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Scalable SOCP-based localization technique for wireless sensor network

Randa M. Abdelmoneem[1], Eman Shaaban[1,*]

[1]Department of Computer Systems, Faculty of Computer and Information Sciences, Ain-Shams University, Egypt

Abstract

Node localization is one of the essential requirements to most applications of wireless sensor networks. This paper presents a detailed implementation of a centralized localization technique for WSNs based on Second Order Cone Programming (SOCP). To allow scalability, it also proposes a clustered localization approach for WSNs based on that centralized SOCP technique. The cluster solves the SOCP problem as a global minimization problem to get positions of the cluster sensor nodes. To enhance localization accuracy, a cluster level refinement step is implemented using Gauss-Newton optimization. The initial position for the Gauss-Newton optimization is the position drawn from the preprocessor SOCP localization. The proposed approach scales well for large networks and provides a considerable reduction in computation time while yielding good localization accuracy.

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1. Introduction

A wireless sensor network (WSN) is a group of a few to several hundreds or even thousands of sensor nodes deployed over a significant area. WSNs have a wide range of applications which include environmental monitoring, target tracking, home automation, military applications and others. To process sensor data in WSN, it is imperative to know where the data is coming from. So, knowledge of nodes locations is an essential requirement for many location-aware applications including aforementioned applications, location-based services (LBS) and location-based routing. Several surveys discussed localization strategies and attempted to classify different localization techniques like [1],[2],[3]. Localization techniques could be classified according to all calculations being performed on a single node or distributed on all the network sensor nodes to centralized localization technique (e.g. MDS-MAP [4] and Semi-Definite programming (SDP) [5]) and distributed localization technique (e.g. APIT [6]). Another new approach in this classification is called locally centralized or cluster-based localization techniques which are distributed techniques that achieve a global goal by communicating with nodes in some neighbourhood only. According to the process of estimating node-to-node distances or angles, the localization techniques are classified to range-based and range-free localization techniques. Range-based localization techniques are based on distances measurements between the nodes using Time of Arrival (TOA), time difference of arrival (TDOA) and received signal strength (RSS) (e.g. Trilateration) or based on angles between the nodes like angle of arrival (AOA) (e.g. Triangulation) [7]. Range-free localization techniques depend on network connectivity (e.g. DV-HOP) [6] to indirectly obtain the distances between the nodes. Also, localization techniques can be classified to anchor-based or anchor-free localization techniques. Anchor-based localization techniques usually provide absolute positions for the nodes whereas anchor-free localization techniques provide relative positions. Biswas and Ye who proposed a semi-definite programming (SDP) relaxation of the localization problem which has various nice properties [5]. SOCP-based localization technique was studied

* Corresponding author. Email: eman.shaaban@cis.asu.edu.eg

by Tseng [8] who provided a second order cone programming (SOCP) relaxation of localization problem, motivated by its simpler structure and its potential to be solved faster than SDP. This paper proposes a locally centralized technique for solving the sensor network localization problem. It is a Refined Clustered technique based on Second-Order Cone Programming (RC-SOCP). The proposed approach divides the large network into smaller sub networks. For each cluster, the cluster solves the SOCP problem as a global minimization problem to get initial positions of the cluster sensor nodes. To enhance localization accuracy, a cluster level refinement step is implemented using Gauss-Newton optimization. The initial position for the GaussNewton optimization is the position drawn from the preprocessor SOCP localization. Rest of the paper is organized as follows: Section 2 introduces the centralized SOCP localization technique. Proposed technique is presented in section 3. Simulation results and evaluations are presented in section 4 and we conclude in section 5.

2. Centralized SOCP localization technique

This section discuses a detailed description of SOCP problem formulation for Castalia wireless sensor networks simulator, providing the formulation algorithm and solver tools. It also investigates the results obtained.

2.1. Problem formulation

Second-order cone programming relaxation method for wireless sensor network localization was first studied by Tseng [8]. In this method **n** is the total number of nodes in R^d (d ≥ 1), **m** are the nodes whose locations $x_i \in R^2$, i =1,...,m are to be determined given **n-m** nodes called anchors with known locations and $\mathbf{d_{ij}}$ which is the euclidean distance between nodes i and j where (i,j) $\in \mathcal{A}$. $\mathcal{A}$ is the undirected neighbour set defined as $\mathcal{A} := \{(i,j) : \|x_i - x_j\| \leq RadioRange\}$. The problem is formulated as the non-convex minimization equation

$$v_{opt} = min \sum_{(i,j)\in\mathcal{A}} |y_{ij} - d_{ij}^2| \tag{1}$$

$$s.t. \quad y_{ij} = \|x_i - x_j\|^2 \quad \forall (i,j) \in \mathcal{A}$$

Where ‖ . ‖ denotes the euclidean norm. Then in order to yield a convex-problem, the equality constraints are relaxed to non-equality constraints, the problem becomes

$$v_{opt} = min \sum_{(i,j)\in\mathcal{A}} |y_{ij} - d_{ij}^2| \tag{2}$$

$$s.t. \quad y_{ij} \geq \|x_i - x_j\|^2 \quad \forall (i,j) \in \mathcal{A}$$

Equation (2) can be rewritten as

$$min \sum_{i,j\in\mathcal{A}} U_{ij} \tag{3}$$

$$\begin{aligned} s.t. \quad & y_{ij} \geq \|x_i - x_j\|^2 \quad \forall (i,j) \in \mathcal{A} \\ & u_{ij} \geq |y_{ij} - d_{ij}^2| \quad \forall (i,j) \in \mathcal{A} \\ & u_{ij} \geq 0 \end{aligned}$$

2.2. Centralized SOCP localization implementation

Given a wireless sensor network with size n sensors, m are sensors with unknown locations, $n - m$ are sensors with known locations (anchors). To solve this localization problem, the centralized SOCP localization technique is performed in three phases. In the first phase, nodes estimate their distances with sensor and anchor neighbours which are within their communication ranges. The second phase involves wireless communication and routing between the nodes. In the third phase, the positions matrix and the lower triangle of the distances matrix are created with sizes 2×n, n×n respectively and filled with their relevant data received from the nodes.
Distances Matrix

$$\begin{pmatrix} 0 & 0 & 0 & .. & 0 \\ d_{10} & 0 & 0 & .. & 0 \\ d_{20} & d_{21} & 0 & .. & 0 \\ .. & .. & .. & .. & 0 \\ d_{n0} & d_{n1} & .. & .. & 0 \end{pmatrix}$$

where d_{ij} is the euclidean distance between nodes i,j.

