the receiver.

By applying the equation $\mathrm{vec}(\mathbf{XZ}) = (\mathbf{I}_k \odot \mathbf{X})\,\mathrm{vec}(\mathbf{Z})$ for any $\mathbf{X} \in \mathbb{C}^{m\times n}$, $\mathbf{Y} \in \mathbb{C}^{n\times n}$ and $\mathbf{Z} \in \mathbb{C}^{n\times k}$ where $\mathbf{Y}$ is a diagonal matrix and $\mathrm{vec}(\mathbf{XYZ}) = \left(\mathbf{Z}^T \odot \mathbf{Z}\right)\mathrm{diag}(\mathbf{Y})$. Vectorizing $\mathbf{Y}_\mathrm{p}$, we obtain $\mathbf{y}_\mathrm{p} = \mathrm{vec}(\mathbf{Y}_\mathrm{p}) \in \mathbb{C}^{N_\mathrm{p}K_\mathrm{p}\times 1}$, which is given by

$$\begin{aligned}\mathbf{y}_\mathrm{p} =&\bar{\xi}\bar{\alpha}\mathrm{vec}\left(\mathbf{W}_\mathrm{p}^H\hat{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\mathrm{diag}(\mathbf{z}_\mathrm{p})\right) + \bar{\xi}\alpha\mathrm{vec}\left(\mathbf{W}_\mathrm{p}^H\tilde{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\mathrm{diag}(\mathbf{z}_\mathrm{p})\right)\\ &+\bar{\xi}\mathrm{vec}\left(\mathbf{W}_\mathrm{p}^H\mathbf{N}_\mathrm{p}\right) + \mathrm{vec}\left(\mathbf{W}_\mathrm{BB}^H\mathbf{E}_\mathrm{A}\right)\\ &=\underbrace{\bar{\xi}\bar{\alpha}\left(\mathbf{I}_{K_\mathrm{p}} \odot \left(\mathbf{W}_\mathrm{p}^H\hat{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\right)\right)}_{\mathbf{G}_1}\mathbf{z}_\mathrm{p} + \underbrace{\bar{\xi}\alpha\left(\mathbf{I}_{K_\mathrm{p}} \odot \left(\mathbf{W}_\mathrm{p}^H\tilde{\mathbf{H}}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p}\right)\right)}_{\mathbf{G}_2}\mathbf{z}_\mathrm{p}\\ &+\underbrace{\bar{\xi}\left(\mathbf{I}_{K_\mathrm{p}} \odot \mathbf{W}_\mathrm{p}^H\right)\mathrm{vec}(\mathbf{N}_\mathrm{p})}_{\mathbf{g}_3} + \underbrace{(\mathbf{I}_{K_\mathrm{p}} \otimes \mathbf{W}_\mathrm{BB}^H)\mathrm{vec}(\mathbf{E}_\mathrm{A})}_{\mathbf{g}_4}\\ =&\mathbf{G}_1\mathbf{z}_\mathrm{p} + \mathbf{G}_2\mathbf{z}_\mathrm{p} + \mathbf{g}_3 + \mathbf{g}_4. \end{aligned} \tag{17}$$

Due to the above mentioned assumptions on the beamforming matrices, and on the training signal matrix, the receiver knows $\mathbf{G}_1$ perfectly while it has only statistical information on $\mathbf{G}_2, \mathbf{g}_3$ and $\mathbf{g}_4$, since they are random vectors. Moreover, since all the entries of the random vectors/matrices in $\mathbf{N}_\mathrm{p}$ and $\mathbf{E}_\mathrm{A}$ have zero mean, then we have $\mathbb{E}[\mathbf{G}_2] = \mathbb{E}[\mathbf{g}_3] = \mathbb{E}[\mathbf{g}_4] = \mathbf{0}$.

3.2. Proposed Least Squares estimation solution

Define $\hat{\mathbf{z}}_\mathrm{LS}$ as the linear LS estimate of $\mathbf{z}_\mathrm{p}$. Given that the receiver knows $\mathbf{G}_1$ perfectly and by ignoring the terms with unknown coefficients in $\mathbf{y}_\mathrm{p} = f(\mathbf{z}_\mathrm{p})$, we formulate a linear LS estimation problem of $\mathbf{z}_\mathrm{p}$ as follows [22]

$$\hat{\mathbf{z}}_\mathrm{LS} = \underset{\mathbf{z}_\mathrm{p}\in\mathbb{C}^{K_\mathrm{p}\times 1}}{\arg\min}\left\|\mathbf{y}_\mathrm{p} - \mathbf{G}_1\mathbf{z}_\mathrm{p}\right\|_F^2 \tag{18}$$

