Reliability and Mean Time to System Failure of a Parallel System' by Using One or Two Decimal Random Data Points

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Abstract

Focusing on Weibull failure rules, which govern the stopping of components, this work evaluates reliability metrics such as stability and the mean time to system failure (MTSF) of a structure that is parallel. These metrics' behaviour has been seen for one or two decimal random values of component failure rates, operation times, form parameters, and the total quantity of components used in the parallel structure. In order to analyze the variation in the ethics of reliability as well as MTSF, the particular case of the Weibull distribution has also been taken up.

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1. Introduction

Particular reliability issues began to draw a lot of attention from numerical researchers and designers throughout the military-modern complex, especially those relating to life assessment and digital and rocket reliability issues. The various articles on factual technique in dependability that have been written provide proof of the close connection connecting measurements and reliability. To evaluate the entire reliability situation and provide recommendations for changes that would improve hardware reliability and decrease support, the Air Force called a meeting with a specific purpose to discuss the dependability of electronic equipment. The guard division created the Consulting Group regarding Reliability with Electronic Equipment (AGREE). provided is a concur distributed reliability report. This corporate least adequacy report includes requirements for ferrule ability tests and the effect of throughput on reliability.

The reliability hypothesis is worried about arbitrary event of bother some occasions or failures during the life of a physical or natural system suggested [1], [2]. Reliability is an inborn property of a system similarly just like the system's ability or force rating. The idea of reliability has been known for various years however have more noteworthy essentialness and significance during the previous decade, especially, because of the effect of automation, improvement of complex rocket and spaces of aware engineers. The theory of reliability concerns the determination of the chance with the intention of a system, perhaps. It consists of lots of gears, its determination work. Assume if otherwise not, the function of the system is determined exclusively by the understanding of which mechanism they work. On behalf of example, a serial system will work if and only if it's all modules will work, while a parallel system will work if and only if at least one of its components works. Then we discuss the relationship between the distribution of a system's operating time and the distribution of component lives. In particular, it appears that if the amount of time a component works has an increasing rate of failures in the average distribution (IFRA), the same

is true for the distribution of the life of the system discussed by [3], [4], [5]. A well-connected industrial process is designed to deliver a high-quality product at the lowest possible cost. However, these solutions are subject to errors caused by a variety of factors. The majority of economic procedures are designed to have parallel redundancy with reduced plant capability in order to reduce losses during process corrective protection. As a result, the entire device is operating at varying levels of performance. Multi-nation factors (MSE) are the names for these states, and a device like this is known as a multi-state system (MSS). However, most real-world international systems are complex, and they fall into the MSS category because they move through various stages of decline, from a perfectly functioning kingdom to a failed state [6], [7].

2. Systems

Reliability theory is based on the idea of comprehending the dependability of systems and their many components. We are considering the likelihood of the person surviving. The likelihood is that the system, comprising its individual parts and the system as a whole will function. To begin, we must assume that our system is sensitive, which means it will function as it does until one of its components fails [8], [9]. Assume additionally that if the system malfunctions, the turned-off component won't switch the system on. With this concept, it's crucial to understand that it assumes monotony because a system of this kind has an increasing structural function. For this kind of system, we can use the terms reasonable or consistent [10].

3. Symbols

The various functions are represented by the numerals shown below:
- $R_s(t)$ denote structure consistency;
- $R_i(t)$ represents ith module consistency;
- $h(t)$ means system's instant failure rate;
- $h_i(t)$ is instant ith module failure rate;
- λ is Steady failure rate;
- T depicts structure survival point;
- T_i means ith module survival point.

4. System Explanation

Consider a parallel system with two parts. When both parts are operational, the system is operational. Even if either part 1 or 2 fails, the system will still function. If and only if the two parts fail, the system is rendered useless. For illustration, (i) a plane with four engines; (ii) A processor with a power resource and a succession [11], [12], [13].
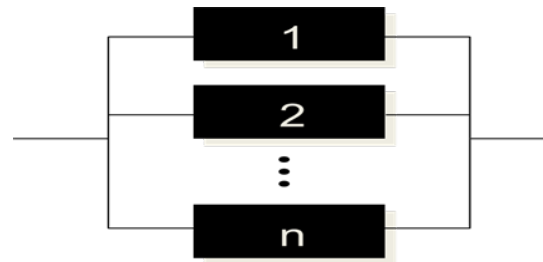

Figure1. Represent the parallel System of n components

In above Figure1. shows the Parallel system with n components. It means that system fails if all components fail. If the event of system is satisfying and an insufficient operation, respectively, are E_j and E_j'. The union of E is currently the condition for system success of the component i. $E_1,...,E_n$. The system's reliability is the likelihood that this event will be successful and is supplied by [14], [15], [16].

