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Optimal ZF Precoder for MU massive MIMO Systems over Ricean Channel with per Antenna Power Allocation

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Abstract

Multiuser massive multiple-input multiple-output systems have the potential to increase the data rate. However, with a large base station (BS) antenna, the non-square channel matrix restricts the zero-forcing (ZF) precoder rotations to obtain the best optimal solution with the per-antenna power allocation. In this paper, we propose the beamforming and lattice reduction (LR) approach to restrain the channel matrix and transform the lattice of the channel vectors to be near orthogonal. Numerical results show that the LR-based ZF precoder outperforms other ZF precoder schemes, such as, the norm approximation of the beamforming matrix. In particular, the sum rate of the proposed optimal ZF precoder requires a small number of BS antenna. Subsequently, with the strong line of sight (LoS) channel, the optimal power allocations in the subchannels depend on the dominance of the users in order to achieve substantial multiplexing and diversity gains. Specifically, the Ricean channel gain with the water-filling allocation at high SNR is non-negligible.

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1. Introduction

The unprecedented demand in services and applications calls for a radical melioration in the evolving wireless networks. To meet this demand, next-generation communication systems e.g. LTE-Advanced have started incorporating MIMO systems to improve on the network capacity [1, 2]. In recent times, the benefit of multiplexing and diversity gains in MIMO [3] has been extended to the multi-user MIMO (MU-MIMO) system where base stations equipped with multiple antennas serve multiple users simultaneously. Such that, random channel vectors for the different users are near orthogonal, this makes MU massive MIMO robust to the propagation environments as compared to the conventional MIMO [4, 5]. Additionally, MU massive MIMO can resist the effect of ill-conditioned propagation (insufficient scattering), e.g. LoS paths [4]. MU massive MIMO robustness to LoS propagation is realized through: proper scheduling on the same time-frequency resource, this is made possible by precoding and detection schemes [6], and channel-dependent scheduling, which enables multiuser interference particularly from other served users to be canceled, where the multiple transmit antenna induces large channel fluctuations [7, 8].

However, in a typical urban wireless network, the MU massive MIMO channels have both LoS channel and non-paltry scattering channel. In this paper, we primarily concentrate on Ricean channel (combination of LoS and scattering channels) in actualizing efficient beamforming for the MU massive MIMO system. One vital point is that LoS channel directs signal energy in a selected angular direction to specific terminals. By physical beamforming, the antenna element sets the beam signals to focus on a particular direction in order to avoid interference from other directions [9, 10]. As introduced in [11], the optimal beam subset and orthogonal random beamforming with user beam selection achieved diversity gain and multiplexing gain.

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Unfortunately most related literature [12–17] have focused on Rayleigh fading channel [1] in substantiating beamforming, this falls short of investigating the fading power variations of specular or LoS component, which is a cardinal component for the millimeter wave (mmWave) communication in 5G networks [18].

Although the overall performance of MU massive MIMO requires efficient multi-user interference (MUI) elimination, the transmit precoding is a strategy to study. Linear precoding such as zero forcing (ZF), minimum mean square error (MMSE) is often used for the interference control. The ZF precoder is more efficient for multiuser interference (MUI) suppression, that is, with the singular value decomposition (SVD) and block diagonalization (BD), the ZF precoder can search domains of MU massive MIMO transmission over entire nullspace of other users [12, 13, 19]. Afterwards, the optimal ZF precoder is maximized under two conditions: by transmitting on the right eigenchannel (the parallel non-interfering subchannels) and by power allocations through optimization on each non-interfering subchannel [20] - [22]. In [13, 23], square and non-square channel matrices are studied, respectively, under sum power constraint. Nevertheless, the optimal ZF precoder under per-antenna power allocation obtains higher sum rate than sum power allocation in [14, 24]. This is attributed to efficiently transmit power through per-antenna, as a result, bounds the allocated power to each of the power amplifiers (PA), which limits the independent linearity of the PA [25]. Thus, the associated hardware serves each antenna effectively as compared to the sum power allocation where power is arbitrarily distributed. The per-antenna power allocation also provides the required power for the antenna beamforming [26, 27]. However, by assuming rank-one optimal precoder solution with the norm approximation, the sum power allocation outperformed the per-antenna power allocation in [28], since the norm approximation allows equal power allocation on the channel. This limitation in per-antenna power is however resolved in this paper with the proposed beamforming approach.

Moreover, MU massive MIMO system with large non-square channel matrix where the BS antennas M are more than the combined user antennas K and users N (i.e. $M \geq NK$), the antenna beamforming vectors are less orthogonal. This limits the matrix norm approximation in accessing all the diagonal elements. To improve on the MU massive MIMO channel matrix, Lattice Reduction (LR) technique is incorporated in [12] to constraint the channel matrix dimension. By utilizing the complex Lenstra, Lenstra and Lovasz (CLLL) algorithm in the LR [29], the basis of the channel vectors can be transformed. This ameliorates the orthogonality of basis vectors. As shown in [15], the sum rate of precoder can achieve the maximum diversity gain with the transceiver. In this paper, we employ the LR to transform the precoder vectors, such that, the channel vectors can be used to focalize the beamforming.

The goal of this work is to analyze the performance of MU massive MIMO downlink system with non-square channel matrix under Ricean channel. We design the optimal ZF precoder and adopt the per-antenna power allocation at the BS. Further, we incorporate the LR to transform the channel lattice of the precoder, and then evaluate the sum rate of the optimal ZF precoder with beamforming in the downlink MU massive MIMO systems. The sum rate of the MU massive MIMO systems under per-antenna power allocation is a great contribution.

The rest of the paper is outlined as: Section 2 designs the system model of the MU massive MIMO system over Ricean Fading channel, then Section 3 presents the Optimal ZF Precoder and the optimization Designs and the numerical results and Discussions are provided in Section 4. Finally, Section 5 draws the conclusions of the study.

2. System Model

We consider a downlink MU massive MIMO system with a BS equipped with M-array antennas, and N users, where each user is equipped with K ($K \geq 1$) antennas. Assuming the MU massive MIMO channel between the BS and nth user is modeled with Ricean channel, the Ricean channel matrix $\mathbf{H}_n \in \mathbb{C}^{K \times M}$ is decomposed into deterministic LoS channel matrix $\mathbf{H}_{L,n}$, which has arbitrary rank mean [30] and scattering channel matrix $\mathbf{H}_{S,n}$. The Ricean channel matrix is written as [18, 30]

$$\mathbf{H}_n = \mathbf{\Lambda}_{L,n}\mathbf{H}_{L,n} + \mathbf{\Lambda}_{S,n}\mathbf{H}_{S,n}, \qquad (1)$$

where $\mathbf{\Lambda}_{L,n} \in \mathbb{C}^{K \times K}$ and $\mathbf{\Lambda}_{S,n} \in \mathbb{C}^{K \times K}$ are diagonal matrices for LoS and scattering channels, respectively, with entries $\mathbf{\Lambda}_{L,n} = diag\left\{\sqrt{\frac{\kappa}{\kappa+1}}\right\}$, $\mathbf{\Lambda}_{S,n} = diag\left\{\sqrt{\frac{1}{\kappa+1}}\right\}$ and $\kappa \in [0, \infty]$ as the Ricean factor[2]. The diagonal channel elements easily support the per-antenna power allocation. Therefore, the $K \times 1$ received signal vector of

[1] Note that this current journal paper extends the conference version in [17], the cases of Ricean Channel, Norm approximation and Optimal power Allocation are included.