Positions Matrix

$$\begin{pmatrix} x_0 & x_1 & x_2 & .. & x_n \\ y_0 & y_1 & y_2 & .. & y_n \end{pmatrix}$$

where x_i,y_i are the x,y co-ordinates of an anchor node or 0,0 if the sensor node is not an anchor. Algorithm 1 shows the pseudo-code of formulating the SOCP-localization problem according to equation (3). For a network of n total nodes and m non-positioned nodes, there are Ω(m) variables and Ω(m) inequality constraints using position and distance matrices [8].

2.3. Simulator and Solver Tools

Several surveys discussed various Simulators like [12, 14] that are used for simulation of ad-hoc networks specially wireless sensor networks. Castalia is an open-source simulator which is based on OMNet++ simulation environment and was developed at the National ICT Australia, It was developed for networks

Algorithm 1 : formulating the SOCP-localization problem

1: **procedure** CREATEMODEL(distanceMatrix, positionsMatrix)
2: Let model = model of the problem , vars = array of variables
3: **for all** d_{ij} in $distanceMatrix$ **do**
4: #Read positions of nodes i,j from positions matrix
5: $x1pos \leftarrow positionsMatrix[0][i]$
6: $y1pos \leftarrow positionsMatrix[1][i]$
7: $x2pos \leftarrow positionsMatrix[0][j]$
8: $y2pos \leftarrow positionsMatrix[1][j]$
9: **if** $x1pos! = 0$ or $y1pos! = 0$ **then** #Check node i being anchor
10: *node i isAnchor*
11: **else**
12: # Search for variables in vars array and if not found create, add them
13: **if** variables of i in $vars$ **then**
14: $x_i \leftarrow vars[i].x$, $y_i \leftarrow vars[i].y$
15: **else**
16: *create vars* x_i, y_i
17: $vars.add(x_i)$, $vars.add(y_i)$
18: **end if**
19: **end if**
20: *Repeat steps 7-17 for the second node j*
21: # If at least one of the nodes is not an anchor complete model formulation
22: **if** *node i isAnchor==false Or node j isAnchor==false* **then**
23: *create variables* u_{ij}, y_{ij}
24: $vars.add(u_{ij})$, $vars.add(y_{ij})$
25: $objectiveExpression \leftarrow objectiveExpression + u_{ij}$
26: *create constraint c*
27: $c.expression \leftarrow y_{ij} - u_{ij}$
28: $c.lowerbound \leftarrow 0$
29: $c.upperbound \leftarrow d_{ij} * d_{ij}$
30: $model.add(c)$
31: **end if**
32: # Create quadratic constraint. Substitue with position values for anchor node(s) if found
33: **if** *node i isAnchor==false And node j isAnchor==false* **then**
34: $expr \leftarrow y - (x_i - x_j) * (x_i - x_j) - (y_i - y_j) * (y_i - y_j)$
35: **else if** *node i isAnchor==true And node j isAnchor==false* **then**
36: $expr \leftarrow y - (x1pos - x_j) * (x1pos - x_j) - (y1pos - y_j) * (y1pos - x_j)$
37: **else if** *node i isAnchor==false And node j isAnchor==true* **then**
38: $expr \leftarrow y - (x_i - x2pos) * (x_i - x2pos) - (y_i - y2pos) * (y_i - y2pos)$
39: **end if**

Algorithm 1 Continue

40: *create constraint q*
41: $q.expression \leftarrow expr$
42: $q.lowerbound \leftarrow 0$
43: $q.upperbound \leftarrow \infty$
44: $model.add(q)$
45: **end for**
46: *create objective obj*
47: $obj.fn \leftarrow minimization$
48: $obj.expression \leftarrow objExpression$
49: $model.add(obj)$

of low power embedded devices such as wireless sensor nodes [13]. Our simulation study is carried out using version 3.2 of Castalia which build upon version 4.1 of OMNet++.
The implementation of our SOCP-based localization technique uses IBM ILOG CPLEX. IBM LOG Concert Technology (modelling layer) C++ Interface Of OPL was used to integrate our problem model in Castalia with the CPLEX solver[15].

2.4. Performance Evaluation

We evaluate the performance of the centralized SOCP-based localization by measuring localization accuracy, computation time and problem size.
The mean error between the estimated and the true location of the non-anchor nodes in the network is adopted as the performance metric. It is defined as follows

$$LE = \frac{1}{N}.\sum_{i=1}^{N} \|\hat{x}_i - x_i\|$$

Where LE denotes a localization error, N denotes the number of nodes in a network whose location is estimated, x_i is the true position of the node i in the network , $\hat{x}_i$ is estimated location of the node i (solution of the location system).
Computation time is defined as the time spent for formulating and solving the SOCP problem at the sink node and it is measured using C++ std.clock() function. The time needed for computing the relative distances at the nodes and communication or message exchanges time is excluded. Problem size corresponds to number of variables and constraints for the SOCP-localization problem formulated at the sink node.

We evaluate performance of the centralized SOCP-based localization technique by studying the effect of varying Network size (number of nodes), Anchors percentage, Communication radio range and Noise value added to distances measurements.

Anchors

Anchors are chosen to form a convex hull distribution around the sensor nodes in the network. This distribution was chosen to assure good localization accuracy when non-anchor nodes are in the convex hull of the anchors [11].

Radio range

Radio range is specified by replacing the path loss parameter $PL(d)$ of the log-normal shadowing model in equation (4)

$$PL(d) = PL(d_0) + 10.\eta.log(\frac{d}{d_0}) + X_\sigma \tag{4}$$

with its equivalent

$$PL(d) = P_{Tx} - P_{Rx}$$

Equation(4) calculates the average path loss in the channel model in Castalia [13]. Setting the random variable X_σ to 0 for the case of no fading, thus simulating general environment conditions where stationary WSN is deployed in fairly static environments and fading is not very significant, thus it is ignored. η is substituted with 2.4 which is typical default value that will produce results similar to many outdoors (and sometimes indoors) environments. d is the path length allowing communications between two paired nodes, thus representing the radio range. So, the radio range can be specified by

$$RR = 10^{P_{Tx} - P_{Rx} - PL(d_0)/24} \tag{5}$$

P_{Rx} is inferred from $ReceiverSensetivity$ value of the radio card chosen (radio model for CC2420 chip by Texas instrument) and is qual to -95 dBm. So the radio range is mainly controlled by the transmit power and the path loss at reference distance values specified according to

$$RR = 10^{P_{Tx} + 95 - PL(d_0)/24} \tag{6}$$

The degree of connectivity is controlled by the radio range specified. It is calculated as the average connectivity of all the nodes which is equivalent to the average number of neighbours for all of the sensor nodes in the network for a specific radio range and environment area.

