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Formation control of multiple unmanned vehicles based on graph theory: A Comprehensive Review

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Abstract

In recent years, formation control for multiple unmanned vehicles becomes an active research topic that has received a lot of attention from scientists due to its superior advantages compared with other conventional systems. Algebraic graph and graph rigidity theories are the two main mathematical backgrounds of the formation control theory. The graph theory is used to describe the interconnections among vehicles in formation while rigid graph theory - an important subset of graph theory - ensured that the inter-vehicle distance constraints of the desired formation are enforced via the graph rigidity. This paper provides a comprehensive review of graph theory supporting formation control for groups of unmanned aerial vehicles (UAV) or swarm UAVs. The background of the theory and the recent developments of graph-theory-based formation control are reviewed. We provide a cohesive overview of the formation control and coordination of multiple vehicles. Finally, some challenges and future potential directions in formation control are discussed.

Keywords: Graph theory, Rigid graph theory, Formation control, Multiple unmanned vehicles, Formation stability

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1. Introduction

With the advanced development of automation, mechatronics, computing, and communication in recent decades, multiple unmanned vehicles (MUV) are becoming an active research topic and applied in various fields [1–5], such as military surveillance, search and rescue operations [6–8], remote sensing operations [9–13], inspect buildings and infrastructure [14–17], manage and monitor crops in agriculture [10], [18–20], transporting goods [21]. In practice, moving and working in MUV has many significant advantages over a single unmanned vehicle, for example, MUV can expand the region of surveillance and reduce the expense of missions, increase the anti-interference performance and efficiency, improve the probability of success in search tasks, increase the robustness and efficiency of the system while reducing system costs, providing redundancy, and completing complex tasks in a vast area.

While working in a group, each unmanned vehicle travels to different places and collaborates with its neighbours to complete a given mission. MUV needs to avoid collisions with obstacles and also among the other partners. In some tasks, MUV may be required to autonomously operate in dangerous environments that easily cause failures of systems and communication interruption. Recently, the formation control problem is received great attention from many researchers to develop effective algorithms that hopefully overcome the challenges. Hence, formation control plays a crucial role in coordinated control of a group of unmanned vehicles. This controlling problem required a group of autonomous vehicles to follow a predefined trajectory while maintaining a desired spatial pattern. In many application scenarios, a team of vehicles needs to follow the pre-set trajectory while maintaining a specific geometric shape.

In general, the four main formation control problems can be summarized as: i) formation generation and maintenance; ii) formation tracking; iii) detecting and obstacle avoidance with formation; and iv) task assignment

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[22, 23]. For the aforementioned problems, it is really necessary to choose an appropriate control strategy. A suitable formation control strategy not only tackles the main formation control problems but also maintains the formation stability. The control strategies have to ensure that the desired shape of the formation must be retained while the team performs the operations. In formation control for MUV, various strategies have been proposed for achieving distributed formation including behaviour-based [24 - 27], leader-follower [28 - 31], virtual structure [32 - 35], graph-based [36 - 38], and artificial potential approaches [39 - 42], etc. Even though these strategies are with different characteristics and considerations, they can be concluded in a graph theory framework, because vehicles in the group must sense some aspect of the formation geometry to maintain a formation shape. Hence, almost all the formation control issues can be contained in a unified framework and studied by using the graph theory.

Graph theory has a crucial mathematical base in formation control for the information exchange between autonomous vehicles, to achieve the control law, and perform the formation stability analysis of the formation. In this approach, algebraic graph theory is exploited for modelling the communication topology of MUV where each vehicle is represented as a vertex, and the edges that connect the vertices represent the information flow from one vertex to another. The outstanding advantage of this theory is decentralization, by which the network can keep a suitable behaviour with varying communication topology. Besides, by using the graph-based approach, formation stability can be achieved if the information flow is stable as long as the local controller stabilizes formation dynamics.