[2] Defined as the normalized power of the specular and scattered components

nth user is modeled as

$$\begin{aligned} \mathbf{y}_n &= \mathbf{H}_n\mathbf{x} + \mathbf{z}_n \\ &= (\mathbf{\Lambda}_{L,n}\mathbf{H}_{L,n} + \mathbf{\Lambda}_{S,n}\mathbf{H}_{S,n})\,\mathbf{x} + \mathbf{z}_n \\ &= \left(\sqrt{\frac{\kappa}{\kappa+1}}\,\mathbf{h}_{L,n}^{\mathrm{H}} + \sqrt{\frac{1}{\kappa+1}}\,\mathbf{h}_{S,n}^{\mathrm{H}}\right)\mathbf{x} + \mathbf{z}_n, \end{aligned} \qquad (2)$$

where $\mathbf{h}_{L,n} \in \mathbb{C}^{M\times 1} = [1\ e^{j\frac{2\pi d}{\lambda}\sin(\theta_n)}, ..., e^{j\frac{2\pi d}{\lambda}(M-1)\sin(\theta_n)}]^T$ and $\mathbf{h}_{S,n} \in \mathbb{C}^{M\times 1}$ are column vectors of $\mathbf{H}_{L,n}$ and $\mathbf{H}_{S,n}$, respectively, $\mathbf{z}_n \in \mathbb{C}^{K\times 1}$ is the (i.i.d) complex Gaussian noise vector and $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal vector. Henceforth, the transmitted signal vector can be formulated as

$$\mathbf{x} = \sum_{n=1}^{N} \mathbf{T}_n\mathbf{s}_n, \qquad (3)$$

where $\mathbf{T}_n \in \mathbb{C}^{M\times K}$ is the precoder matrix and $\mathbf{s}_n \in \mathbb{C}^{K\times 1}$ denotes transmit data vector, thus $\mathbb{E}\left[\left(\mathbf{s}_n\mathbf{s}_n^{\mathrm{H}}\right)\right] = \mathbf{I}_K$. In this case, the total power P_T radiated from the BS antenna array is written as

$$\mathbb{E}\left[\left(\mathbf{x}\mathbf{x}^{\mathrm{H}}\right)\right] = \left[\sum_{n=1}^{N} \mathrm{tr}(\mathbf{T}_n\mathbf{T}_n^{\mathrm{H}})\right] \le P_T\ , \qquad (4)$$

and the power radiated by each BS antenna element from the precoder is as

$$\mathbb{E}\left[\left(\mathbf{x}\mathbf{x}^{\mathrm{H}}\right)\right]_{ii} = \left[\sum_{n=1}^{N} \mathrm{tr}(\mathbf{T}_n\mathbf{T}_n^{\mathrm{H}})\right]_{ii} \le p_i \quad \forall_i = 1, ..., M \qquad (5)$$

where p_i is power of ith transmit antenna. From (3), the nth user received signal $\mathbf{y}_n$ can be expanded by the help of the precoded transmitted signal as [17]

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{T}_n\mathbf{s}_n + \underbrace{\sum_{\substack{j=1\\ j\neq n}}^{N} \mathbf{H}_n\mathbf{T}_j\mathbf{s}_j + \mathbf{z}_n}, \qquad (6)$$

where the underlined term denotes the interference plus noise. Note that the desired signal, interference signals and noise are uncorrelated. It is significant to adopt a model that removes the interference and then use water-filling to control the noise. In (6), the BS transmits to different user terminals, as a result, each user terminal receives all the transmitted signals, the user terminal, therefore, has to extract the desired signal $\mathbf{s}_n$ and avoid interference.

3. Optimal ZF Precoder Design

We assume the transmitters have perfect CSI for transmit precoding, the estimation of the nth user effective channel $\mathbf{H}_n\mathbf{T}_n$ is obtained by precoding the pilots of $\mathbf{T}_n$. This is used to mitigate the nth user downlink multiuser interference (MUI) in (6). To avoid MUI, we enforce the multiuser ZF condition on the interference in (6) as

$$\mathbf{H}_n\mathbf{T}_j = 0 \quad \text{for} \quad j \neq n. \qquad (7)$$

Remark 1. *The suppression of the inter-user interference by ZF condition further reduces as the number of antennas at the BS increases, in this sense, the loss in the desired signal gain reduces as the user channels become more orthogonal.*

Moreover, (7) completely zeros the interference component in (6). By invoking condition (7) into (6), we arrive at

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{T}_n\mathbf{s}_n + \mathbf{z}_n. \qquad (8)$$

Now, the columns of $\mathbf{H}_n\mathbf{T}_n$ correspond to the singular values of the the non-interference. That is, the condition (7) forced $\mathbf{T}_n$ to be located in the nullspace of $\bar{\mathbf{H}}_n = \left(\mathbf{H}_1^{\mathrm{H}}, \mathbf{H}_2^{\mathrm{H}}, \mathbf{H}_{n-1}^{\mathrm{H}}\quad \mathbf{H}_{n+1}^{\mathrm{H}}, ..., \mathbf{H}_N^{\mathrm{H}}\right)^{\mathrm{H}}$ from reception by the nth user against other users transmissions. Here, BD is required to eigendecompose the MU massive MIMO channel into multiple parallel subchannels. Assuming the $M \ge NK$ regime, the singular value decomposition (SVD) is performed as [31]

$$\bar{\mathbf{H}}_n = \mathbf{U}_n\ \Sigma_n\ \mathbf{V}_n^{\mathrm{H}}, \qquad (9)$$

where $\mathbf{U}_n$ and $\mathbf{V}_n$ are $(N-1)K \times (N-1)K$ and $(M \times M)$ unitary matrices respectively, Σ_n is $(N-1)K \times M$ component of diagonal matrix consisting of the ordered singular values.

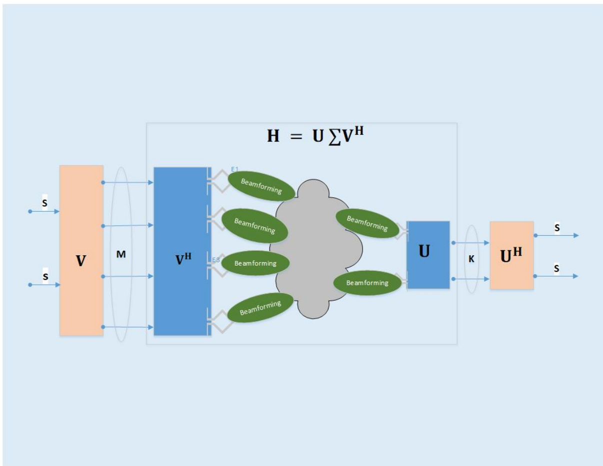

Figure 1. System design with the precoding and beamforming

For $rank\left(\bar{\mathbf{H}}_n\right) = (N-1)K$, columns of $\bar{\mathbf{H}}_n$ can be constructed in $\mathbf{V}_n$ for the precoder $\mathbf{T}_n$. This enables the transmitted signal to correspond to the received signal. To model the matrix space of the orthonormal basis that maps the transmit precoder $\mathbf{T}_n$ to the channel $\bar{\mathbf{H}}_n$, we decompose the aggregated transmit precoder matrix. By applying QR decomposition on the precoder matrix $\mathbf{T}_n$