After some manipulation, we obtain the linear LS estimate as

$$\begin{aligned}\hat{\mathbf{z}}_\mathrm{LS} =&\mathbf{G}_1^\dagger\mathbf{y}_\mathrm{p}\\ =&(\mathbf{G}_1^H\mathbf{G}_1)^{-1}\mathbf{G}_1^H\mathbf{y}_\mathrm{p}. \end{aligned} \tag{19}$$

Define $\mathbf{H}_\mathrm{LS} = \mathbf{W}_\mathrm{p}^H\mathbf{H}\mathbf{F}_\mathrm{p}\mathbf{S}_\mathrm{p} = [\mathbf{h}_{\mathrm{LS},1}\ \ \mathbf{h}_{\mathrm{LS},2}\ \ \cdots\ \ \mathbf{h}_{\mathrm{LS},K_\mathrm{p}}] \in \mathbb{C}^{N_\mathrm{p}\times K_\mathrm{p}}$ as the baseband equivalent end-to-end channel estimate. Given the channel $\mathbf{H}$, $\mathbf{H}_{\mathrm{LS}}$ and hence $\mathbf{h}_{\mathrm{LS,k}} \in \mathbb{C}^{N_{\mathrm{p}}\times 1}$ for all $k \in \{1, 2, \cdots, K_{\mathrm{p}}\}$ can be computed exactly in advance at the receiver. Thus, we have

$$\hat{\mathbf{z}}_{\mathrm{LS}} = (\mathbf{G}_1^H\mathbf{G}_1)^{-1}\mathbf{G}_1^H\mathbf{y}_{\mathrm{p}} = \left(\bar{\xi}\right)^{-1}\begin{bmatrix} \frac{\mathbf{h}_{\mathrm{LS},1}^H\mathbf{y}_{\mathrm{p}}[1]}{|\mathbf{h}_{\mathrm{LS},1}^H|_2^{-2}} \\ \frac{\mathbf{h}_{\mathrm{LS},2}^H\mathbf{y}_{\mathrm{p}}[2]}{|\mathbf{h}_{\mathrm{LS},2}^H|_2^{-2}} \\ \vdots \\ \frac{\mathbf{h}_{\mathrm{LS},K_{\mathrm{p}}}^H\mathbf{y}_{\mathrm{p}}[K_{\mathrm{p}}]}{|\mathbf{h}_{\mathrm{LS},K_{\mathrm{p}}}^H|_2^{-2}} \end{bmatrix} \tag{20}$$

where $\mathbf{y}_{\mathrm{p}}[k] = \mathbf{y}_{\mathrm{p}}[(k-1)*N_{\mathrm{p}}+1 : k*N_{\mathrm{p}}]$ is the vector of the elements from the $((k-1)*N_{\mathrm{p}}+1)$-th index to the $k*N_{\mathrm{p}}$-th index. Thus, the LS estimate of the phase impairment at the pilot symbol k, for $k = 1, 2, \cdots, K_{\mathrm{p}}$, is given by

$$\hat{\mathbf{z}}_{\mathrm{LS},k} = \left(\bar{\xi}\right)^{-1}\frac{\mathbf{h}_{\mathrm{LS},k}^H\mathbf{y}_{\mathrm{p}}[k]}{||\mathbf{h}_{\mathrm{LS},k}^H||_2^{-2}}. \tag{21}$$

This means that the K_{p} phase impairment estimates $\hat{\mathbf{z}}_{\mathrm{LS},k}$, $k = 1, 2, \cdots, K_{\mathrm{p}}$ can be computed in parallel, thus reducing the computational complexity significantly. Since $\mathbf{G}_1^H\mathbf{G}_1$ is always invertible, the LS estimation is always feasible.

After that, we propose an estimate of the phase impairment value at training symbol $k \in \mathcal{K}_{\mathrm{p}}$, which is denoted as $\hat{\beta}_{\mathrm{LS}}[k], \forall k \in \mathcal{K}_{\mathrm{p}}$ and is computed as the angle of $\hat{\mathbf{z}}_{\mathrm{LS},k}$. We also propose the following estimate of the CFO Δf

$$\Delta f_{\mathrm{LS}} = \frac{1}{2\pi(K_{\mathrm{p}}-1)}\sum_{k=1}^{K_{\mathrm{p}}-1}\left(\hat{\beta}_{\mathrm{LS}}[k+1] - \hat{\beta}_{\mathrm{LS}}[k]\right). \tag{22}$$

As a result, the following estimate of phase noise $\theta[k], \forall k \in \mathcal{K}_{\mathrm{p}}$ can be obtained as

$$\hat{\theta}_{\mathrm{LS}}[k] = \hat{\beta}_{\mathrm{LS}}[k] - 2\pi\Delta f_{\mathrm{LS}}k. \tag{23}$$

From equation (21) we can estimate phase impairment elements of $\hat{\mathbf{z}}_{\mathrm{LS}}$ in parallel, thus reducing the computational complexity significantly. For the complexity analysis of the proposed method, we count the number of multiplications and additions used in each step of algorithms from equation (21). Table 1 shows the complexity of the method that mentioned at the abstract and introduction using $\mathcal{O}$ notation. As a result, our proposed method has lower order of computational complexity than [8],[12] and does not require Fourier transform as in [17].