Comparative studies of the major formation control strategies show that the graph-based approach solved most basic formation problems well, such as formation shape generation, trajectory tracking, reconfiguration, and task assignment [43 - 46]. Besides, this method also has the ability to overcome one of the main challenges of formation control which is formation stability [22].

In this work, we mainly pay attention to formation control based on graph theory. Instead of synthesizing strategies and control laws for vehicles in a formation, which typically serves multiple objectives such as shape control, collision avoidance, and motion to the desired trajectory, we focus on reviewing the architectures behind the control laws. Accordingly, the recent developments in formation control for MUV are summarized in a graph-theory-based framework. This literature provides a cohesive overview for developing advanced research in the formation control and coordination of multiple vehicles.

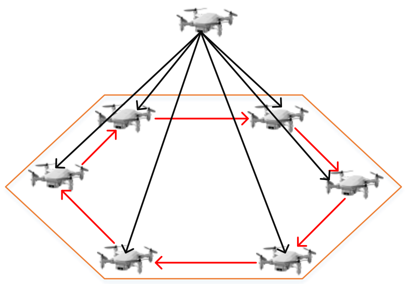
The rest of the paper is organized as follows: Section 2 presents algebraic graph-based formation control, this part provides a preliminary on the basics of graph theory, the mathematical description models of vehicles, and reviews of studies of graph-based approach in formation control. Section 3 and section 4 give an important subset of the algebraic graph-rigid graph theory and formation stability problems, respectively which clarifies recent research in these fields. Section 5 discusses and evaluates more about the graph theory issues to point out research directions. Finally, the conclusions and future developments are provided in Section 6.

2. Algebraic Graph Based Formation Control

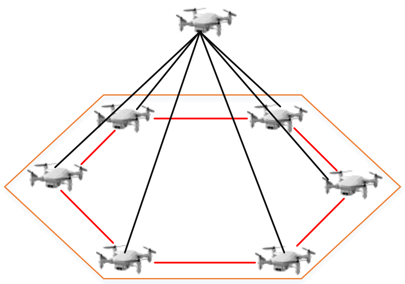
2.1. Basic of Graph Theory

The algebraic graph is the main background of the formation control theory. The basic concepts which are normally used in formation control are covered such as topology, graph theory, and consensus. A tool commonly used to analyse consensus control strategies is the graph theory, where the topology indicates a potential interaction between neighbouring vehicles and is described by the graph [47].

The graph is a network structure consisting of vertices and edges connecting vertices. A graph theory makes no sense about not only the length of the segment but also the position of vertices. Each node represents a vehicle of MUV, and the edge represents the information flow from one vertex to another. It can be a directed graph (digraph) $\vec{G} = (V, \vec{E})$ or an undirected graph $G = (V, E)$ (Figure 1). An undirected graph $G = (V, E)$ where $i, j \in V$ is a non-empty set of vertices $V = \{1,2, \dots, n\}$, $(i, j) \in E$ is a non-empty set of edges $E \subset V \times V$. The cardinality of V and E be $|V| = n$ and $|E| = m$. An undirected graph does not have loops or multiple edges in a pair of vertices. Each edge can move in both directions, so the edges represent a bidirectional relationship.



a) Directed graph



b) Undirected graph

Figure 1. Two types of graphs

If an edge has a direction as $i \rightarrow j$, when connecting two vertices i and j, it is called a directed edge and is denoted as $(i,j)^{\bar{e}}$. If a part or all of edges have directions, it is called a directed graph $\vec{G} = (V, \vec{E})$.

The set of all vertices that are connected to vertex i with an edge, it is called set of neighbours of vertex i and is represented by $N_i = \{j \in V | (i,j) \in E\}$

A graph is called connected if any two different vertices i and j in V there exists at least one path from i to j. This distance that is the maximum between any two vertices is called the diameter Υ [48].