Noise to distance

We added normally distributed measurement noise to the true distance according to the equation

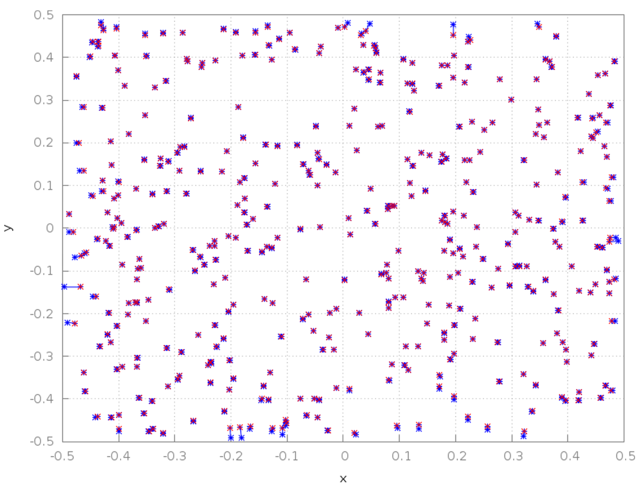
$$d_{ij} = \|x_i - x_j\|.max\ \{\ 0, 1 + \epsilon_{ij}.nf_d\ \} \quad \forall (i,j) \in \mathcal{A}$$

Where ϵ_{ij} is a normal random variable N(0, 1) representing measurement noise and $nfd \in [0,1]$ is the noise factor (standard deviation of the distance error in percentage) for the distance measurements [9].

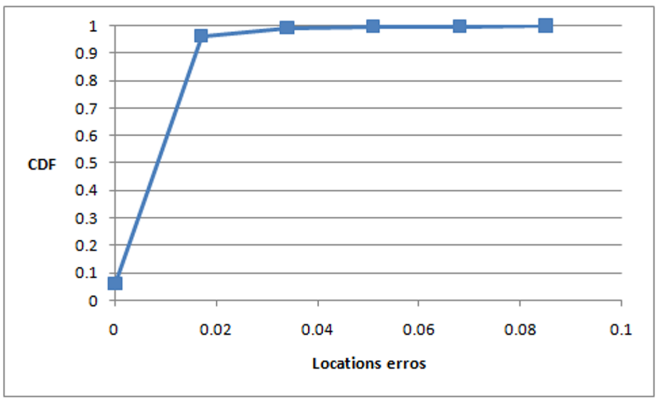
2.5. Simulation Results

Simulations are conducted for a randomly generated 500 nodes uniformly distributed on the unit square area $[-0.5, 0.5]^2$ with noise factor (nfd)=0.05, radio range (rr)=0.17 and Degree of Connectivity=38. Anchors are chosen to form a convex hull distribution around the sensor nodes in the network with percentage (p) of 15%. Simulations are averaged over 10 runs with confidence level of 95%. Simulations were carried out on a PC with 2.4 GHz Quad-Core processor and 4 GB RAM using Castalia simulator integrated with IBM CPLEX solver using C++ interface.

Fig.**??** shows a snapshot of the true sensor positions and the estimated positions. True positions of the sensors are depicted in blue coloured points and the estimated node positions are depicted in red coloured points, solid lines indicate the error between the estimated and true sensor positions. A close match is observed between the estimated and true positions. The estimated positions become less accurate as we move towards the boundary. Fig.1b shows the Commulative distributive function (CDF) of localization errors. The



(a) Snapshot of the true vs estimated nodes locations



(b) CDF of localization errors

Figure 1. Centralized SOCP results for uniform topology: n = 500, $RadioRange$ =0.17, p =0.15 and nfd =0.05.

total mean localization error (LE) for the network in

Fig.1 is 0.02 with confidence interval (-0.002 , 0.04) and standard deviation 0.04 .

Effect of Varying Network Size. Fig.2 shows the CDF of localization errors (log scale) for different network sizes with the same radio range in the same area for a network with nfd=0.05, rr=0.11 and anchor percentage of 15%. Fig.2 shows that when the network size increases in the same area, the degree of connectivity is increased and this leads to larger communications between nodes. Consequently, more distances are obtained between nodes which directly improves the localization accuracy. On the other hand, it must be noted that increasing node density and hence communications between nodes increases network overhead and energy consumption.

The mean localization errors (LEs), and their confidence intervals are listed in table 1 which shows a decrease in mean localization error when increasing the network size. Table 2 shows the number of variables, number of constraints, and the computation runtime spent for solving the networks in Fig.2. Computation runtime grows with increasing network size, thus limits the scalability of the technique for large networks.

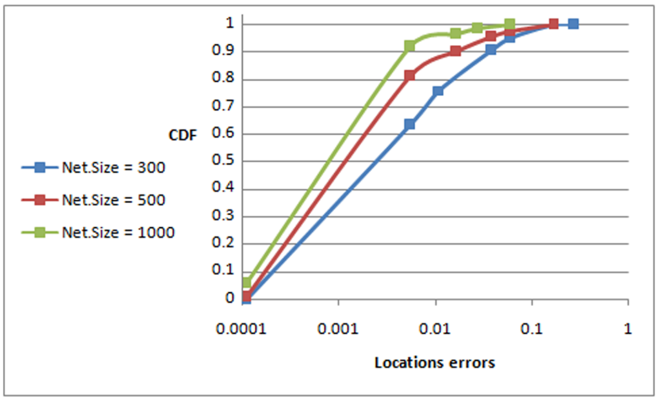


Figure 2. CDF of localization errors (log scale) for different network sizes

Table 1. Mean localization errors for different network sizes

network size	mean error	standard deviation	confidence interval
300	0.01	0.022	(-0.001 , 0.025)
500	0.006	0.015	(-0.003 , 0.016)
1000	0.002	0.018	(-0.009 , 0.014)

Effect of Varying Anchors Percentage. Fig.3 shows the effect of changing anchor percentage on the localization accuracy for a network of 500 nodes, rr=0.17 and nfd=0.05. We changed number of anchor nodes while maintaining number of non-positioned nodes the same (500 non-positioned nodes). Fig.3 shows that increasing anchor percentage considerably increases the localization accuracy. For a network of 20% of anchors, 80% of the nodes have error less than 0.0017. For anchor percentage greater than 20%, slight improvement in localization accuracy is achieved. This means that the implemented technique requires a proper setting of percentage of anchors to achieve good localization accuracy. The mean localization errors (LEs) and their confidence intervals are listed in table 3. However, increasing anchors percentage increases the total complexity of the computations. This is shown in table 4.