R = Prob. $(E_1 \cup E_2 \cup \ldots\ldots\ldots\ldots \cup E_n)$

= 1- Prob. $(E_1' \cap E_2' \cap \ldots\ldots\ldots\ldots \cap E_n')$

$\quad$ = 1- Prob. (E_1') Prob. (E_2') $\ldots\ldots\ldots\ldots$ Prob. (E_n') $\quad$ (1)

Now we consider probability of events according to success or failure with time t.

$$\text{Prob. } (E_i) = p_i \text{ and Prob. } (E_i') = q_i$$

$$R_s(t) = 1 - \prod_{i=1}^{n} q_i(t) \quad (2)$$

We know that probability of success $p_i(t)$ + probability of failure $q_i(t) = 1$ then we can write

$$R_s(t) = 1 - \prod_{i=1}^{n}(1 - p_i(t))$$
$$R_s(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t)) \quad (3)$$

The mean time to system failure is given by [17]

$$MTSF = \int_0^{\infty} 1 - \prod_{i=1}^{n}(1 - R_i(t)) \quad (4)$$

5. Reliability Evaluation

The parallel system's reliability along with mean time to system failure have been determined through taking the component failure rate's Weibull distribution into consideration. The prominence of these reliability statistics has also been assessed for certain Weibull distribution situations. Given that it can be used as a stand-in for a wide variety of patterns of distribution, including regular and exponential distributions. Parameters are used to evaluate the accuracy of the Weibull distribution. Scale and form are treated independently in the Weibull density function of

probability (pdf). In this study, we only pay attention to the form parameter to examine the reliability [1], [18], [19]. Assume that the Weibull failure law governs the failure rates of every component.

$$h_i(t) = \lambda_i t^{\beta_i} \tag{5}$$

The component reliability is provided by the following:

$$R_i(t) = e^{-\int_0^t h_i(v)dv} = e^{-\int_0^t \lambda_i v^{\beta_i}dv}$$

$$= e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}} \tag{6}$$

As a result, these factors guarantee the system's dependability: [16]

$$R_s(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

$$= 1 - \prod_{i=1}^n \left(1 - e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)$$

$$= 1 - \left(1 - e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)^n \tag{7}$$

And last one is

$$MTSF = \int_0^\infty R(t)dt$$

$$= \sum_{i=1}^n (-1)^{i+1} \binom{n}{i} \frac{\Gamma(1/\beta+1)}{[n\lambda_i((\beta+1)^\lambda\beta)]^\lambda(1/\beta+1)} \tag{8}$$

6. Reliability Metrics for random parameter ranges

Numerous performance factors, such as failure rate (λ), element running time (t), design element (β), dependability, including the average time among failures (MTSF) about the structure (n), have all been estimated [20]. The outcomes are represented statistically and aesthetically in the way that follows:

6.1 Case 1:

When the operational period of the module (t) is different, but the failure rate (f) and shape factors (s) are both the same.

By using equations (7) and (8), we calculate below table values according to Case 1

Table 1: Component Count vs. Reliability

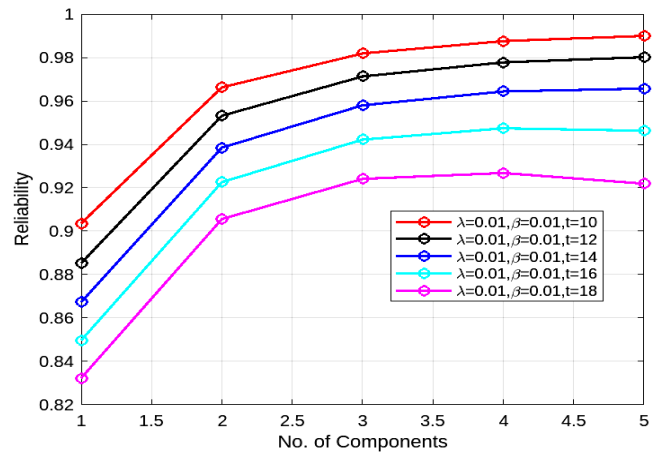| Component Count	Reliability				
	$\lambda=0.01$, t=10, $\beta=0.01$	$\lambda=.01$,t=12 , $\beta=.01$	$\lambda=.01$,t=14 , $\beta=.01$	$\lambda=.01$,t=16 , $\beta=.01$	$\lambda=.01$, t=18 , $\beta=.01$
1	.9036	.8853	.8673	.8497	.8323
2	.9663	.9532	.9385	.9226	.9056
3	.9819	.9713	.958	.9422	.9241
4	.9876	.9778	.9644	.9474	.9268
5	.99	.9802	.9657	.9463	.9219

Figure 2. Component Count vs. Reliability

Table 2: Component count versus (MTSF)