The adjacency matrix and the Laplacian matrix are the most important concepts of graph theory. The adjacency matrix of G: $A = [a_{ij}] \in R^{n\times n}$ is a square matrix of the size $|V|$ [49].

$$a_{ij} = \begin{cases} 1 \text{ ,if } (i,j) \in E \\ 0 \text{ ,otherwise} \end{cases} \quad a_{ij} = a_{ji}, i \neq j \text{ , } a_{ii} = 0 \quad (1)$$

The degree of vertex i is defined as the number of its neighbouring vertices. The degree matrix D of graph G is also a square and diagonal matrix with diagonal entries

$$d_{i,i} = |\{j \in V: \{i,j\} \in E\}| \ (i \in V) \quad (2)$$

The matrix D is invertible because we assume that the graphs are connected. Laplacian of graph G is the symmetric positive semi-definite matrix, it is defined as:

$$L = D^{-1}(D - A) \quad (3)$$

Some properties of the Laplacian matrix [50]

1) All of the eigenvalues of L are nonnegative real numbers less than or equal to 2
2) Zero is one of the eigenvalues of L
3) The zero eigenvalue occurs with multiplicity one whenever G is connected graph
4) If G is connected graph, each nonzero eigenvalue λ of L:

$$\lambda \geq \frac{1}{\Upsilon \sum_{i\in V} d_{i,i}}$$

2.2. System Description

In general, when studying the formation control for the multi-vehicle system, we will focus on the interaction between the vehicles, the mathematical model of each individual vehicle in the system description can be simplified by the single-integrator model or double-integrator model.

Consider a group of n kinematic vehicles operating in R^d ($d -$ dimensional Euclidean space, $d = 2,3$)

$$\dot{p}_i = f_i\,(p_i, u_i, w_i), \text{ for } i = 1,2,3\ldots,n \quad (4)$$

In which, $p_i \in R^d$ denote the position of vehicle i, and we assume that the absolute position of vehicle is in descartes coordinate referenced as the center point of the vehicle and each vehicle i has only access to the relative position $\hat{p}_i^j = p_j - p_i, j \in N_i$ (N_i is the set of neighbour vehicles who have relationships with vehicle i); The control input $u_i \in R^d$ denotes the velocity of each vehicle: $u_i = h_i(\hat{p}_i^j | j \in N_i)$; $w_i \in R^d$ is the disturbance signals if exists. The dynamics with control laws are rewritten by:

$$\begin{cases} \dot{p}_i = f_i\,(p_i, u_i, w_i) \\ u_i = h(\hat{p}_i^j | j \in N_i) \end{cases} \quad (5)$$

The pre-specified geometric pattern (desired formation) defined by $p_{ij}^* = p_j^* - p_i^*, j \in N_i$, the formation generation and maintenance can be achieved in the corresponding consensus based on formation. In general, the objective of the formation control design is to drive the desired relative positions, that is $\hat{p}_i^j = p_{ij}^*$. If the distance constraint is the objective of control design, that is $\|p_j - p_i\| = d_{ij}^*$ for $(i,j) \in E$, the scalar parameter $d_{ij} = d_{ji} > 0$ represents the distance at which vehicles i, j should converge to (more details will be provided in section 3)

Figure 2 shows a system description for formation control based on graph theory when the objective of the control design is relative positions [23]. For the single-order kinematic models case $\dot{p}_i = u_i$ with $e := p^* - p$. In this construction, the feedback connection between the vehicles and the integrator represents the dynamics with control laws while the feedback connection between the integrator and G represents a relation as:

$$\dot{e} = -(L_G \otimes I_n)e \quad (6)$$

in which, $\otimes$ denotes the Kronecker product, I_n is the identity matrix.