Table 2. Performance and computation runtime comparison of different network sizes.

network size	$\|\mathcal{A}\|$	# of variables	# of constraints	comp. run time (sec.)
300	1498	3472	1481	1.279
500	4388	9460	4305	5.936
1000	17051	35132	16716	152.044

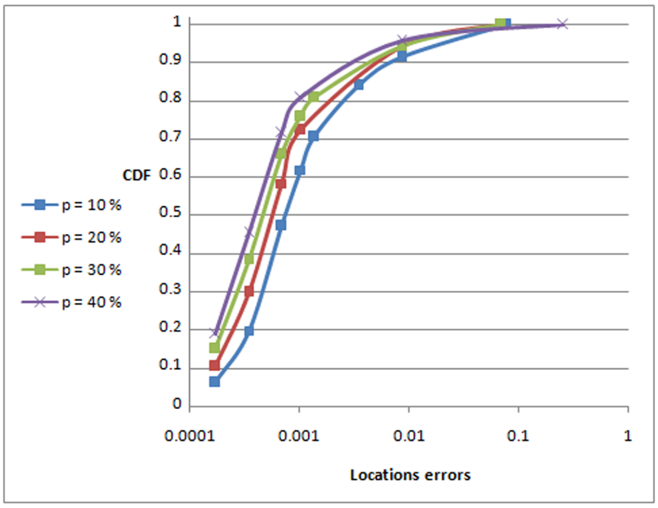


Figure 3. CDF of localization errors (log scale) for different percentage of anchors

Table 3. Mean localization errors for different anchor percentages

anchor percentage	mean error	standard deviation	confidence interval
10%	0.003	0.006	(-0.001 , 0.006)
20%	0.002	0.005	(-0.001 , 0.005)
30%	0.0018	0.0045	(-0.001 , 0.0048)
40%	0.0015	0.0046	(-0.001 , 0.0044)

Effect of Varying Communication Radio Range. To study the effect of changing radio range on localization

Table 4. Performance and computation runtime comparison of different anchor percentages.

anchor percentage	$\lvert\mathcal{A}\rvert$	# of variables	# of constr-aints	comp. run time (sec.)
10%	12590	25970	12485	35.32
20%	15013	30370	14685	66.39
30%	17611	34566	16783	94.56
40%	20198	38306	18653	124.71

accuracy, we set rr to 0.08, 0.1 and 0.15. Fig.4

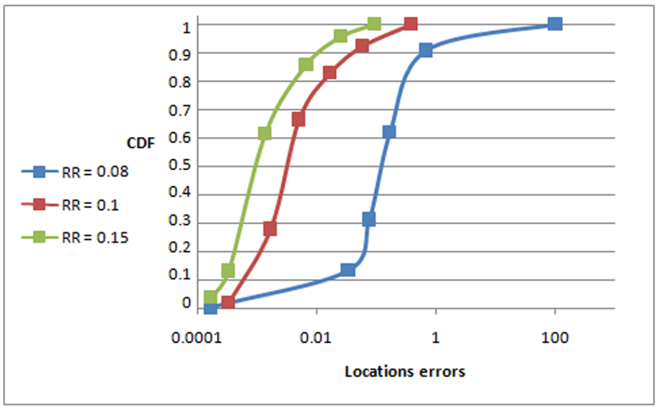


Figure 4. CDF of localization errors (log scale) for different radio ranges

shows that networks with lower radio range have higher localization error. This is due to less inter-node communications and hence distances information between nodes. It must be noted that for rr=0.08, 97% of total nodes are localized (413 node) whereas 100% of total nodes are localized for rr=0.1, 0.15. However, increasing radio range increases power consumption and adds more communication overhead to the network. The mean localization errors (LEs) and their confidence intervals are listed in table 5. Performance and computation runtime comparison is shown in table 6.

Table 5. Mean localization errors and computation time for different radio ranges

radio range	mean error	standard deviation	confidence interval
0.08	4.2	16.7	(-6.15 , 14.5)
0.1	0.015	0.034	(-0.006 , 0.036)
0.15	0.004	0.009	(-0.002 , 0.01)

Effect of Varying Noise Value Added to Distances Measurements. Fig.5 shows the effect of changing nfd on the localization accuracy. Fig.5 shows that there is no

Table 6. Performance and computation runtime comparison of different radio ranges.

radio range	$\lvert\mathcal{A}\rvert$	# of variables	# of constraints	comp. run time (sec.)
0.08	982	2750	965	0.62
0.1	3668	8052	3601	4.58
0.15	7819	16172	7661	29.82

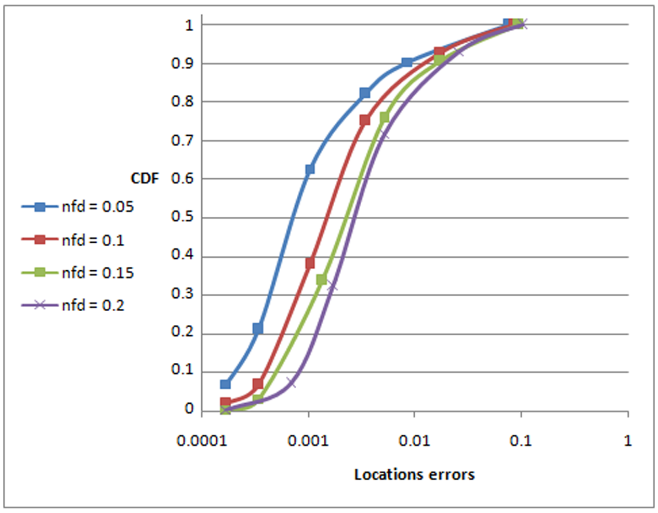


Figure 5. CDF of localization errors (log scale) for different nfd

significant improvement in localization accuracy when decreasing nfd. Thus, the implemented localization technique solves the localization problem with good localization accuracy in the presence of inaccuracies in distance measurements. The mean localization errors (LEs) and their confidence intervals for the networks in Fig.5 are listed in table 7.

Table 7. Mean localization errors for different nfd

nfd	mean error	standard deviation	confidence interval
0.05	0.003	0.007	(-0.001 , 0.007)
0.1	0.005	0.009	(-0.0008 , 0.01)
0.15	0.006	0.01	(-0.0005 , 0.012)
0.2	0.007	0.012	(-0.0001 , 0.014)

3. RC-SOCP: Proposed refined clustered SOCP localization approach

RC-SOCP is a refined clustered SOCP localization technique to allow scalability of the centralized SOCP-localization technique for large dense networks with thousands of nodes. This technique achieves better performance by reducing computation time, energy consumption, and communication overhead resulting from numerous communications overhead between the nodes in centralized localization approach. Its architecture is divided into three phases as shown in Fig.6: clustering phase, localization phase and refinement phase. In the first phase, Min-Max d clustering algorithm is used to divide the network and select the CH according to the IDs of the nodes. In the second phase, each cluster implements SOCP-based localization technique to localize the member nodes including the CH itself. In the third phase, each cluster implements Gauss Newton local algorithm to refine the estimated locations obtained.

