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Extreme Value Distributions in Hydrological Analysis in the Mekong Delta: A Case Study in Ca Mau and An Giang Provinces, Vietnam

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Abstract

Climate change poses a critical risk to the sustainable development of many regions in Vietnam, especially in the Mekong River. In this paper, we show the specific extreme value distributions of rainfall, flow, and crest of salinity based on the hydrological data from 1975 to 2017 in An Giang and Ca Mau provinces in the Mekong Delta. We also derive a theoretical model and validate its accuracy compared to the empirical data over the years. The results demonstrate that the extremely high flows increase in both magnitude and frequency, while the extremely low ones are projected to occur less often under the climate change. The results can further help the local governments reduce the risk of lack water in dry season, control the salinization, and avoid the threat of flooding in the downstream of the Mekong Delta.

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1. Introduction

The Mekong Delta is located at the end and in the lowest region of the Mekong river, in the far south of Vietnam. The delta consists of 13 provinces in a triangle form of 3.9 million hectares beginning from Tien Giang province in the east, to An Giang and Kien Giang provinces in the northwest, and down to Ca Mau province in the southernmost tip of Vietnam. The delta is very flat and its average elevation is about 0.8m above the mean sea level. The Mekong river gets through a network of canals and runs into the East Sea and the Gulf of Thailand (West Sea). In the wet season, most of the water discharge (80-85%) run into the two main branches, i.e., Bassac and Mekong. The remaining portion (15-20%) spreads over the overland.

In this region, agriculture plays an important role and employs most of the total workforce. However, about 50% portion of the delta area and more than 2 million people, especially in the Plain of Reeds and the Long Xuyen quadrangle, are affected by seriously seasonal flooding of 3m depth. In addition, in the dry season, over 1.4 million hectares of the coastal regions in the delta are under the effect of salt water intrusion. The Mekong Delta has been facing many challenges due to not only climate change but also hydropower causing more droughts and saltwater intrusion [1].

Under climate change scenarios, the temperature in the Mekong Delta region is projected to increasing rapidly. Meanwhile, the increase in precipitation is not significant in all the seasons of the year. Especially in summer, the temperature is very high but a little bit increase in precipitation [2–4]. Consequently, there is a high risk of longer droughts in summer. Moreover, the Mekong Delta is the region mostly affected by the highest sea level rise in the 21st century in Vietnam. Both the problems of climate change and sea level rise are able to make more droughts and salinization in the Mekong Delta [5, 6].

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Recently, many dams for hydropower plants have been built in the upstream of the Mekong river as planned by Laos, China, and Thailand [7]. Under the effect of many hydropower plants, the flow water quantity of the Mekong river coming to the Mekong Delta is reduced significantly. The flooding season cannot be utilized because of very short period and the lack of water. Taking into account all the aforementioned problems, the droughts and saltwater intrusion sooner or later becomes serious disasters.

In this paper, we study the specific extreme value distributions of rainfall, flow, and crest of salinity based on the hydrological data from 1975 to 2017 in two typical provinces: An Giang and Ca Mau. These two coastal provinces have been mostly affected by the lack of fresh water from the Mekong River and the seawater intrusion, in the Mekong Delta, Vietnam. The results show that the magnitude and the frequency of extremely high flows increase, while extremely low ones are projected to be less frequency. Importantly, a theoretical model is derived and compared to the empirical data over the years to evaluate its accuracy. These findings can help the local governments reduce the risk of lack water in dry season, control the salinization, and avoid the threat of flooding in the downstream of the Mekong Delta in future.

The rest of this paper is organized as follows. In Section II, we introduce the extreme value distributions. Section III is dedicated to applying the extreme value distributions to the hydrological model and analysis in Ca Mau and An Giang provinces. Finally, we conclude the paper in Section IV.