as

$$\mathbf{T}_n = \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n, \quad (10)$$

where $\bar{\mathbf{V}}_n \in \mathbb{C}^{M\times m}$ is the orthonormal basis matrix, with $m = M - (N-1)K$ as columns conditioned on $\mathbf{T}_n$, and upper triangular matrix $\hat{\mathbf{V}}_n \in \mathbb{C}^{m\times K}$ denotes arbitrary matrix of the power constraint over the $\mathbf{T}_n$, this assumes computation of the diagonal elements. Then, plugging (9) and (10) into (8), the information signal transmitted through the eigenchannels can be received. Accordingly, the estimated received signal for the nth user is expressed as

$$\hat{\mathbf{s}}_n = \mathbf{U}_n^{\mathrm{H}} \mathbf{y}_n = \mathbf{U}_n^{\mathrm{H}} \mathbf{U}_n \, \Sigma_n \, \mathbf{V}_n^{\mathrm{H}} \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n \mathbf{s}_n + \tilde{\mathbf{z}}_n, \quad (11)$$

where $\tilde{\mathbf{z}}_n = \mathbf{U}_n^{\mathrm{H}} \mathbf{z}_n$ is the additive Gaussian noise and the $\mathbf{U}_n \; \Sigma_n \; \mathbf{V}_n^{\mathrm{H}} \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n \mathbf{U}_n^{\mathrm{H}}$ provides the parallelized non-interfering SU-MIMO channels. In MU massive MIMO systems, the parallelized channels provide several independent parallel subchannels within the eigenstructure to enhance the multiplexing gain. On the other hand, in order for the precoder to be optimal, the subchannels must be properly aligned with the precoder rotation $\mathbf{T}_n = \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n$, so as to extract the transmitted power. The channel in (11) over the $\hat{\mathbf{V}}_n$ often assumes water-filling to provide the equivalent power to the parallelized eigenchannels.

3.1. Optimal ZF Precoder Optimization

To construct the $\mathbf{T}_n = \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n$ precoder rotations, we set the power rotation around $\hat{\mathbf{V}}_n$ as $\hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^{\mathrm{H}} = \Theta_n \; (m \times m)$, whereby Θ_n is a *positive semi-definite* (PSD) matrix and its rank is lower than M. However, the sum rate maximization problem with the per-antenna power allocation is formulated as

$$\begin{aligned} \max_{\boldsymbol{\Theta}_\mathbf{n}} \text{imize } C_n(\mathbf{P}_n) &= \sum_{n=1}^{N} \log\det(\mathbf{I} + \mathbf{B}\mathbf{P}_n) \\ \text{subject to } & \left[\sum_{n=1}^{N} \operatorname{tr} \left| \bar{\mathbf{V}}_\mathbf{n} \boldsymbol{\Theta}_\mathbf{n} \bar{\mathbf{V}}_n^{\mathrm{H}} \right| \right]_{ii} \le p_i \quad \forall_i = 1, \ldots, M \\ & \boldsymbol{\Theta}_\mathbf{n} \ge 0 \quad n = 1, \ldots., N \\ & \text{rank } (\boldsymbol{\Theta}_\mathbf{n}) \le K \,, \end{aligned} \quad (12)$$

where $\mathbf{P}_n = \left| \mathbf{U}_n \; \Sigma_n \; \mathbf{V}_\mathbf{n} \bar{\mathbf{V}}_\mathbf{n} \boldsymbol{\Theta}_\mathbf{n} \bar{\mathbf{V}}_\mathbf{n}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \Sigma_n^{\mathrm{H}} \mathbf{U}_n^{\mathrm{H}} \right|$ and $\mathbf{B}$ is any arbitrary matrix in the objective function. We observe that under the per-antenna power allocation, the sum rate maximization is over the diagonal entries of $\boldsymbol{\Theta}_\mathbf{n}$. As a consequence of the $M \ge NK$ (non-square) regime, the dimensions of the $\bar{\mathbf{V}}_n(M \times m)$ become larger than $\hat{\mathbf{V}}_n(m \times K)$, this makes the optimization problem in (12) difficult or impossible to achieve best optimal solution. That is because the domain search for the optimization in (12) limits the span of the diagonal $[.]_{ii}$ in choosing the $\boldsymbol{\Theta}_\mathbf{n}$ entries. Hence the rotations around the nullspace in $\left(\bar{\mathbf{V}}_n \hat{\mathbf{V}}_n\right)$ results in rank deficiency since the rank$(\boldsymbol{\Theta}_\mathbf{n}) = M - (N-1)K$ is lower than M. Nonetheless, assuming the matrix is square ($M = NK$), the precoder is easily optimized under sum power allocation [23] since the matrix diagonalization is not required. To resolve the precoder rotation problem, matrix determinant maximisation solution is discussed in [13], besides, we propose a new beamforming focalization approach (Figure 1) with the channel matrix in next subsection.

3.2. Optimal SVD-ZF with Beamforming (BF)

Hereafter, we modify the previous approach under per-antenna power allocation, by designing an optimum transmission strategy. That is, beamforming approach to resize the matrix dimension and facilitate cohesion for the channel matrix between the transmit and receive antennas. We define the $(N-1)K \times m$ channel matrix as

$$\mathbf{X}_n = \Sigma_n \mathbf{V}_\mathbf{n} \bar{\mathbf{V}}_n, \quad (13)$$

as a result, define the beamforming matrix $\mathbf{W}_n \in \mathbb{C}^{M \times (N-1)K}$ by

$$\mathbf{W}_n = \bar{\mathbf{V}}_n \mathbf{X}_n^{\dagger}, \quad (14)$$

where $\mathbf{X}_n^{\dagger} = \mathbf{X}_n^{\mathrm{H}} \left(\mathbf{X}_n \mathbf{X}_n^{\mathrm{H}} \right)^{-1}$ is the Moore-Penrose inverse of the channel matrix $\mathbf{X}_n$ with the precoder and $\bar{\mathbf{V}}_n$ is defined by $\mathbf{V}_\mathbf{n}$ in (9). It is worthnoting that the beamforming $\mathbf{W}_n$ taps only a single eigenmode of the channel $\mathbf{X}_n$ since the channel matrix $(N-1)K \times m$ is rank deficient. By dropping the $\mathbf{U}_n$ matrix in the sequel and capitalizing on $\mathbf{P}_n = \Sigma_n \, \mathbf{V}_\mathbf{n} \bar{\mathbf{V}}_\mathbf{n} \boldsymbol{\Theta}_\mathbf{n} \bar{\mathbf{V}}_\mathbf{n}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \Sigma_n^{\mathrm{H}}$ and (13), we recompute PSD matrix $\boldsymbol{\Theta}_\mathbf{n} = \hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^{\mathrm{H}}$ as

$$\begin{aligned} \boldsymbol{\Theta}_\mathbf{n} &= \left(\Sigma_n \mathbf{V}_\mathbf{n} \bar{\mathbf{V}}_\mathbf{n} \right)^{\dagger} \mathbf{P}_n \left(\bar{\mathbf{V}}_\mathbf{n}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \Sigma_n^{\mathrm{H}} \right)^{\dagger} \\ &= \left(\mathbf{X}_n^{\dagger} \right) \mathbf{P}_n \left(\mathbf{X}_n^{\dagger} \right)^{\mathrm{H}}. \end{aligned} \quad (15)$$