4. Numerical results

This section provides numerical results to investigate the performance of the proposed phase impairment estimation method. The proposed method is evaluated by using the mean square error (MSE) of the phase impairment $\beta[k]$. The resulting MSE in the case of LS estimation is computed as follows

$$\mathrm{MSE} = \mathbb{E}\left(\frac{1}{K_{\mathrm{p}}}\sum_{k=1}^{K_{\mathrm{p}}}\left\|\beta[k] - \hat{\beta}_{\mathrm{LS}}[k]\right\|_2^2\right). \tag{24}$$

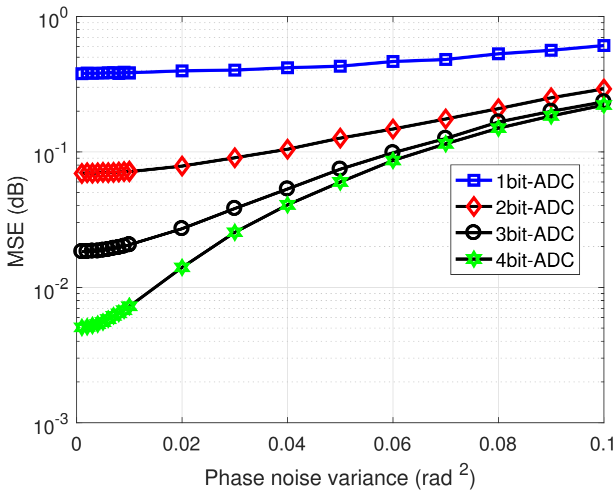

Figure 2. MSE as a function of phase noise variance, the level of CSI is 0.2.

Consider the following simulation scenario: 1) the number of transmit antennas $N_t = 128$, which is connected to 8 RF chains, 2) the number of receiver antennas $N_r = 64$, which is connected to 4 RF chains, 3) antenna array of transceiver is Uniform Linear Array (ULA) with the same distance $d = \lambda/2$ between adjacent antenna elements, 4) the number of pilot streams $N_d = 2$, the carrier frequency is 28 GHz, bandwidth is 100 MHz [23]. Monte-Carlo simulation results are obtained based on 100.000 realizations of channel vectors, phase noise, and quantization noise. We also consider the mmWave channel model with the number of paths $L = 3$ and AoAs/AoDs are uniformly distributed in $[0, 2\pi]$ [24]. Furthermore, analog precoding/combining matrices ($\mathbf{F}_{\mathrm{RF}}$, $\mathbf{W}_{\mathrm{RF}}$) and digital precoding/combining matrices ($\mathbf{F}_{\mathrm{BB}}$, $\mathbf{W}_{\mathrm{BB}}$) are designed using fully-array connected architecture [25]. The training symbol matrix is generated by Zadoff-Chu sequences [8].

Firstly, we show the changing MSE according to different phase noise variance and the resolution of ADC for CSI = 0.2 as shown in Fig. 2. As expected that the MSE value decreases with the resolution of the ADCs and increases with the values of phase noise variance. Moreover, when the values of phase noise variance is large the MSE values depend significantly on the value of phase noise variance.

Table 1. COMPARISON OF COMPUTATIONAL COMPLEXITY

Estimation	Method	Complexity
Channel and phase noise [12]	EKF	$\mathcal{O}(N_t^3.N_r^3)$
CFO and channel gain [8]	PBiGAMP	$\mathcal{O}(N_r^2.N_p^2).N_p.L$
CFO and channel gain [17]	W-OMP	$\mathcal{O}(M.M_r(K_p + N_{FFT}.log_2(N_{FFT})))$
phase noise and CFO	proposed estimation	$\mathcal{O}(N_p^2.K_p^2)$

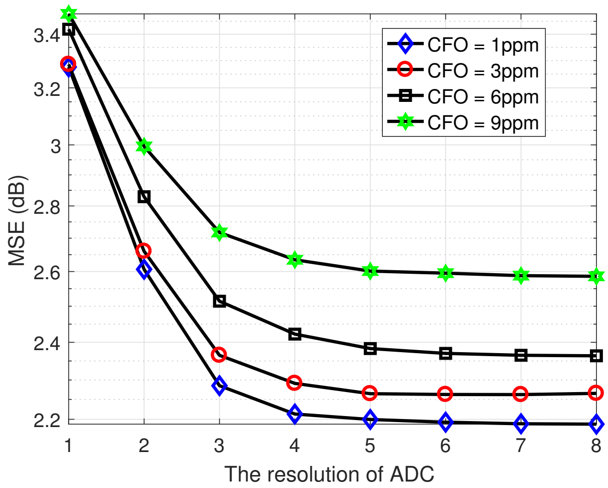

Figure 3. MSE as a function of the resolution ADC, phase noise variance is $10^{-2}rad^2$.