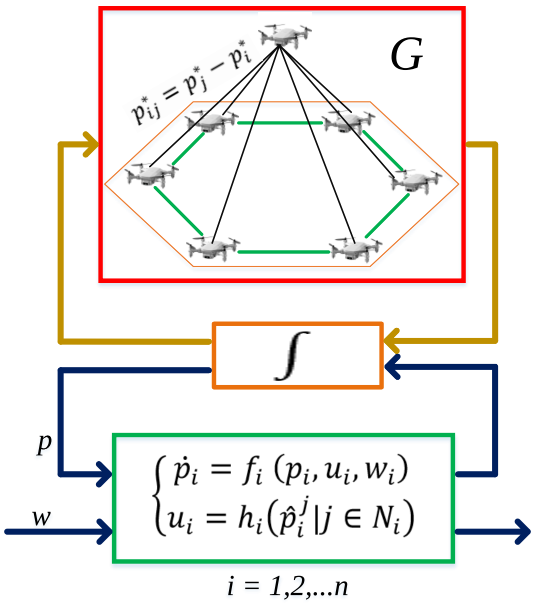


Figure 2. System description with the relative position graph-based formation control

The control law u_i evolves according to $\hat{p}_i^j$ .The formation controllers can be designed using the properties of the Laplacian matrix, and their stabilities can be verified by the eigenvalue of the Laplacian matrix [23, 51, 52].

2.3. Graph Based Approach in Formation Control

Formation control of multiple vehicles can operate together to perform global tasks. The main objective of formation is to maintain a certain shape with constant relative distances between vehicles during the movement toward a specific goal. Hence, shape and position are two important factors of formation control. Algebraic graph theory is the main mathematical base of the formation control theory. An outstanding advantage of the graph-based approach in formation control is decentralization, by which the multi-vehicle team can keep an appropriate behaviour even in the presence of varying communication topologies. However, the main disadvantage of this approach is that vehicles can only receive information from their neighbours.

A distributed formation control strategy for multi-vehicles based on a double-graph model is presented in [53]. In this literature, each vehicle adjusts its behaviour in terms of the leader vehicle and its neighbours. The direct connection of the network is established between the performance of a linear consensus protocol and the algebraic connectivity.

In [54], digraphs are utilized to represent information exchanges between vehicles, taking into account the general case of unidirectional information exchange. The information consensus among vehicles is considered in the presence of dynamically changing topology. Information consensus can be obtained asymptotically if the union of the digraphs has a spanning tree.

Dong et al. [55] propose two different formation control strategies by using graph theory. In the first strategy, the model of the robot is transformed into a linear system by dynamic feedback linearization. Then, the controller is designed based on the graph theory. In the second strategy, a time-varying parameter is introduced in the control law by means of the time-scaling technique.

Motion planning is studied for multiple robots subject to constraints which are modelled using an algebraic graph in [56]. In this approach, each edge is associated with the interaction between two robots describing a constraint on relative configurations. The weighted graphs are proposed to maintain the formation shape and avoid collisions [57]. A multi-layer formation control scheme is presented by Li et al [58]. In there, a layered finite-time estimator is studied for multi-agents in each layer to achieve their target positions and velocities based on the information of agents in their prior layers. Subsequently, a model-based control law is proposed to obtain a multi-layer formation.

3. Rigid graph based formation control

The rigidity theory is an important subset of graph theory. The formation control theory is developed on the base of mathematical concepts from these theories. Rigid graph theory ensured that the inter-vehicle distance constraints of the desired formation are enforced via the graph rigidity.

To model a physical structure of n-vehicles, we can use a framework $F = (G, p)$, where graph $G = (V, E)$ and coordinates $p = [p_1, p_2, \dots, p_n] \in R^{n\times d}$. Framework F is a realization of a graph G at given points in Euclidean space. The rigid graph theory studies conditions for a unique framework of MUV when the inter-vehicle constraints between neighbour vehicles are specified by some scalar or vector magnitude. The magnitudes of distances in undirected edges are called graph rigidity or distance rigidity and distances in directed edges are called persistence. In short, an undirected graph is rigid if the only possible continuous moves are those which preserve every inter-vehicle distance. In general, the concept of rigidity is mainly for the undirected graph. For the directed graph, we have the concept of persistence [59].