2. Extreme Value Distributions

Let $X_1, X_2, ..., X_n$ be a sequence of independent random. The maximum value of this sequence, $\max\{X_1, X_2, \ldots, X_n\}$, has a cumulative distribution function given by

$$\begin{aligned} H_n(x) &= P(\max\{X_1, X_2, ..., X_n\} \leq x) \\ &= P\{X_1 \leq x, X_2 \leq x, ..., X_n \leq x\} \\ &= P\{X_1 \leq x\}P\{X_2 \leq x\}...P\{X_n \leq x\} \\ &= F_{x_1}(x)F_{x_2}(x)...F_{x_n}(x). \end{aligned} \tag{1}$$

If the random variable X_i $(i = 1, 2, ..., n)$ is independent and has the same distribution $F_X(x)$, the cumulative distribution functions of maximum H_n and minimum $L_n = \min\{X_1, X_2, \ldots, X_n\}$ drawn from a population with cumulative distribution function $F_X(x)$ are respectively expressed as

$$H_n(x) = P(max\{X_1, X_2, ..., X_n\} \leq x) = (F_X(x))^n \tag{2}$$

and

$$L_n(x) = P(min\{X_1, X_2, ..., X_n\} \leq x) = 1 - [1 - F_X(x)]^n. \tag{3}$$

To avoid degeneracy, we find a linear transformation such that

$$\lim_{n\to\infty} H_n(a_n + b_n x) = \lim_{n\to\infty} (F_X(a_n + b_n x))^n = H(x) \tag{4}$$

and

$$\lim_{n\to\infty} L_n(c_n + d_n x) = \lim_{n\to\infty} \{1 - [F_X(a_n + b_n x)]^n\} = L(x). \tag{5}$$

2.1. Definition 1

A given distribution F(x) is said to belong to the maximal (or minimal) domain of attraction of $H(x)$ (or $L(x)$) if (4) (or (5)) holds for at least one pair of sequences $\{a_n\}$ and $\{b_n > 0\}$.

Feasible limit distribution for maxima [8, 9]. The only nondegenerate family of distributions that satisfies (4) is

$$H_\beta(x, \lambda, \delta) = exp\left\{-\left[1 - \beta\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\beta}}\right\}, \tag{6}$$
$$\beta \neq 0, 1 - \beta\left(\frac{x-\lambda}{\delta}\right) \geq 0,$$

where $x \leq \lambda + \frac{\delta}{\beta}$ if $\beta > 0$ and $x \geq \lambda + \frac{\delta}{\beta}$ if $\beta < 0$. In addition, if $\beta = 0$, the family of distributions is obtained by taking the limit of (6) as $\beta \to 0$, we have

$$H_0(x, \lambda, \delta) = exp\left[-exp\left(\frac{x-\lambda}{\delta}\right)\right], x \in R. \tag{7}$$

The distributions in (6) and (7) are called the maximal generalized extreme value distributions (GEVDs). This maximal GEVD family includes the well-known Fréchet, Weibull, Gumbel families for maxima, which are respectively given by

$$H_{GD}(x) = exp\left[-exp\left(\frac{x-\lambda}{\delta}\right)\right], -\infty < x < +\infty, \tag{8}$$

$$H_{WD}(x) = \begin{cases} exp\left[-\left(\frac{x-\lambda}{\delta}\right)^\beta\right], \text{if } x \leq \lambda, \beta > 0, \\ 1, \text{ otherwise.} \end{cases} \tag{9}$$

and

$$H_{FD}(x) = \begin{cases} exp\left[-\left(\frac{\delta}{x-\lambda}\right)^\beta\right], \text{if } x \geq \lambda, \beta > 0, \\ 0, \text{ if } x < \lambda. \end{cases} \tag{10}$$

Feasible limit distribution for minima. The only nondegenerate family of distributions that satisfies (5) is

$$L_\beta(x,\lambda,\delta) = 1 - exp\Big\{-\Big[1+\beta\Big(\frac{x-\lambda}{\delta}\Big)^{\frac{1}{\beta}}\Big]\Big\}, \quad (11)$$
$$\beta \neq 0, 1-\beta\Big(\frac{x-\lambda}{\delta}\Big) \geq 0,$$

where $x \geq \lambda + \frac{\delta}{\beta}$ if $\beta > 0$ and $x \leq \lambda + \frac{\delta}{\beta}$ if $\beta < 0$. In addition, if $\beta = 0$, the family of distributions is obtained by taking the limit of (11) as $\beta \to 0$, we have

$$L_0(x,\lambda,\delta) = 1 - exp\Big[-exp\Big(\frac{x-\lambda}{\delta}\Big)\Big], x \in R. \quad (12)$$