Subsequently, by substituting (15) into (14), the optimal SVD-ZF with beamforming (BF) can be obtained. Therefore, the optimization problem in (12) is rewritten as

$$\begin{aligned} \max_{\mathbf{P}_n} \text{imize } C_n(\mathbf{P}_n) &= \sum_{n=1}^{N} \log\det(\mathbf{I} + \mathbf{B}\mathbf{P}_n) \\ \text{subject to } & \sum_{n=1}^{N} \operatorname{tr} \left| \mathbf{W}_n \mathbf{P}_n \mathbf{W}_n^{\mathrm{H}} \right|_{ii} \le p_i \quad \forall_i = 1, ..M \\ & \mathbf{P}_n \ge 0, \mathbf{W}_n \ge 0 \\ & \text{rank } (\mathbf{W}_n) = \text{rank } (\mathbf{P}_n) \le K \,. \end{aligned} \quad (16)$$

Note that the optimal solution always has $rank(\mathbf{P}_n) \ge 1$ for $K \ge 1$ with the user terminals. In the same line of discussion, rank relaxation approach is considered in [32]. Intuitively, assuming the $\mathbf{W}_n \geq 0$ satisfies $\sum_{i=1}^{M} |\mathbf{W_n}|_{ii} \geq 0$ for $p_i \geq 0$, the beamforming channel matrix align the mapping of the transmit antennas onto the receive antennas, then the beam pattern focus directly in the optimal direction. Then the beamforming matrix ($\mathbf{W}_n$) becomes suboptimal as the channel ($\mathbf{X}_n$) turns orthogonal, thus maximizes the achievable sum rate for n user.

Optimal Power Allocation. As the parallel channels have different channel quality, optimal allocation power over the parallel channels is performed by the water-filling. From (9), the Ricean channel Σ_n $(N-1)K \times M$ have non-negative entries, with diagonal elements in the descending order in the form $\left(\sqrt{\frac{\kappa}{\kappa+1}}\,\lambda_{L,\max(k,i)} + \sqrt{\frac{1}{\kappa+1}}\,\lambda_{S,\max(k,i)}\right) \geq \left(\sqrt{\frac{\kappa}{\kappa+1}}\,\lambda_{L,\min(k,i)} + \sqrt{\frac{1}{\kappa+1}}\,\lambda_{S,\min(k,i)}\right)$. Thus, the water-filling power allocation over the channel is given as

$$p_l = \left(\nu - \frac{1}{\left(\frac{\kappa}{\kappa+1}\,\lambda_{L,l}^2 + \frac{1}{\kappa+1}\,\lambda_{S,l}^2\right)}\right)^+ \qquad 1 \leq l \leq \Delta_{\mathbf{H}_n} \quad (17)$$

where p_l is the power used to transmit the information, ν is the parameter chosen to fulfill the water-fill level with the power allocation $\sum_{l=1}^{\Delta_{\mathbf{H}_n}} p_l = p_i \quad \forall_i$, $\Delta_{\mathbf{H}_n}$ is the number of positive singular values p_l in the water-filled sub-streams and $(x)^+$ is given as $\max(x, 0)$. The per-antenna power allocation provides single measure that reflects on the individual power for each antenna [26]. Allocating power to each eigenchannel with water-filling achieves the optimality in the channel sum rate. In the case of per-antenna power allocation with strong LoS channel ($\kappa \geq 1$), the optimal solution is not proved to be globally optimal [27]. This is attributed to the similarity between the channel paths, where the collinearity between channels is ([0 1]) [33]. Reducing the channel collinearity ($\kappa \geq 1$) improves the channel sum rate.

3.3. Optimal SVD-ZF with norm beamforming approximation

To evaluate the inequality constraint (16) in the fixed point p_i which is accomplished in the undetermined $|\mathbf{P_n}|_{ii}$, we let eigenvector of the $\mathbf{P_n}$ be $\mathbf{p}_n = (k, 1)$ for $1 \leq k \leq K$. Besides, the beamforming vector $\mathbf{w}_n = (w_{1,k}, .., w_{M,k})$ with entry (i, k) forms the Hermitian matrix $\mathbf{W_n}$ as the k-dimensional volume of the parallelepiped forms the vectors over the M antennas. From Shur's inequality [31], the beamforming vector coefficient is $|\mathbf{w}_n|^2 \leq \left(\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n\right)$, thus the bounds of the optimization is $\left[\mathbf{p}_n\,|\mathbf{w}_n|^2\right]_{ii}$ for the ith transmit antenna.

Hence, the norm approximation of the inequality constraint in (16) with $|\mathbf{w}_n|^2 = 1$ is a convex problem and is solvable with at least one optimal value [21]. We adopt the norm approximation for the power allocation follows:

Proposition 1. Suppose $\|\cdot\|_q$ and $\|\cdot\|_r$ are norms on $\mathbb{C}^M$ and $\mathbb{C}^K$, respectively, where $1 \leq q, r \leq \infty$ we define the operator norm of $\mathbf{W}_n \in \mathbb{C}^{M \times K}$, then the corresponding norm of the mapping $\mathbf{W}_n$ is as

$$\begin{aligned}
&\|\mathbf{W}_n\|_{q,r} \\
&= \max_{\mathbf{p}_n} \left\{ \|\mathbf{W}_n\mathbf{p}_n\|_q : \|\mathbf{p}_n\|_r \;\leq 1,\, \mathbf{p}_n \in \mathbb{C}^K \right\} \\
&\overset{(a)}{=} \max_{\mathbf{p}_n} \left\{ \sqrt{\left\|[|\mathbf{W}_n\mathbf{p}_n|]^2\right\|_{q/2}} : \|\mathbf{p}_n\|_r \;\leq 1 \right\} \\
&\overset{(b)}{=} \max_{\mathbf{p}_n} \left\{ \left(\left(\mathbf{W}_n\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\mathbf{W}_n^{\mathrm{H}}\right)^{\frac{q}{2}}\right)^{\frac{1}{q}} : \sqrt{\left\|\left(\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\right)\right\|_{\frac{r}{2}}} \;\leq 1 \right\} \\
&= \max_{\mathbf{p}_n \in \mathbb{C}^K} \left\{ \left\|\left(\mathbf{W}_n\left(\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\right)\mathbf{W}_n^{\mathrm{H}}\right)\right\|_{\frac{q}{2}} : \left\|\left(\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\right)\right\|_{\frac{r}{2}} \;\leq 1 \right\} \\
&= \max_{\mathbf{P_n}} \left\{ \left\|\left(\mathbf{W}_n\mathbf{P_n}\mathbf{W}_n^{\mathrm{H}}\right)\right\|_{\frac{q}{2}} : \|(\mathbf{P_n})\|_{\frac{r}{2}} \;\leq 1 \right\} \\
&\leq \max_{\mathbf{P_n}} \left\{ \left\|\mathrm{tr}\left(\mathbf{W}_n\mathbf{P_n}\mathbf{W}_n^{\mathrm{H}}\right)\right\|_{\frac{q}{2}} : \left\|\mathrm{tr}\left(\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\right)\right\|_{\frac{r}{2}} \;\leq 1,\;\; \mathbf{P}_n \geq 0 \right\},
\end{aligned} \quad (18)$$