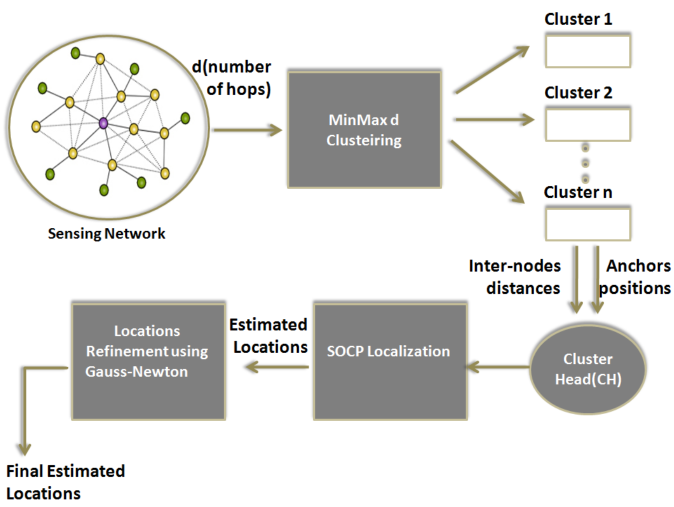


Figure 6. Refined Clustered SOCP Architecture

3.1. Clustering

Min-Max d clustering algorithm is used to divide the network and select the cluster head CH according to the IDs of the nodes. It was chosen because it is a simple, less computationally demanding and thus doesn't add extra complexity to the localization technique. This clustering algorithm runs asynchronously eliminating overhead of synchronizing the clocks of the nodes. Also, a low variance in cluster sizes leads to better load balancing among the cluster-heads [10].

3.2. Implementing SOCP on cluster-head

At the end of the clustering phase, the network is divided into multiple independent clusters. Each cluster of size CN sensors, CM sensors with unknown locations and $CN - CM$ sensors with known locations (anchors). Member nodes send their relevant distance information with their neighbours to their elected CHs. Each CH constructs the positions matrix and the lower triangle of the distance matrix of sizes 2×CN, CN×CN respectively as in centralized technique then formulates SOCP localization problem for its cluster member nodes according to algorithm 1 and uses CPLEX optimization solver to estimate the positions of the nodes.

3.3. Refinement phase

To enhance localization accuracy of clustered SOCP (C-SOCP), a cluster level refinement step is implemented. Each cluster head solves the network localization problem using Gauss-Newton (iterative least-squares) algorithm [11]. The initial position guess for the Gauss-Newton optimization is the positions drawn from the preprocessor SOCP solver which is close to the global solution. Given m residual functions $r = (r_1, ..., r_m)$ of n variables $X = (X_1, ..., X_n)$, with $m \geq n$, a non-linear least square problem is an unconstrained optimization problem of the form

$$min \sum_{i=1}^{m} r_i(\mathbf{X})^2. \tag{7}$$

The Gauss-Newton algorithm iteratively finds the minimum of the sum of the squares in equation(7). For implementing the Gauss-Newton method as a refinement step, where X refers to the position coordinates of the sensor node (x,y), and hence n is set to 2. m is the number of residuals, and is equal to the number of anchor neighbours to that sensor node. The residual function r_i is defined as

$$r_i = \|X - a\|^2 - d^2 \tag{8}$$

Where a represents the positions of the anchor node (a_x, a_y) and d is the estimated distance between the sensor node and the non-positioned node. Starting with an initial guess $\mathbf{X}^{(0)}$ for the minimum, the method proceeds by the iterations

$$\mathbf{X}^{(s+1)} = \mathbf{X}^{(s)} - \left(\mathbf{J_r}^{\mathrm{T}}\mathbf{J_r}\right)^{-1}\mathbf{J_r}^{\mathrm{T}}\mathbf{r}(\mathbf{X}^{(s)}) \tag{9}$$

Where the symbol $^{\mathrm{T}}$ denotes the matrix transpose. If r and X are column vectors, the entries of the Jacobian matrix are

$$(\mathbf{J_r})_{ij} = \frac{\partial r_i(\mathbf{X}^{(s)})}{\partial X_j} \tag{10}$$

The derivative of the residual function to the first variable x is:

$$\frac{\partial r_i(\mathbf{X}^{(s)})}{\partial x} = \frac{(x - x0)}{r_i(\mathbf{X}^{(s)})}$$

And the derivative of the residual function to the second variable y is:

$$\frac{\partial r_i(\mathbf{X}^{(s)})}{\partial y} = \frac{(y - y0)}{r_i(\mathbf{X}^{(s)})}$$

Algorithm 2 : Gauss-Newton Algorithm

```
procedure GAUSSNEWTON(distanceMatrix, positionsMatrix)
Let iters = number of iterations , neighbours = array of anchor neighbours
for all node_i do
    i_x ← initial guess of x coordinate for node i
    i_y ← initial guess of y coordinate for node i
    n ← number of neighbours of node i
    neighbours ← getNeighboursOfNode(i)
    Let distanceEstimate = array of distances estimations with anchor neighbours of size n × 1
    Let distanceNoise = array of distances noises between estimated distances and distances of initial guess n × 1
    Let J = jaccobian matrix of size n × 2
    Let J_T = transpose Jacobian matrix of size 2 × n
    Let delta = matrix of size 2 × 1 of values for updating x,y coordinates of node i
    for j = 0 to iters do
        for k = 0 to n do
            xdiff ← i_x − neighbours[k].x
            ydiff ← i_y − neighbours[k].y
            distance ← pow(xdiff, 2) + pow(ydiff, 2)
            distanceEstimate(k, 0) ← sqrt(distance)
            J(k, 0) ← xdiff/distanceEstimate(k, 0)
            J(k, 1) ← ydiff/distanceEstimate(k, 0)
            distance ← distanceMatrix[i, k]
            distanceEstimate(k,0) ← distanceNoise(k,0) - distance
        end for
        delta ← ( ( ( ( J_T J )J_T ).inverse() )J_T ) distanceNoise
        i_x ← i_x − delta(0, 0)
        i_y ← i_y − delta(1, 0)
    end for
end for
```

Algorithm 2 shows the pseudo-code of the Gauss-Newton algorithm for refining the locations obtained from C-SOCP.