Obviously, adding edges to a graph does not destroy rigidity whereas removing edges can affect to ensure rigidity, hence, the concept of minimal rigidity was presented in [60,61], an undirected graph is minimally rigid if it is rigid and if a single edge removes, it will lose rigidity of graph. This has a crucial role in practice because it ensures the formation of multiple vehicles with the minimum number of sensing and communication links.

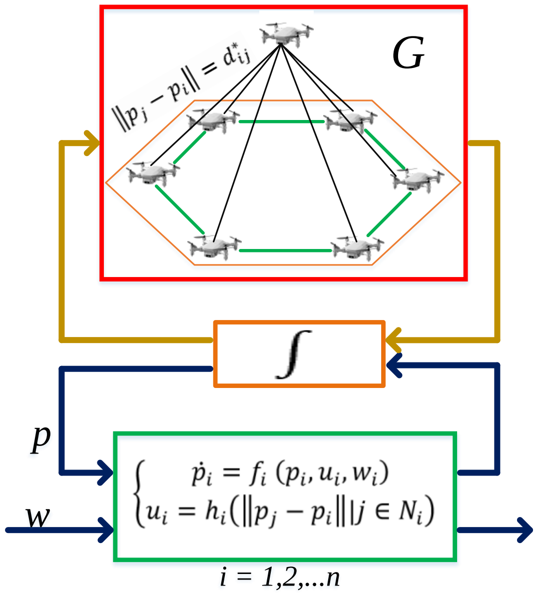


Figure 3. Distance rigid graph based formation control

Depending on an arbitrary ordering of the edges in E, the edge function ϕ_G for framework $F = (G, p)$ is defined by

$$\phi_G(p) = \left[\dots, \|p_j - p_i\|^2, \dots\right]^T, (i,j) \in E \qquad (7)$$

The rigidity property of framework F is characterized by a rigidity matrix $R(p) \in R^{m\times dn}$. The rigidity matrix $R(p)$ can be used to determine the infinitesimal rigidity of a framework F.

The rigidity matrix is given by

$$R(p) = \frac{1}{2}\frac{\partial \phi_G(p)}{\partial p} \qquad (8)$$

In rigidity graph-based formation control, the pre-specified geometric pattern is given by a set of distance constraints $\|p_j - p_i\| = d_{ij}^*$ rather than the desired relative positions. Therefore, the control laws are designed by using distance terms instead of relative positions. The dynamics with control laws are rewritten as (9) and the system description for formation control based on rigidity theory is illustrated in Figure 3 [23].

$$\begin{cases} \dot{p}_i = f_i\,(p_i, u_i, w_i) \\ u_i = h_i(\|p_j - p_i\| | j \in N_i) \end{cases} \tag{9}$$

The rigid graph-based formation control is heavily dependent on the gradients of the potential functions closely related to the distance constraints between the neighboured vehicles [62].

In [63,64], the control law is proposed to eliminate the reported inconsistency-induced orbits in rigidity formation. In this research, the distance causes the rigid formation to converge exponentially fast to a closed circular orbit in R^2 and the orbit becomes helical in R^3. Other main tools of rigidity graphs are the Henneberg sequence and Laman's theorems [65]. The Henneberg sequence constructed two-dimensional minimally rigid graphs, and Laman's theorems verified if a two-dimensional graph was rigid [66]. By using Henneberg sequence and Laman's theorems, the primitive operators are defined to deal with the transformations such as restructuring and splitting of rigid formation [67, 68].

Zen and co.[69] expand the rigidity graph theory to the weighted framework and propose the rigidity eigenvalue, this eigenvalue is used as the algebraic characterization of the infinitesimal rigidity. In there, a fully decentralized approach for maintaining the formation rigidity is studied by estimating the common relative position reference frame of a multi-robot system with only range measurements. In other literature, Eren et al. [70] present the rigid formations with leader-follower architecture for information structures to secure control.