The distributions in (11) and (12) are called the minimal generalized extreme value distributions (GEVDs). This minimal GEVD family includes the well-known Fréchet, Weibull, Gumbel families for maxima, which are respectively given by

$$L_{GD}(x) = 1 - exp\Big[-exp\Big(\frac{x-\lambda}{\delta}\Big)\Big], -\infty < x < +\infty, \quad (13)$$

$$L_{WD}(x) = \begin{cases} 0, \text{ if } x < \lambda, \\ 1 - exp\Big[-\Big(\frac{x-\lambda}{\delta}\Big)^\beta\Big], \text{ otherwise.} \end{cases} \quad (14)$$

and

$$L_{FD}(x) = \begin{cases} 1 - exp\Big[-\Big(\frac{\delta}{x-\lambda}\Big)^\beta\Big], \text{if } x \leq \lambda, \\ 1, \text{ otherwise.} \end{cases} \quad (15)$$

2.2. Theorem 1

Let $\{\xi_i; i = 1, 2, ...\}$ be a sequence of independent distributed random variables belonging to the maximal domain of attraction of $H_{\beta_i}(x,\lambda_i,\delta_i) \equiv H_i$ and let $\{\eta_i; i = 1, 2, ...\}$ be a sequence of independent distributed random variables belonging to the minimal domain of attraction of $L_{\beta_i}(x,\lambda_i,\delta_i) \equiv L_i$, we have

$$\sum_{i=1}^{n}(E(H_i)+E(L_i)) = 2\sum_{i=1}^{n}\lambda_i \quad (16)$$

and

$$\sum_{i=1}^{n}(Var(H_i)+Var(L_i)) =$$
$$\begin{cases} \frac{\pi^2}{3}\sum_{i=1}^{n}\delta_i^2, \text{ if: } H_i \sim MaxGD; L_i \sim MinGD, \\ 2\sum_{i=1}^{n}\delta_i^2\Big[\Gamma\Big(1+\frac{2}{\beta_i}\Big)-\Gamma^2\Big(1+\frac{1}{\beta_i}\Big)\Big], \\ \quad \text{if: } H_i \sim MaxWD; L_i \sim MinWD, \\ 2\sum_{i=1}^{n}\delta_i^2\Big[\Gamma\Big(1-\frac{2}{\beta_i}\Big)-\Gamma^2\Big(1-\frac{1}{\beta_i}\Big)\Big], \\ \quad \text{if: } H_i \sim MaxFD; L_i \sim MinFD. \end{cases} \quad (17)$$

Proof. of (16)
Let $H_i \sim MaxGD$ and $L_i \sim MinGD$, we have

$$E(H_i) = \int_{-\infty}^{+\infty} exp\Big[-exp\Big(\frac{x-\lambda_i}{\delta_i}\Big)\Big]xdx = \lambda_i - \delta_i\Gamma'(1). \quad (18)$$

and

$$E(L_i) = \int_{-\infty}^{+\infty}\Big\{1-exp\Big[-exp\Big(\frac{x-\lambda_i}{\delta_i}\Big)\Big]\Big\}xdx = \lambda_i + \delta_i\Gamma'(1). \quad (19)$$

Similarly, let $H_i \sim MaxWD$ and $L_i \sim MinWD$, we have

$$E(H_i) = \int_{-\infty}^{\lambda_i} exp\Big[-\Big(\frac{x-\lambda_i}{\delta_i}\Big)^{\beta_i}\Big]xdx = \lambda_i - \delta_i\Gamma'(1+1/\beta_i). \quad (20)$$

and

$$E(L_i) = \int_{-\infty}^{\lambda_i}\Big\{1-exp\Big[-\Big(\frac{x-\lambda_i}{\delta_i}\Big)^{\beta_i}\Big]\Big\}xdx \quad (21)$$
$$= \lambda_i + \delta_i\Gamma'(1+1/\beta_i).$$