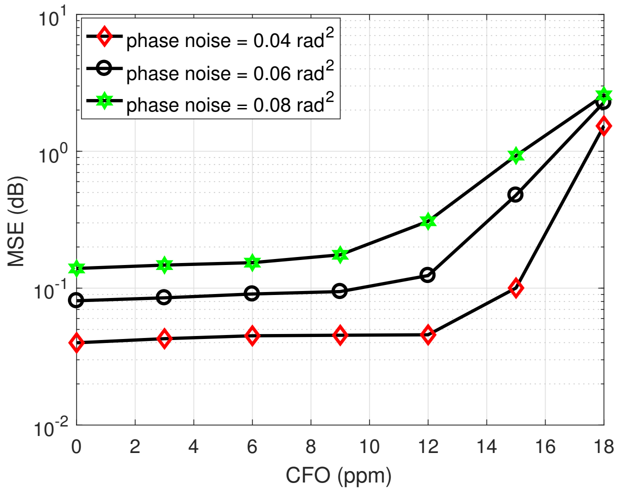

Figure 4. MSE as a function of CFO, 4 bit ADC.

Next, Fig. 3 presents the MSE in dB as a function of the resolution of ADC for various CFO values [1, 3, 6, 9] ppm. The simulation results pointed out that when increasing the number of bit ADC, the MSE decreases, especially with 1-4 bit ADC. When the resolution is large (>5 bit), the MSE will be saturation. It is due to quantization noise is assumed as an additive noise and depend on the number of bit ADC.

Third, Fig. 4 shows the changing MSE according to CFO for various phase noise variances [0.04 0.06 0.08], 4bit-ADC, CSI = 0.5. It is obvious that that MSE performance of proposed system is extremely affected with changing the CFO.

Finally, in Fig. 5 we investigate the MSE versus the level of channel estimation accuracy. In Fig. 5 shows the different values of phase noise variance $[0.1, 0.2, 0.3, 0.4]rad^2$, CFO = 10ppm and 4-bit ADC. We can conclude that MSE performance of proposed system is extremely affected and it varies with changing the phase noise and CFO. As a result, we must design tranceiver to obtain good MSE performance.

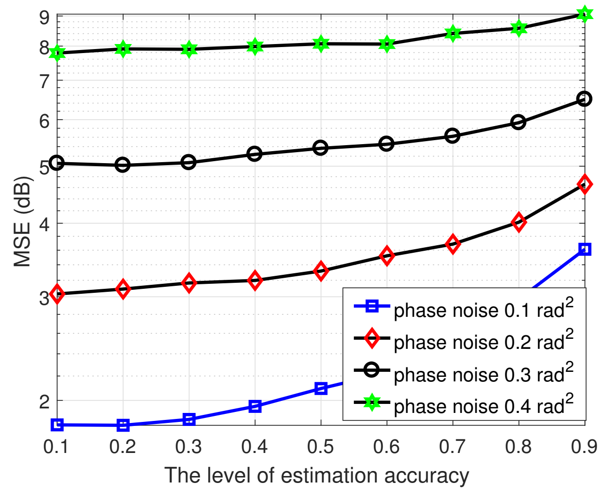

Figure 5. MSE as a function of channel estimation accuracy for the different values of phase noise variance, 4-bit ADC.

5. Conclusion

In this paper, we have considered millimeter-wave MIMO systems with more realistic assumptions on hardware components, signal processing techniques and imperfect CSI, than prior work. We have investigated the effect of carrier frequency offset, imperfect CSI, quantization noise and phase noise on the hybrid analog and digital beamforming architecture mmWave MIMO systems. We formulated the linear CLO-based phase impairment estimation, which results from varying phase noise and constant carrier frequency offset. A very low complexity linear LS method was proposed to estimate the phase impairment values. Numerical results have shown that the proposed method works well in wide ranges of ADC resolutions, CFO values and phase noise variances. Future work may focuses on the development of MMSE method with higher computational complexity and potentially better MSE performance. Future work may also focus on deriving a solution of joint phase noise and channel estimation problem with the effect of quantization noise on hybrid beamforming architecture mmWave MIMO systems.

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