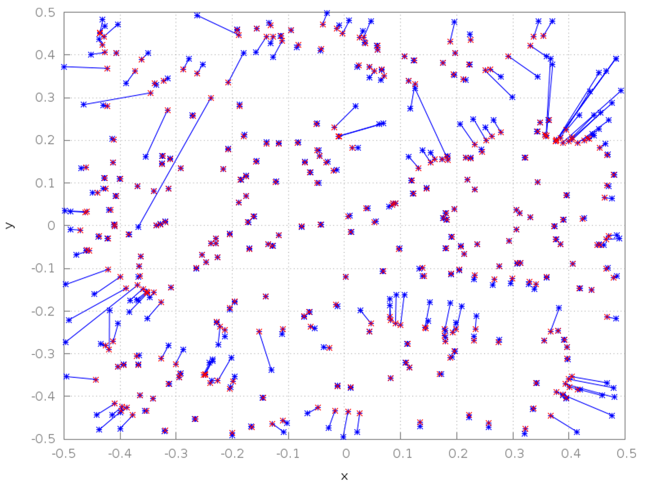
3.4. Performance evaluation of RC-SOCP

RC-SOCP performance is evaluated by measuring localization accuracy and computation time. The localization accuracy is calculated as defined in section 2.4. To calculate RC-SOCP computation time, we calculate C-SOCP runtime and Gauss-Newton refinement algorithm runtime on each CH. The computation time metric is measured as the maximum RC-SOCP computation time among CHs. C-SOCP runtime is the time spent for formulating and solving the SOCP problem on the CH using C++ std.clock() function. The time needed for computing the relative distances at the nodes and communication or message exchanges time is excluded. We evaluate the performance of RC-SOCP localization technique by studying the effect of varying: cluster size , anchor percentage, communication radio range, noise value added to distances measurements.

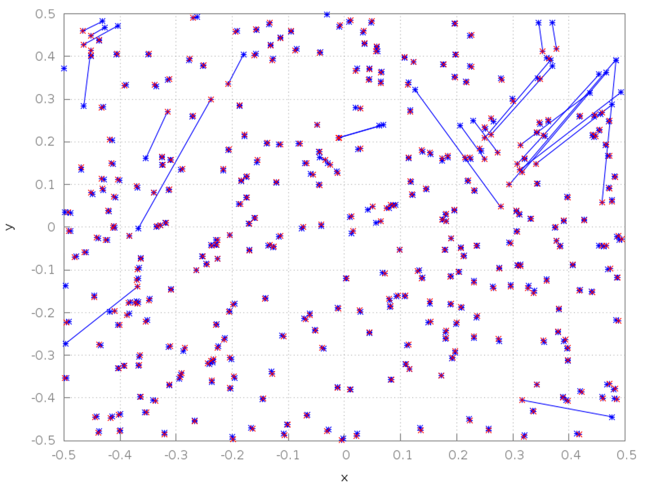
4. Simulation results

Simulations are conducted for a randomly generated 500 nodes uniformly distributed on the unit square area $[-0.5, 0.5]^2$ with noise factor(nfd) = 0.05, radio range(rr) = 0.17, d = 3 and Degree of Connectivity = 37. Anchors are chosen to be uniformly distributed throughout the network with percentage(p) of 30%. Number of Gauss-Newton iterations is 25. Simulations are averaged over 10 runs with confidence level of 95%. Figs. (7a) and (7b) show a snapshot of the true sensor positions and the estimated positions of the C-SOCP and RC-SOCP respectively. True positions of the sensors are depicted in blue colored points and the estimated node positions are depicted in red colored points, solid lines indicate the error between the estimated and true sensor positions. An improvement in localization accuracy is observed for RC-SOCP compared to C-SOCP. It is also observed that the estimated positions become less accurate as we move towards the boundaries of the clusters in C-SOCP which is greatly refined in RC-SOCP. The mean localization error (LE) for the network in fig. 7 using C-SOCP is 0.02 and standard deviation 0.04, and LE is 0.01 and standard deviation 0.04 for RC-SOCP. This clarifies that the mean error is reduced by 50% using RC-SOCP. Fig.7c shows a comparison between the Commulative distributive function (CDF) of localization errors resulting from centralized-SOCP localization implemented in section 2, C-SOCP and RC-SOCP. The detailed CDF of localization errors for the results in fig. 7c are listed in table 8.

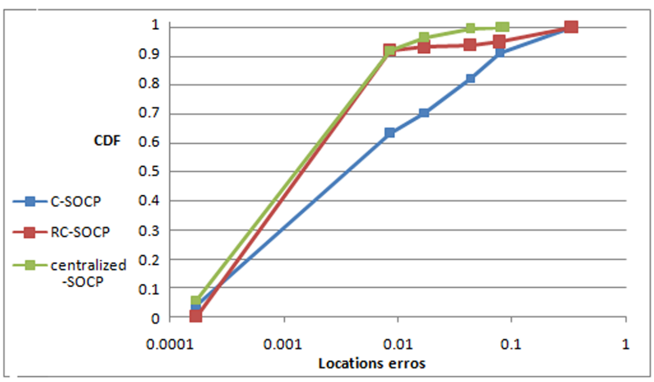
Fig.7c shows that RC-SOCP and centralized-SOCP lead to improved results for localization error compared to C-SOCP. Table 8 shows that, in RC-SOCP, a high percentage of nodes (90%) have a small localization error (<0.006), while the corresponding percentage of nodes is 59.9% in C-SOCP. However, due to the increase in the localization error of some of boundary nodes (outliers) of the network in RC-SOCP, the maximum localization error for both C-SOCP and RC-SOCP is the same(<0.35). It is worthy noting that RC-SOCP and centralized-SOCP has almost the same localization accuracy for most of nodes (>90%) which is (<0.006). For less than 10% of the nodes, there is a slight improvement in accuracy for centralized-SOCP. RC-SOCP provides a considerable reduction in computation time compared to centralized-SOCP due to clustering while still yielding good localization accuracy and scalability.



(a) Snapshot of real vs estimated nodes locations of C-SOCP localization



(b) Snapshot of real vs estimated nodes locations of RC-SOCP localization



(c) CDF results of localization errors(log scale) of centralized SOCP, C-SOCP and RC-SOCP sizes

Figure 7. C-SOCP and RC-SOCP results for uniform topology: $n = 500, rr = 0.17, p = 0.15, d = 3 and nf d = 0.05$.