And finally, let $H_i \sim MaxFD$ and $L_i \sim MinFD$, we have

$$E(H_i) = \int_{\lambda_i}^{+\infty} exp\Big[-\Big(\frac{\delta_i}{x-\lambda_i}\Big)^{\beta_i}\Big]xdx = \lambda_i + \delta_i\Gamma'(1-1/\beta_i). \quad (22)$$

and

$$E(L_i) = \int_{\lambda_i}^{+\infty}\Big\{1-exp\Big[-\Big(\frac{\delta_i}{x-\lambda_i}\Big)^{\beta_i}\Big]\Big\}xdx \quad (23)$$
$$= \lambda_i - \delta_i\Gamma'(1-1/\beta_i).$$

Based on the results from (18) to (23), we obtain (16). □

Proof. of (17)
Let $H_i \sim MaxGD$ and $L_i \sim MinGD$, we have

$$Var(H_i) = Var(L_i) = \frac{\pi^2\delta_i^2}{6}. \quad (24)$$

Similarly, let $H_i \sim MaxWD$ and $L_i \sim MinWD$, we have

$$Var(H_i) = Var(L_i) = \delta_i^2\Big[\Gamma\Big(1+\frac{2}{\beta_i}\Big)-\Gamma^2\Big(1+\frac{1}{\beta_i}\Big)\Big]. \quad (25)$$

And finally, let $H_i \sim MaxFD$ and $L_i \sim MinFD$, we have

$$Var(H_i) = Var(L_i) = \delta_i^2\Big[\Gamma\Big(1-\frac{2}{\beta_i}\Big)-\Gamma^2\Big(1-\frac{1}{\beta_i}\Big)\Big]. \quad (26)$$

Based on (24), (25), and (26), we obtain (17). □

3. Applications to Ca Mau and An Giang Provinces

In this section, we apply the extreme value distributions to the two particular Ca Mau and An Giang provinces. We first present the extreme value distributions in hydrological model in terms of problems and solutions of maximum distribution functions and then analyze the hydrological results in these provinces in the sequel.

3.1. Extreme Value Distributions in Hydrological Models: Problems and Solutions

There are three problems studied in this paper including 1) finding the maximum Gumbel distribution function for maximum rainfall in Ca Mau based on the data at the hydrological station, i.e., Ca Mau hydrological station located at the left side of Ganh Hao river, Ca Mau town, Ca Mau province, from 1976 to 2017); 2) finding the maximum Gumbel distribution function for maximum water lever in Tien river based on the data at the hydrological station in Tan Chau district, An Giang province, from 1976 to 2017; and 3) finding the maximum Gumbel distribution function for maximum of salinity peak in Ca Mau based on the data at the hydrological station in Ca Mau province, from 2000 to 2017. The experimental data was provided by the Southern Regional Hydrometeorological Center. The problem are solved specifically as below.

First, we derive the expectation and the variance of Gumbel distribution given as follows:

$$\mathrm{EX} = \mu + 0,577216\sigma \tag{27}$$

and

$$VarX = \frac{(\pi\sigma)^2}{6}. \tag{28}$$

where $0,577216 \equiv$ Euler constant.

By using moment method, we find the statistical estimation expressed as

$$\begin{cases} \overline{X} = \mu + 0,577216\sigma, \\ S^2 = \frac{(\pi\sigma)^2}{6}. \end{cases} \tag{29}$$

And then we have

$$\begin{cases} \overline{\mu} \approx \overline{X} - 0,4501S, \\ \overline{\sigma} \approx 0,7797S. \end{cases} \tag{30}$$

where $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ and $S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$.

Second, we use maximum likelihood method for parameter estimation in distribution maximum to illustrate the use of actual data through Newton – Raphson algorithm. To building the likelihood function, we have

$$\begin{aligned} L(\mu,\sigma) = f(x_1, x_2, ..., x_n \mid \mu, \sigma) &= \prod_{i=1}^{n} f(x_i \mid \mu, \sigma) \\ &= \prod_{i=1}^{n} \frac{1}{\beta} exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\} exp\Big(-exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}\Big) \\ &= \Big(\frac{1}{\sigma}\Big)^n exp\Big\{-\sum_{i=1}^{n}\Big[\frac{x_i-\mu}{\sigma} + exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}\Big]\Big\} \end{aligned} \tag{31}$$