where step (a) and step (b) follow from $\|\mathbf{z}\|_d = \sqrt{\left\|[|\mathbf{z}|]^2\right\|_{\frac{d}{2}}} = \sqrt{\left\|(\mathbf{z}\mathbf{z}^{\mathrm{H}})\right\|_{\frac{d}{2}}}$, it can be concluded that the $\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}$ is symmetric PSD [21], so the maximization only increases by the optimal value with $q, r \in [1, \infty)$. To this end, the (16) is convex w.r.t inequality (18) where $1 \leq q \leq 2 \leq r \leq \infty$, thus the optimal solution has $\mathrm{rank}(\mathbf{p}_n) \geq 2$, as $k \geq 2$ user antennas by the convex constraint $\left\|\mathrm{tr}\left(\mathbf{p}_n\mathbf{p}_n^{\mathrm{H}}\right)\right\|_{\frac{r}{2}}$. Subsequently, the best transmission strategy is to employ the water-filling to allocate power on the channel with high gain.

However, the bounds of $\|\mathbf{W}_n\|_{q,r}$ is not tight under large M BS antenna in the $(M \geq NK)$ regime, assuming the maximum $rank(\mathbf{W_n}) = (N-1)K$ and $\bar{\mathbf{V}}_n \in \mathbb{C}^{M \times m}$ for $m = M - (N-1)K$ obtains the singular value. In this case, the beamforming $\mathbf{W_n}$ consists of long $\mathbf{w}_n$ row vectors. And, the maximum number of uncoupled equivalent beamforming is $(N-1)K < M$, with remaining $M - (N-1)K$ transmit antennas become redundant with no receive antennas. This allows off diagonal elements to appear in the main diagonal $\mathbf{P_n}$. To validate this reason in the zero-noise MU-MIMO system, the expected received signal in (11) can be rewritten as

$$\begin{aligned}\hat{\mathbf{y}}_n &= \mathbf{U}_n^{\mathrm{H}}\mathbf{U}_n\ \Sigma_n\ \mathbf{V}_n^{\mathrm{H}}\bar{\mathbf{V}}_n\hat{\mathbf{V}}_n\mathbf{s}_n \\ &\overset{(a)}{=} \Sigma_n\ \mathbf{V}_n^{\mathrm{H}}\bar{\mathbf{V}}_n\hat{\mathbf{V}}_n\mathbf{s}_n \\ &\overset{(b)}{=} \mathbf{X}_n\hat{\mathbf{V}}_n\mathbf{s}_n \\ &\overset{(c)}{=} \mathbf{W}_n^{\mathrm{H}}\bar{\mathbf{V}}_n\hat{\mathbf{V}}_n\mathbf{s}_n \\ &= \mathbf{W}_n^{\mathrm{H}}\mathbf{T}_n\mathbf{s}_n, \end{aligned} \tag{19}$$

where step(a) follows $\mathbf{U}_n^{\mathrm{H}}\mathbf{U}_n = \mathbf{I}_K$, step (b) is from $\mathbf{X}_n = \Sigma_n\ \mathbf{V}_\mathbf{n}^{\mathrm{H}}\bar{\mathbf{V}}_n$ and step (c) obtains from change of variable in (14). From (8) and (19), the quality of the beamforming $\mathbf{W_n}$ is determined by finding the Frobenius (error) norm of the received signals, i.e. $\|\mathbf{y}_n - \hat{\mathbf{y}}_n\|_2$. The result is shown in Figure 2 with $M - K$ (increase M while K is fixed). With large BS antennas, the steering beamforming $\mathbf{W_n}$ in (19) may not point directly to the direction of the nth user but towards other users, as in (6) and (8). Hence, the nth user receives a small part of the transmit power. To resolve this problem, the massive MIMO matrix dimension constrained is discussed in [15], this involves the user antennas, channel matrix $\mathbf{X}_n$ and $\bar{\mathbf{V}}_n$ precoder power matrix. In the next subsection, we determine the tightness of $\mathbf{W_n}$ by reducing the basis of $\mathbf{w}_n$ consisting of short vectors. Intuitively, the short vectors correspond to the subchannels that actually participate in the information signal transmission and beamforming.

3.4. Optimal SVD-ZF with Lattice Reduction based BF

In this subsection, the precoder $\mathbf{T}_n = [\mathbf{t}_1, \mathbf{t}_2, ...\mathbf{t}_M]$ transmits to the users with the lattice reduction based beamforming. Lattice reduction (LR) incorporated with the complex LLL (Lenstra, Lenstra and Lovasz) algorithm [12] is efficient in transforming the columns of the $\mathbf{W_n}$ beamforming matrix. The algorithm is designed with the Gram-Schmidt Orthogonalization (GSO) to project channel $\mathbf{X}_n$ and orthonomal basis matrix $\bar{\mathbf{V}}_n$ to be more orthogonal. To transform beamform matrix $\mathbf{W_n}$ (14), we decompose the complex lattice of $\mathbf{T}_n$ by

$$\mathbf{T}_n^* = \bar{\mathbf{V}}_n^*\hat{\mathbf{V}}_n^* \tag{20}$$

where $\bar{\mathbf{V}}_n^* \in \mathbb{C}^{M\times m}$ denotes a unimodular transformation matrix with complex integers. Ordinarily, the GSO is initiated by setting column vectors of $\hat{\mathbf{V}}_n^*$ and $\bar{\mathbf{V}}_n^*$ as $\hat{\mathbf{v}}_n^* = [v_{1,k}^*, v_{2,k}^*, ...v_{M,k}^*]$ and $\xi_i^* = \frac{\langle v_{i,k}, v_{i,k}^*\rangle}{\|v_{i,k}^*\|^2}$, respectively, whereby the GSO coefficient ξ_i^* is the vector collinearity [0 1] used to determine the similarity between two channel vectors and bound the orthogonality defect[3]. Thus, evaluates the vector subspaces and the correlation of the vector distance. In this case, the dimension span in vector space of the channel basis is to eliminate vectors that are linear combinations of other vectors. In (20), each column vector of $\mathbf{T}_n^*$ is $\mathbf{t}_i^* = \left(\xi_i^* v_{i,k}^*\right)$ for $1 < k \leq K$ and $1 \leq i \leq M$, and orthonormal basis for the ith BS antenna and the kth user antenna is given by [29]

$$\mathbf{t}_M^* = \mathbf{t}_M^* - \sum_{i=1}^{k-1} \xi_i^* v_{i,k}^* \quad \text{for } 1 \leq i < k \leq M \tag{21}$$