Table 8. CDF results of localization errors

Localization error	technique	nodes percentage	mean error	stand. dev.
< 0.004	C-SOCP	55.5%	0.001	0.001
	RC-SOCP	70%	0.002	0.0009
	centralized-SOCP	87%	1.07e-1	0.0001
< 0.006	C-SOCP	59.9%	0.001	0.0016
	RC-SOCP	90%	0.0029	0.0013
	centralized-SOCP	90%	0.001	0.121
< 0.07	C-SOCP	91%	0.01	0.016
	RC-SOCP	95%	0.0039	0.005
	centralized-SOCP	99.5%	0.0025	0.118
< 0.2	C-SOCP	98%	0.019	0.03
	RC-SOCP	98%	0.0077	0.02
	centralized-SOCP	100%	0.0026	0.118
< 0.35	C-SOCP	100%	0.02	0.038
	RC-SOCP	100%	0.01	0.04
	centralized-SOCP	100%	0.0026	0.118

4.1. The effect of varying cluster Size

To study the effect of varying cluster size on localization error and computation time, we set the number of consecutive hops (d) to 1, 2, 3 and 4. Fig.8 shows CDF of localization errors (log scale) for C-SOCP and RC-SOCP with different d. Table 9 shows number of clusters, C-SOCP runtime, Gauss-Newton runtime and RC-SOCP runtime of networks. Fig.8 shows that the refinement phase of RC-SOCP enhances the result of the preprocessor C-SOCP. C-SOCP is highly dependent on cluster size compared to RC-SOCP. As the cluster size increases, the localization accuracy is improved since more nodes distances information is obtained at the CH. These distances information directly affect and enhance the formulation of the sub problems and hence improve the result of the SOCP problem solver. However, increasing cluster size increases the problem size and computation time and communication overhead of the cluster. For RC-SOCP, the localization accuracy is increased when d is increased from 1 to 2. For $d \geq 2$, there is not significant difference in localization accuracy for different cluster size (d), since refinement algorithm reaches the optimal solution for the estimate of locations of the nodes. So, adjusting cluster size using $d = 2$ which is approximated to number of clusters around 11 in this network size(500 nodes) as shown in table 9 yields good accepted localization accuracy. Table 9 shows that incrementing the cluster size d reduces the number of clusters approximately to the

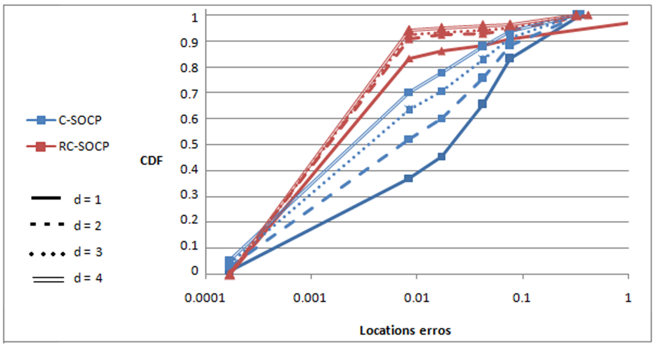


Figure 8. CDF reaults of localization errors (log scale) for different cluster sizes

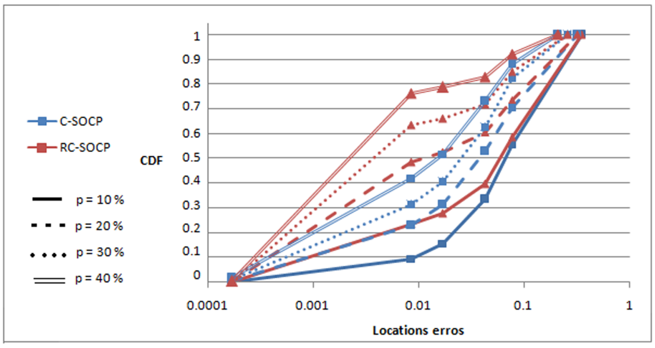


Figure 9. CDF results of localization errors(log scale) for different anchors percentage

half. As a result of increasing cluster sizes, C-SOCP and RC-SOCP run times are increased while Gauss-Newton runtime has no significant change. Table 10

Table 9. Performance and computation runtime comparison of different cluster sizes.

d	# of clusters	C-SOCP runtime (sec.)	Gauss-Newton runtime	RC-SOCP runtime (sec.)
1	24	2.117	0.014	2.128
2	11	6.468	0.011	6.475
3	7	14.761	0.019	14.791
4	5	20.554	0.016	20.57

Table 10. Mean localization errors for different cluster sizes for C-SOCP and RC-SOCP

d	technique	mean error	standard deviation	confidence interval
1	C-SOCP	0.03	0.04	(0.009 , 0.06)
	RC-SOCP	0.02	0.13	(-0.05 , 0.11)
2	C-SOCP	0.028	0.04	(0.002 , 0.05)
	RC-SOCP	0.015	0.05	(-0.016 , 0.047)
3	C-SOCP	0.02	0.038	(-0.002 , 0.045)
	RC-SOCP	0.013	0.04	(-0.01 , 0.04)
4	C-SOCP	0.016	0.033	(-0.004 , 0.037)
	RC-SOCP	0.01	0.036	(-0.01 , 0.03)

also shows that increasing the cluster size reduces LE in C-SOCP significantly for all d whereas it leads to slightly improved LE in RC-SOCP for $d \geq 2$.

4.2. The effect of varying anchors percentage

To investigate the effect of varying anchors percentage on localization error and computation time, we change number of anchor nodes to 10%, 20%, 30% and 40% of the number of non-positioned nodes while maintaining the number of non-positioned nodes the same (500 non-positioned nodes), $rr = 0.11$, $d = 2$ and $nfd = 0.05$ for C-SOCP and RC-SOCP. Fig.9 shows that increasing anchors percentage considerably increases the localization accuracy for both C-SOCP and RC-SOCP; and RC-SOCP has good localization accuracy compared to C-SOCP. Appropriate percentage of anchors ensures that each cluster has an acceptable number of anchors to perform the localization problem. However, increasing number of anchors often imposes additional costs. Table 11 shows that the number of clusters and computation runtime time are increased with increasing the number of anchors. This happens due to the increase of the total number of nodes in the network. Table 12 shows a reduction in LE when increasing anchors percentages in both C-SOCP and RC-SOCP.