From eq. (31), we further define Λ as

$$\begin{aligned} \Lambda &= ln\Big(L(\mu,\sigma)\Big) \\ &= -nln(\beta) - \sum_{i=1}^{n}\Big[\frac{x_i-\mu}{\sigma} + exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}\Big] \end{aligned} \tag{32}$$

And its partial derivatives with respect to μ and σ are respectively given by

$$\begin{aligned} \frac{\partial\wedge}{\partial\mu} &= -\sum_{i=1}^{n}\Big[\frac{-1}{\sigma} + \frac{1}{\sigma} exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}\Big] \\ &= \frac{n}{\sigma} - \frac{1}{\sigma}\sum_{i=1}^{n} exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}. \end{aligned} \tag{33}$$

and

$$\begin{aligned} \frac{\partial\wedge}{\partial\sigma} &= -\frac{n}{\sigma} - \sum_{i=1}^{n}\Big[-\frac{x_i-\mu}{\sigma^2} + \frac{x_i-\mu}{\sigma^2} exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}\Big] \\ &= -\frac{n}{\sigma} + \sum_{i=1}^{n} + \frac{x_i-\mu}{\sigma^2} - \sum_{i=1}^{n}\frac{x_i-\mu}{\sigma^2} exp\{-\frac{x_i-\mu}{\sigma}\}. \end{aligned} \tag{34}$$

We select $\hat{\mu}$ and $\hat{\sigma}$ such that they satisfy the following system equations:

$$\begin{cases} \frac{\partial\wedge}{\partial\sigma} = 0 \\ \frac{\partial\wedge}{\partial\delta} = 0 \end{cases} \tag{35}$$

So far, we apply the Newton – Raphson algorithm by computing the second-order partial derivatives presented as

$$\begin{aligned} \frac{\partial^2\wedge}{\partial\mu^2} &= -\frac{1}{\sigma}\sum_{i=1}^{n}\Big(\frac{1}{\sigma}\Big) exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\} \\ &= -\frac{1}{\sigma^2}\sum_{i=1}^{n} exp\Big\{-\frac{x_i-\mu}{\sigma}\Big\}. \end{aligned} \tag{36}$$

$$\frac{\partial^2\wedge}{\partial\mu\partial\sigma} = -\frac{n}{\sigma^2} + \frac{1}{\sigma^2}\sum_{i=1}^{n} exp\left\{-\frac{x_i-\mu}{\sigma}\right\} - \frac{1}{\sigma}\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma^2} exp\left\{-\frac{x_i-\mu}{\sigma}\right\} = -\frac{n}{\sigma^2} + \frac{1}{\sigma^2}\sum_{i=1}^{n} exp\left\{-\frac{x_i-\mu}{\sigma}\right\} - \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu) exp\left\{-\frac{x_i-\mu}{\sigma}\right\}. \quad (37)$$

$$\frac{\partial^2\wedge}{\partial\sigma^2} = \frac{n}{\sigma^2} - \frac{2}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu) + \frac{2}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu) exp\left\{-\frac{x_i-\mu}{\sigma}\right\} - \frac{1}{\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 exp\left\{-\frac{x_i-\mu}{\sigma}\right\}. \quad (38)$$

and finally, by setting

$$f = \begin{bmatrix} \frac{\partial\wedge}{\partial\mu} \\ \frac{\partial\wedge}{\partial\sigma} \end{bmatrix}, \quad K = \begin{bmatrix} \frac{\partial^2\wedge}{\partial\mu^2} & \frac{\partial^2\wedge}{\partial\mu\partial\sigma} \\ \frac{\partial^2\wedge}{\partial\mu\partial\sigma} & \frac{\partial^2\wedge}{\partial\sigma^2} \end{bmatrix},$$

we yield

$$\begin{bmatrix} \alpha^{(j+1)} \\ \sigma^{(j+1)} \end{bmatrix} = \begin{bmatrix} \alpha^{(j)} \\ \sigma^{(j)} \end{bmatrix} - K^{-1}\left(\alpha^{(j)}, \sigma^{(j)}\right) f\left(\alpha^{(j)}, \sigma^{(j)}\right). \quad (39)$$