In [71], the case of a single-integrator rigid formation system, undesired rigid motions will occur if there exist inconsistent distances perceived by neighboured agent pairs. The main approaches in this research include linearization analysis and exponential stability. To investigate whether such rigid motions still occur in double-integrator formation systems with distance measurement errors or inconsistent distances, the rigid formation control systems are modelled by double integrators containing the formation stabilization model and flocking control model [72].

4. Formation stability

One of the main goals of formation control is to distribute the control activity while still stabling and achieving a coordinated task. Hence, the stabilization of vehicle formations using techniques from the graph theory is studied extensively and achieved a lot of positive results. Whereby, by using the graph-based approach, formation stability can be achieved if the information flow is stable as long as the local controller stabilizes.

In [73], the authors mention three contents of the stability for formation control: string stability, mesh stability, and leader-to-formation stability. In [74], the authors used the Nyquist-type criterion and solved linear matrix inequalities (LMIs) for multi-agent systems (MAS). In [75], the authors research the communication graph method. Considering a known vehicle model and utilizing state-space techniques, they acquire the stability of a formation.

In [76], D. V. Dimarogonas and K. H. Johansson offered distance-based formation instead of position-based formation. In this research, the author proposed a control law based on the negative gradient of a potential function between each of the pairs of vertices that creates an edge in formation. The results show that the tree graph structure is a necessary and sufficient condition for formation stabilization. The control law is proposed as (10) and then analyse the formation stability by using Lyapunov functions (13), (14).

$$u_i = -\sum_{j\in N_i} \frac{\partial\gamma\left(\beta_{ij}(p)\right)}{\partial p_i} = -\sum_{j\in N_i} 2\rho_{ij}(p_i - p_j) \tag{10}$$

where $\gamma: R^+ \to R^+ \cup \{0\}$ is a function of the distance between i and j; $i, j \in V, i \neq j$;

$\beta_{ij}(p) = \|p_i - p_j\|^2$ is the Euclidean distance of any pair of vehicles in the group

$\rho_{ij} \triangleq \dfrac{\partial\gamma(\beta_{ij})}{\partial\beta_{ij}}$

$\gamma(\beta_{ij})$ is continuously differentiable;

$\rho_{ij} = \rho_{ji}$ for $i, j \in V, i \neq j$

$\gamma(d_{ij}^2) = 0$ and $\gamma(\beta_{ij}) > 0$ for all $\beta_{ij} \neq d_{ij}^2$

The set of control laws is derived by:

$$u = -2(R\otimes I_2)p \text{ ,in which } u = [u_1^T, \dots, u_N^T]$$

In which, the symmetric matrix R is defined by:

$$R_{ij} = \begin{cases} -\rho_{ij}\,, j \in\ N_j \\ \sum_{J\in N_i}\rho_{ij}\sum, i = j \\ 0\,, j \notin N_j \end{cases} \tag{11}$$

Then, the stability of the system is examined by using the candidate Lyapunov function:

$$V_f(p) = \textstyle\sum_i \sum_{J\in N_i}\gamma\left(\beta_{ij}(p)\right) \tag{12}$$

Gradient: $\nabla V_f = 4(R\otimes I_2)p$ (13)

The time-derivative is given by:

$$\dot{V_f} = -4\|(R\otimes I_2)p\|^2 \leq 0 \tag{14}$$

The authors show that the system reaches a static configuration and provide a formation potential (15) that guarantees formation stabilization for a class of graphs.

$$\gamma\left(\beta_{ij}(p)\right) = \frac{\left(\beta_{ij} - d_{ij}^2\right)^2}{\beta_{ij}} \tag{15}$$

In [77], Dimarogonas et al. use the incidence matrix and its spectral properties. This matrix is to determine the convergence characteristics of two formation control problems. First, a communication topology is processed and realizes the result. If vertices arrange into a tree topology, formation control will have the convergence characteristic. The second problem in the same direction as [76] is the distance-based formation control. Chang et al [78] also use the Lyapunov stability theorem in the stability conditions. Then, the combination of fuzzy sliding-mode control (FSMC) and consensus control based on graph theory is to check the stability.