Consequently, the LR process $|v_{1,k}^*|, \ldots, |v_{k-1,k}^*|$ approaches zero if $\xi_i^* = 0$, thus $\mathbf{t}_M^*$ is almost orthogonal in the subspace span $\mathbf{t}_1^*, \ldots, \mathbf{t}_{M-1}^*$ to accomplish linearly independent vectors. Therefore, the lattice basis is reduced in size if $|\xi_i^*| \leq 1/2$ [15], by

$$|v_{i,k}^*| = \frac{1}{2}|v_{i,i}^*| \text{ for } 1 \leq i < k \leq M \tag{22}$$

where the reduced basis ensures off-diagonal elements of the channel vectors are almost half the diagonal elements. This however does not guarantee minimum basis for the lattice. The general size-reduced basis using Lovasz condition [29] is achieved by subtracting a suitable linear combination $\left(\rho - |\xi_{k-1}^*|^2\right)$ in the consecutive basis $v_{k,k}^*$ and $v_{k-1,k-1}^*$, and is written as

$$\|v_{k,k}^*\|^2 + \|\xi_{k-1,}^* v_{k-1,k-1}^*\|^2 \geq \rho\, \|v_{k-1,k-1}^*\|^2, \quad 2 \leq k \leq M, \tag{23}$$

where the reduction basis $\rho = \frac{3}{4}$ is standard value ($\frac{1}{4} < \rho < \mathbf{1}$) in achieving a better performance in (22) for large matrices. Note that the new shorter basis $v_{k,k}^* + \xi_{k-1,}^* v_{k-1,k-1}^*$ is the transformation of $v_{k,k}$ onto the orthogonal vector space, similarly $v_{k-1,k-1}^*$ is component of $v_{k-1,k-1}$ beam vector basis. Again, the $\hat{\mathbf{v}}_n^*$ is near orthogonal and shorter projection of $\hat{\mathbf{v}}_n$. As such, the reduced vector $\mathbf{t}_n^* = \xi_n^* v_{M,k}^*$ of $\mathbf{T}_n^* = \bar{\mathbf{V}}_n^*\hat{\mathbf{V}}_n^*$, is used for beamforming $\mathbf{W}_n^* = \bar{\mathbf{V}}_n^*(\mathbf{X}_n^*)^\dagger$, whereby channel $(\mathbf{X}_n^*)^\dagger$ is more orthogonal and shorter as compared to the beamforming $\mathbf{W_n}$ (14). The implementation of the CLLL algorithm requires QR decomposition (i.e. householder Reflections) as $\mathbf{W}_n^* = \mathbf{QR}$, where $\mathbf{Q} = \bar{\mathbf{V}}_n^*$ is $(M \times m)$ matrix and $\mathbf{R} = (\mathbf{X}_n^*)^\dagger$ is $(m \times (N-1)K)$ upper triangular matrix. This follows the iteration over polynomial time, which is presented in the algorithm in Table 1.

The CLLL algorithm swaps pairs of $v_{k,k}$ and $v_{k-1,k-1}$ for $v_{k,k}^*$ and $v_{k-1,k-1}^*$ as the size-reduction steps proceed. Finally, from (16), the optimal precoder achieves the

[3]The orthogonality defect is used to measure the orthogonality of the basis vectors

Table 1. CLLL Algorithm Procedure

1. Initialize the GSO for $v_{1,k}, .., v_{i,k}$, calculate $v^*_{1,k}, .., v^*_{i,k}$ and coefficients ξ^*_i
2. Form size reduction for the pairs $v_{k,k}$ and $v_{k-1,k-1}$ and update ξ^*_{k-1}
3. Use Lovasz condition for the pair $v^*_{k,k}$ and $v^*_{k-1,k-1}$ and update ξ^*_{k-1}
4. Else go to step 2.

maximum sum rate as $C^*_n = \max_{\mathbf{W}^*_n} C_n(\mathbf{P}_n)$ with reduced basis of the transformed beamforming.

To test the quality of the proposed beamforming $\mathbf{W}^*_n$, (20) is considered, then the expected received signal in (11) is rewritten as

$$\begin{aligned}\hat{\mathbf{y}}^*_n &= \mathbf{U}^{\mathrm{H}}_n \mathbf{U}_n \, \Sigma_n \, \mathbf{V}^{\mathrm{H}}_n \bar{\mathbf{V}}^*_n \hat{\mathbf{V}}^*_n \mathbf{s}_n \\ &= \mathbf{X}^*_n \hat{\mathbf{V}}^*_n \mathbf{s}_n \\ &= \mathbf{W}^{*\mathrm{H}}_n \bar{\mathbf{V}}^*_n \hat{\mathbf{V}}^*_n \mathbf{s}_n \end{aligned} \tag{24}$$

Similarly, the quality of beamforming $\mathbf{W}^*_n$ is determined by the Frobenius (error) norm of the received signal as $\|\mathbf{y}_n - \hat{\mathbf{y}}^*_n\|_2$. Figure 2 shows the absolute received error for (19) and (24). The proposed optimal SVD-ZF-LR precoder achieve good gain since the beamforming channels are near orthogonal, hence diversity gain $(M - K)$ compensate the Ricean channel correlation or vector collinearity. Thus, the large BS antennas generate larger DoFs and support the beamforming focalization.

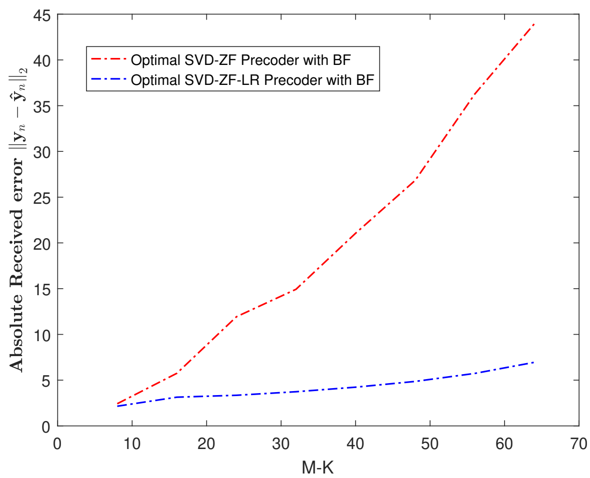

Figure 2. Frobenius norm of the received signals with the $(M - K)$ diversity order

Proposition 2. Considering $(M \geq NK)$ with fixed user antenna $1 < k \leq K$ for all N users, then channel Σ_n depends on user selection $(N-1)K$, with $M \to \infty$, $N \to \infty$ and $M \gg N$, then $0 < k \leq \frac{M}{N} < \infty$ has constant k values by

$$k \leq \frac{M}{N} \tag{25}$$

Therefore, the per-antenna power allocation in the eigenchannels is as optimal as the water-filling in achieving maximum channel capacity in the $M \geq Nk$ regime.

4. Numerical Analysis and Discussions

In this section, numerical analysis and discussions are provided to validate the performance of per-antenna power allocation for MU massive MIMO. We analyze the impact of the channel correlation from the Ricean fading channel. The theoretical tightness of the study is simulated with Monte Carlo of 10000 realizations. The precoder is constructed from the $\bar{\mathbf{V}}_n$ $(M \times m)$, where $m = M - (N-1)K$, the LR standard basis is $\rho = \frac{3}{4}$ and Ricean factor κ is varied. The figures compare schemes such as direct SVD-ZF-BF (16), SVD-ZF-BF with BF $\|\mathbf{W_n}\|$ and the proposed LR-based SVD-ZF-BF, all the schemes are analyzed with the per-antenna power allocation. In Figure 3, the plot demonstrates that the per-antenna power allocation (5 efficiently utilizes the transmit power than the total power allocation (4) power. Therefore, per-antenna power allocation can enhance the beamforming energy.