The aforementioned computations are repeated until the following inequalities hold

$$\left[\begin{bmatrix} \mu^{(j+1)} \\ \sigma^{(j+1)} \end{bmatrix} - \begin{bmatrix} \mu^{(j)} \\ \sigma^{(j)} \end{bmatrix}\right]^T \left[\begin{bmatrix} \mu^{(j+1)} \\ \sigma^{(j+1)} \end{bmatrix} - \begin{bmatrix} \mu^{(j)} \\ \sigma^{(j)} \end{bmatrix}\right] < k \quad (40)$$

and

$$\Delta_j = (\mu^{(j+1)} - \mu^{(j)})^2 + (\sigma^{(j+1)} - \sigma^{(j)})^2 < k = 10^{-4}. \quad (41)$$

3.2. Hydrological Analysis

The problem of maximum rainfall in Ca Mau. Based on the values of $\mu^0 = 97,225$ and $\sigma^0 = 23,688$ calculated by moment method in the maximum likelihood and Newton – Raphson algorithm, we get the detailed step by step results as shown in Table 1 below.

Table 1. Maximum rainfall in Ca Mau province

Step j	μ^j	σ^j	Δ_j	$< k = 10^{-4}$
0	97,225	23,688		
1	96,37	23,464	0,0375	$> 10^{-4}$
2	96,378	23,481	$4,67.10^{-7}$	$< 10^{-4}$

The extreme distribution function in this problem is

$$F_1(x) \approx exp\left[-exp\left(\frac{-(x-96,378)}{23,481}\right)\right]. \quad (42)$$

The problem of maximum water lever in Tien river, An Giang province. Based on the values of $\mu^0 = 97,225$ and $\sigma^0 = 23,688$ calculated by moment method in the maximum likelihood and Newton – Raphson algorithm, we got the detailed step by step results as shown in Table 2.

Table 2. Maximum water lever in Tien river, An Giang province

Step j	μ^j	σ^j	Δ_j	$< k = 10^{-4}$
0	379,37	46,573		
1	376,761	53,52	55,585	$> 10^{-4}$
2	375,862	59,728	39,336	$> 10^{-4}$
3	375,414	62,899	10,2533	$> 10^{-4}$
4	375,317	63,497	0,365	$> 10^{-4}$
5	375,313	63,514	0,000338	$> 10^{-4}$
6	375,313	63,514	$4,93.10^{-7}$	$< 10^{-4}$

The extreme distribution function in this problem is

$$F_2(x) \approx exp\left\{-exp\left\{\frac{-(x-375,313)}{63,514}\right\}\right\}. \quad (43)$$

The problem of maximum salinity peak in Ca Mau province. Similarly, based on the values of $\mu^0 = 30,77$ and $\sigma^0 = 2,377$ calculated by moment method in the maximum likelihood and Newton – Raphson algorithm, we get the detailed step by step results as shown in Table 3.

Table 3. Maximum salinity peak in Ca Mau province

Step j	μ^j	σ^j	Δ_j	$< k = 10^{-4}$
0	30,77	2,337		
1	30,6953	72,549	0,0458	$> 10^{-4}$
2	30,6,778	2,6016	$3,07.10^{-3}$	$> 10^{-4}$
3	30,6767	2,6049	$1,27.10^{-5}$	$< 10^{-4}$

And the extreme distribution function in this problem is

$$F_2(x) \approx exp\left\{-exp\left\{\frac{-(x-30,6767)}{2,605}\right\}\right\}. \quad (44)$$

4. Conclusion

In this paper, we have presented the hydrological impact assessments in An Giang and Ca Mau provinces in Mekong River basin focusing on hydrological extremes. We found the maximum Gumbel distribution function for maximum rainfall in Ca Mau based on the data at the hydrological station Ganh Hao, from 1976 to 2017 and the maximum Gumbel distribution function for maximum water lever in Tien river based on the data at the hydrological station in Tan Chau district, An Giang province, from 1976 to 2017. We also derived the maximum Gumbel distribution function for maximum of salinity peak in Ca Mau based on the data at the hydrological station in Ca Mau province, from 2000 to 2017. It is noted that the extremely high-flow events increase in both magnitude and frequency. In addition, the extremely low flows are projected to occur less often. These findings are useful for the local governments to efficiently reduce the risk of lack water in dry season, control the salinization, and avoid the threat of flooding in the downstream of the Mekong Delta in future.

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