In [79], the authors analyse stability by combining graph-theoretic and system-theoretic. In other literature, Fax et al. [80] set up the relation between the formation controller and the topology of the communication network with the Laplacian matrix, and these authors proved that if the local controller was stable, then the formation stability with linear dynamics based on the stability of the information flow. In [81], the authors can describe how control signals of leader-vehicle and disturbances influence the stability of the formation. They also can control deep into the stability of a particular sub-formation. In other literature, Lin and co. [82] proved that if and only if there is a globally accessible vertex in the perception graph, the formation control is stable by using tools of the graph theory. The formation problem of multiple robots under a directed fixing interaction topology is presented by Wen et al [83]. According to this research, the relative output measurements of neighbour robots are used for a class of distributed information communication protocols. Consensus tracking problems and formation control of single-leader multiple robot systems with general linear node dynamics are solved.

5. Discussion and opening issues

The graph theory acts as an essential tool used in multi-vehicle distributed formation control. Almost all the formation control problems for multi-vehicles can be contained in a unified framework and studied by using the graph theory. By using the properties of the Laplacian matrix, distributed formation controllers can be designed, and their formation stabilities can also be verified by the eigenvalue of the Laplacian matrix. The formation control laws will depend on the distance and angle between vehicles and primarily stabilize the inter-vehicle distance dynamics to desired distances. Most graph-based formation controllers utilized the single-integrator model. However, recent studies proposed results that are based on the double-integrator model or the full dynamic model.

Besides the achievements, graph-based formation control problems remain some open issues. Firstly, existing results of this theory are mainly considered vehicles in a plane, and the research cannot be directly expanded to three-dimensional. Hence, expanding and improving the existing foundation's problem of the graph theory to three-dimensional have not been effectively solved. Secondly, the global stability of the rigid formation control remains open.

Further, the Laplace matrix with constant weights only solves the single-class problems, and it cannot be used in multi-layer formation control due to the existence of cross-interactions. Therefore, one of the current research trends in graph theory-based formation control is focused on multilayer systems. Accordingly, control design problems have become much more complex and need to be further exploited and researched.

With increasing the applications of MUV, especially in the military field and disaster detection systems, the increase of the formation scalability becomes extremely important. Nevertheless, this problem can induce communication delays and decision burdens. Hence, formation topologies and network communication between vehicles have to be further researched.

Many experimental results have been designed to validate the theories. However, we have to recognize that almost the existing theoretical results are only verified by simulations, rather than by actual systems, due to various restrictions of the experiments and the high cost, etc. Therefore, demonstrating and applying the theoretical results to the actual MUV is the most pressing.

6. Conclusions and future work

Although formation control for MUV based on graph theory has technical challenges, its benefits and advantages in formation control theory have inspired tremendous studies. This paper has reviewed the recent research and developments in graph theory-based formation control for MUV. We focus on the main mathematical bases of the algebraic graph and rigidity theories, respectively which clarify recent research in these fields. Besides, the formation stability based on graph theory has also been specifically reviewed and analysed.

Most of the studies mentioned in this paper are formation control problems based on graph theory in normal conditions. In addition to task executions under normal conditions, vehicles must possess a fault tolerance ability to react correspondingly to eliminate the adverse effect on mission completion. A survey of formation control under faulty situations on fault detection and diagnosis will be further researched and perfected in the near future.

The work on formation control problems for MUV includes many practical respects and experiments, and the authors believe that in the future, more issues related to graph theory-based formation control will be effectively solved. With extending the civilian and military applications of the MUV system, the increase in the formation scalability also becomes profoundly important. Formation topology and communication between vehicles have to be considered. Consensus algorithms and graph theory are effective tools for performing distributed computing tasks. Besides, higher levels of decision-making problems become essential, especially in distributed formation control that uses higher levels of decision-making. In practice, vehicles may need to make decisions to tackle real-time situations. Intelligent controllers based on artificial intelligence can be considered a promising solution.

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