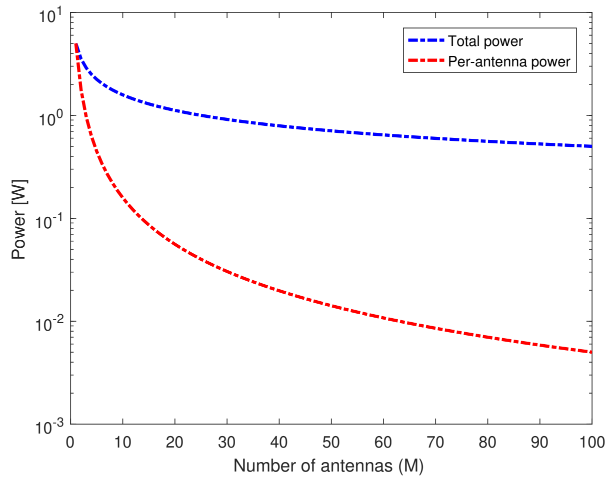

Figure 3. Compares the power utilization of the sum power and per-antenna power allocations

Now, Figure 4 and Figure 5 show the sum rate with the SNR for all the schemes with Ricean factor $\kappa = 0$ and $\kappa = 10$, respectively. Clearly, the LR-based SVD-ZF-BF achieves higher sum rate as (N) users selection increases, this validate tightness of orthogonal channel with the distinct pairs $v_{k,k}$ and $v_{k-1,k-1}$. And the Ricean channel gain with water-filling power allocation at high SNR is non-negligible. However, the direct SVD-ZF-BF improves with user selections whilst SVD-ZF-BF with BF $||\mathbf{W_n}||$ performed poorly. The poor performance is due to the absolute value and rank-one assumption of in $\mathbf{W}_n$, which constrained the orthogonal beamforming in the beam subset. The overall sum rate of our LR-based SVD-ZF-BF scheme improved the precoder performance than in [13, 14]. Conversely, the sum rate reduces as κ increases, although the LR-based SVD-ZF-BF performance improved the sum rate gap as the $\kappa = 10$. Implying that the lattice reduction supported the dominate LoS channel in achieving the diversity gain and multiplexing gain.

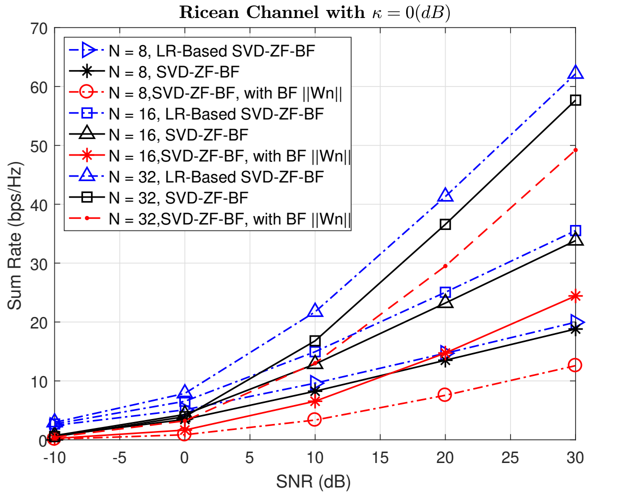

Figure 4. Sum Rate versus the SNR values, with M = 128 and K antennas =2

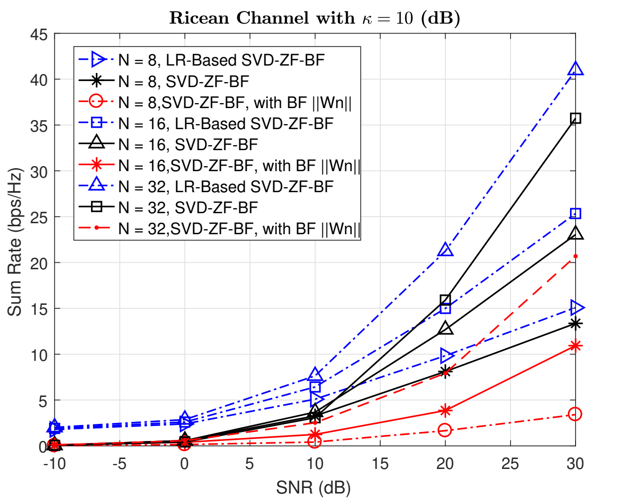

Figure 5. Sum Rate versus the SNR values, with M = 128 and K antennas =2

Then, the sum rate as a function of BS antennas M are presented in Figure 6, and Figure 7. It can be observed that the sum rate increase with M for the LR-based SVD-ZF-BF and SVD-ZF-BF, this argues the channel gain from $M \geq NK$. In particular, as M turns large, the sum rate becomes stable suggesting the limited (saturation) gain due to the spread over the large $\mathbf{H}_n\mathbf{T}_n$ [13]. So far, the rate gain by the LR-based SVD-ZF-BF is due to the elimination of vectors which are linear combinations of others vectors. We also observe that the sum rate reduces as κ increase, this is due to fewer LoS channels among different channel vectors, thus enable diversity reduction in $(M - NK)$. Again, LR-based SVD-ZF-BF effectively improved the sum rate gap in $\kappa = 10$, especially, when the number of BS antenna is small. With small BS antennas and low transmit power, the proposed optimal ZF precoder with LR-based SVD outperform the other schemes. This can be attributed to the beamforming focalization by the distinct lattice vectors and the larger DoFs. Hence lattice reduction compensates the channel correlation without adding more BS antennas. On the other hand, the sum rate of SVD-ZF-BF with BF $||\mathbf{W_n}||$ scheme is constant regardless of channel randomness, that is the norm matrix restricts the beamforming vectors.

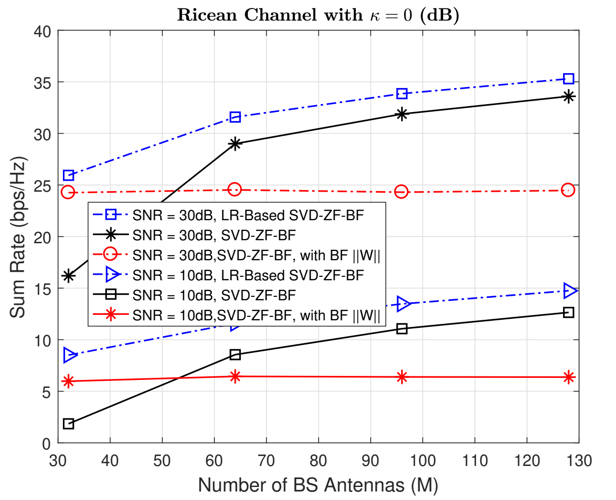

Figure 6. Sum Rate versus the number of BS antenna (M), with N users =16 and K =2