Table 11. Performance and computation runtime comparison of different anchors percentages.

anchor percen-tage	# of clust-ers	C-SOCP runtime (sec.)	Gauss-New ton runtime (sec.)	RC-SOCP runtime (sec.)
10%	20	0.356	0.011	0.365
20%	24	0.393	0.016	0.405
30%	26	0.465	0.012	0.476
40%	27	1.693	0.022	1.71

4.3. The effect of varying communication radio range

To study the effect of changing radio range on localization accuracy, we set rr to 0.08, 0.1, 0.15 and 0.17. Fig.10 shows that for C-SOCP and RC-SOCP, higher radio range leads to higher degree of connectivity and hence results in higher localization accuracy. It is noticed in table 14 that for the network with rr =0.08, C-SOCP has a better localization accuracy for 50% of the nodes (LE =0.032) than in RC-SOCP (LE=0.038). This refers to low degree of connectivity for rr=0.08 and less distances information for C-SOCP resulting in bad initial guess

Table 12. Mean localization errors for different anchors percentages for C-SOCP and RC-SOCP

anchor percentage	technique	mean error	standard deviation	confidence interval
10%	C-SOCP	0.08	0.06	(0.042 , 0.118)
	RC-SOCP	0.083	0.07	(0.035 , 0.1)
20%	C-SOCP	0.05	0.048	(0.02 , 0.08)
	RC-SOCP	0.052	0.137	(-0.03 , 0.137)
30%	C-SOCP	0.03	0.04	(0.01 , 0.06)
	RC-SOCP	0.038	0.1	(-0.036 , 0.1)
40%	C-SOCP	0.02	0.03	(0.007 , 0.04)
	RC-SOCP	0.02	0.07	(-0.02 , 0.06)

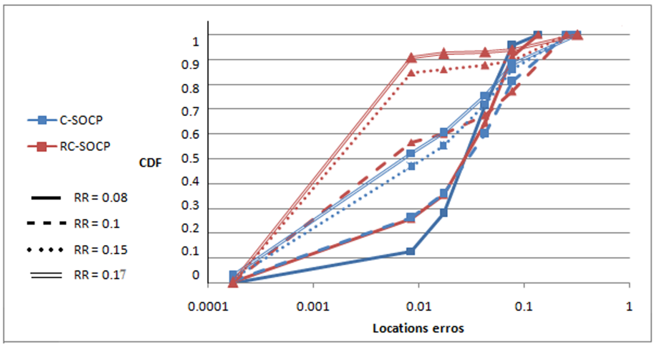


Figure 10. CDF results of localization errors (log scale) for different radio ranges

for Gauss-Newton iterations. Large radio ranges lead to higher localization accuracy. However, increasing radio range increases power consumption, and adds more communication overhead. Average degree of connectivity (*doc*), number of clusters and run-times of networks in fig. 10 are shown in table 13.

Table 13. Performance and computation runtime comparison of different radio ranges.

radio range	*doc*	# of clusters	SOCP runtime (sec.)	Gauss-Newton run time (sec.)	RC-SOCP runtime (sec.)
0.08	3	63	0.148	0.013	0.162
0.1	12	48	0.595	0.013	0.601
0.15	28	14	1.592	0.013	1.605
0.17	37	11	6.468	0.011	6.475

4.4. The effect of varying noise value added to distances measurements

Fig.11 shows that RC-SOCP is highly affected by nfd compared to C-SOCP while RC-SOCP has a good localization accuracy compared to C-SOCP for all nfds.

Table 14. Mean localization errors for different radio ranges for C-SOCP and RC-SOCP

radio range	technique	mean error	standard deviation	confidence interval
0.08	C-SOCP	0.032	0.02	(0.02 , 0.05)
	RC-SOCP	0.038	0.08	(-0.01 , 0.09)
0.1	C-SOCP	0.032	0.04	(0.017 , 0.07)
	RC-SOCP	0.025	0.13	(-0.04 , 0.12)
0.15	C-SOCP	0.032	0.04	(0.005 , 0.06)
	RC-SOCP	0.02	0.05	(-0.01 , 0.05)
0.17	C-SOCP	0.028	0.04	(0.002 , 0.05)
	RC-SOCP	0.015	0.05	(-0.02 , 0.05)

A slight improvement in localization accuracy for C-SOCP is noticed when decreasing nfd while significant improvement in localization accuracy is noticed when decreasing nfd in RC-SOCP. The localization accuracy of RC-SOCP for $nfd = 0.05$, nfd =0.1 gives the same results. This means that Gauss-Newton was able to get the same localization accuracy when nfd is increased from 0.05 to 0.1, but was unable to do same for nfds greater that 0.1. The mean localization errors (*LE*s) for the network in fig. 11 are listed in table 15.

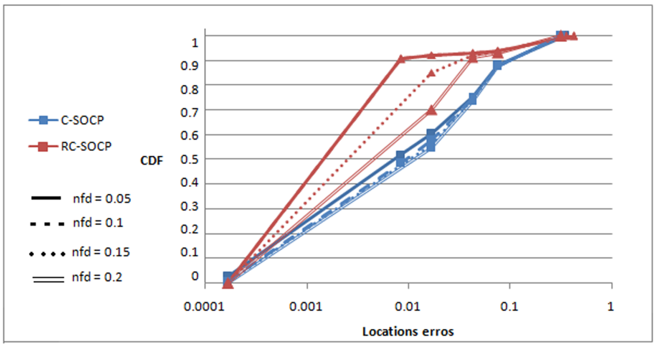


Figure 11. CDF results of localization errors (log scale) for different nfd

5. Conclusion

This paper presented an implementation of localization technique for WSNs based on Second Order Cone Programming (SOCP) to solve the localization problem in WSNs. We used our own experience trying to provide a detailed description of SOCP problem formulation for Castalia wireless sensor networks simulator. To allow scalability, it also proposed a refined clustered localization approach based on the centralized SOCP technique (RC-SOCP). An extensive simulation study of the approach under different

Table 15. Mean localization errors for different nfd for C-SOCP and RC-SOCP

nfd	technique	mean error	stand. dev.	confidence interval
0.05	C-SOCP	0.028	0.04	(0.002 , 0.05)
	RC-SOCP	0.015	0.05	(-0.016 , 0.047)
0.1	C-SOCP	0.029	0.04	(0.0037 , 0.052)
	RC-SOCP	0.0196	0.06	(-0.018 , 0.058)
0.15	C-SOCP	0.030	0.04	(0.0048 , 0.056)
	RC-SOCP	0.022	0.046	(-0.0068 , 0.051)
0.2	C-SOCP	0.032	0.04	(0.0052 , 0.059)
	RC-SOCP	0.026	0.047	(-0.003 , 0.051)

scenarios and with varying several networks parameters has been investigated. Simulation results show that RC-SOCP achieves good performance and acceptable localization accuracy with controlling cluster size (number of hops between the CH and gateway node), communication radio range of the sensor nodes and anchors percentage. Moreover, it shows that the proposed approach solves the localization problem with good localization accuracy in the presence of inaccuracies in distance measurements. The proposed refined clustered SOCP-based localization approach performs almost as well as the centralized-SOCP while providing a considerable reduction in computation time. However, it still yields good localization accuracy and scalability.

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