And, Figure 8 plots the sum rate against the number of users N, i.e. selection of the SU-MIMO channels. The number of users increases with SNR gain, as a result, increases the sum rate in all schemes. Obviously, the proposed LR-based SVD-ZF-BF shows a high gain in the equivalent selection of SU-MIMO channels with the orthogonal basis, but norm approximation $||\mathbf{W_n}||$ obtained less gain, this justifies our argument that $1 < NK \leq rank(\mathbf{W_n})$ is not tight for norm approximation (less orthogonal), which the norm squared suffers from such an assumption. Moreover, the strong LoS channel induces the subchannels with similar channel paths,

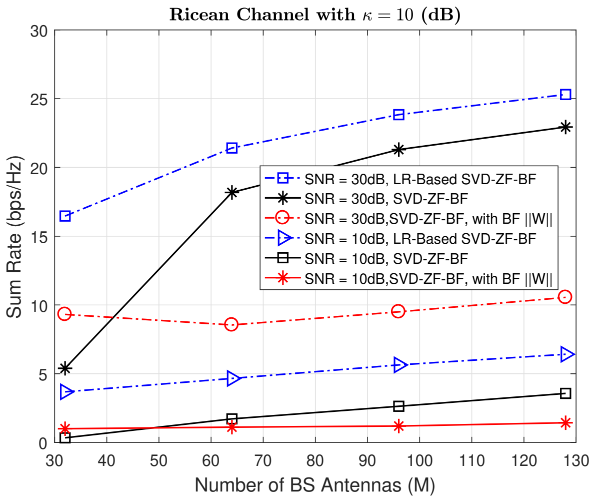

Figure 7. Sum Rate versus the number of BS antenna (M), with N users =16 and K =2

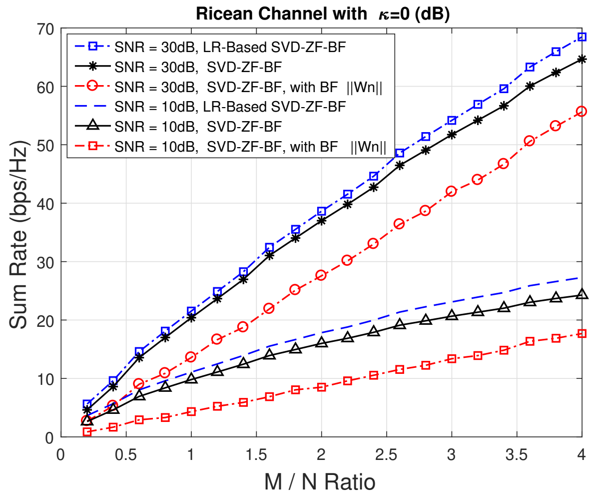

Figure 9. Sum rate versus the $0 < k \leq \frac{M}{N} < \infty$, the ratio k is user antenna

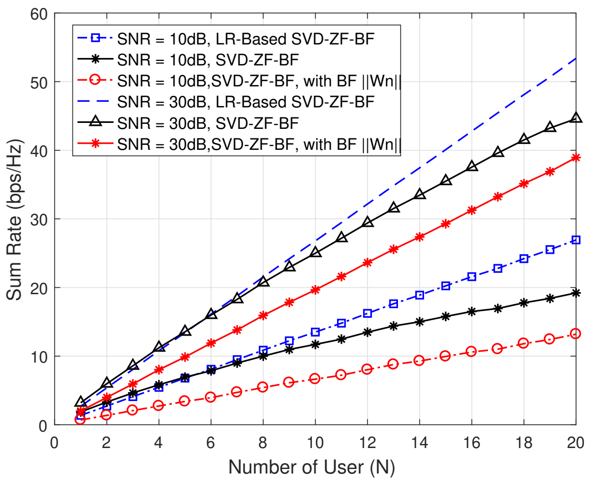

Figure 8. Sum Rate with number of Users (N), with M = 128 and K =2

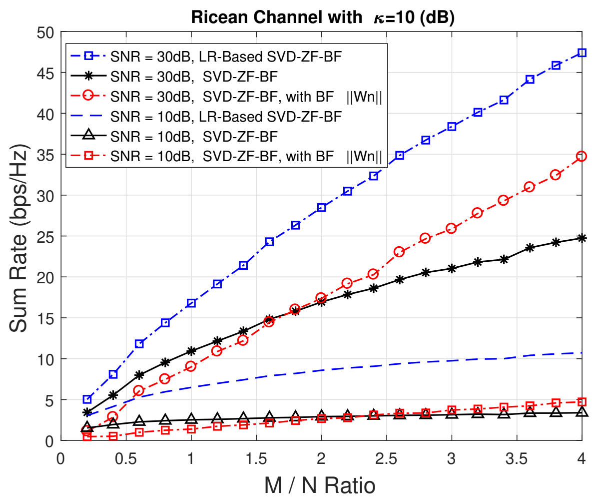

Figure 10. Sum rate versus the $0 < k \leq \frac{M}{N} < \infty$, the ratio k is user antennas

which minimize the singular values, as a result, the Ricean-ness is same as in the Figure 4 and Figure 5.

Finally, the sum rate is compared with the K antennas, i.e $k \leq \frac{M}{N} < \infty$ (as $1 \leq k \leq K$) are presented in Figure 9 and Figure 10. These results depict the impact of the multiplexing gain and diversity gain $(M - NK)$. As $M = (N-1)K$ grows larger, the sum rate due to (25) turns to the dominance of $M - N + 1$ channels, which increases power allocation in the eigenchannels. However, increase in transmit antenna M results in an increase in the multiplexing gain $\Sigma_{\mathbf{n}}$ and $(N-1)K$, and compensate the increase in the optimal power allocation in the proposed LR-based SVD-ZF-BF. This indicates that with large K, the beamforming channels are projected onto the orthogonal projection since the channel vectors of the nth user are near-orthogonal. Therefore, the water-filling power allocation to the eigenchannel is optimal in the sum rate. These results are consistent with Figure 6 and Figure 7 with the $\frac{1}{k}M = N$. It is also observed that the sum rate increases with user antennas for all schemes, but as κ increases, the sum rate of the SVD-ZF-BF reduces drastically. In summary, the closeness of the user antennas lead to channel correlation and few LoS paths, however, the proposed optimal precoder withstands this severity in the Ricean channel. Admittedly, in [32], the received power is carried by the few LoS channel paths, which limit the performance.

5. Conclusion

In this paper, we discuss the optimal ZF precoder over Ricean channel with the per-antenna power allocation in the downlink MU massive MIMO system. By considering non-square massive MIMO channel matrix, a beamforming approach is designed to align the channel matrix to the optimal ZF precoder with the per-antenna power allocation. Further, lattice reduction is introduced to transform the lattice basis of the beamforming channel matrix. Optimal ZF precoder with LR-based beamforming guaranteed higher sum rate (multiplexing and diversity gains) as compared with other precoding schemes e.g. norm approximation beamforming. The numerical results show that the optimal power allocation in the subchannels depends on the number of users to achieve multiplexing and diversity gains. Conversely, the severity of the Ricean channel reduces the sum rate. The theoretical analysis accomplishes practical results for optimal ZF precoder with per-antenna power allocations in MU massive MIMO systems.

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