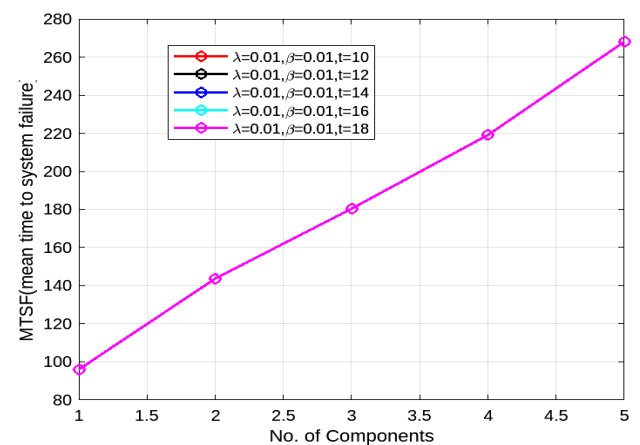| Compo nent Count	The Mean Time to failure of the system(MTSF)				
	$\lambda=0.0$ 1, t=10, $\beta=.01$	$\lambda=.01$,t=12 , $\beta=.01$	$\lambda=.01$,t=14 , $\beta=.01$	$\lambda=.01$,t=16 , $\beta=.01$	$\lambda=.01$, t=18 , $\beta=.01$
1	96.045 08	96.045 08	96.045 08	96.045 08	96.045 08
2	143.71 495	143.71 495	143.71 495	143.71 495	143.71 495
3	180.38 948	180.38 948	180.38 948	180.38 948	180.38 948
4	219.09 437	219.09 437	219.09 437	219.09 437	219.09 437
5	268.02 758	268.02 758	268.02 758	268.02 758	268.02 758

Figure 3. Component Counts versus (MTSF)

6.2 Case: 2

(At what time the element running period (t) as well as form factor (β) are mutually identical, but the malfunction speed (λ) be different)
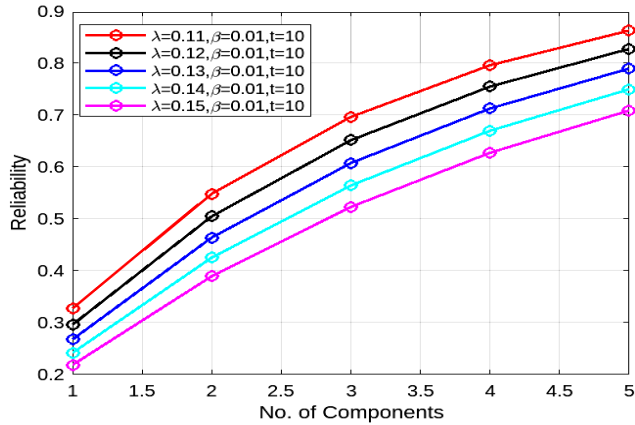

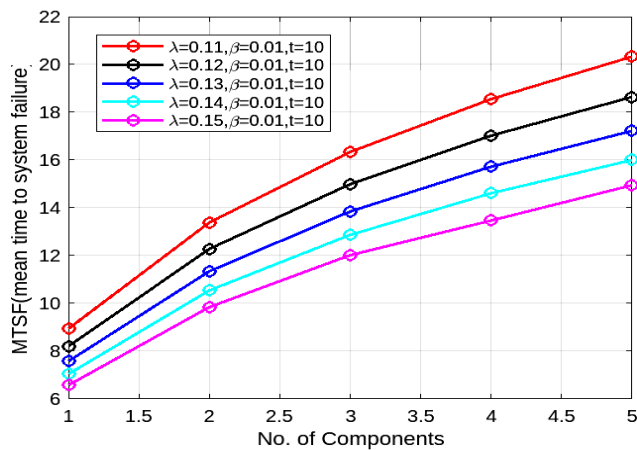**Figure 4.** Component Count vs. Reliability

Figure 5. Component Counts versus (MTSF)

6.3 Case 3:

The shape factor (β) is different but the failure rate (λ) and working point of the part (t) are mutually identical
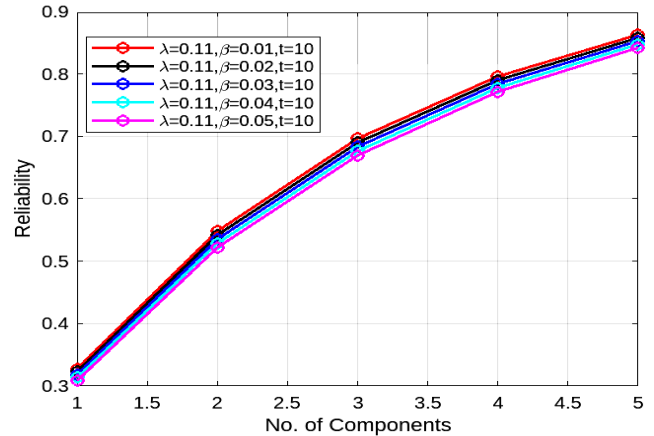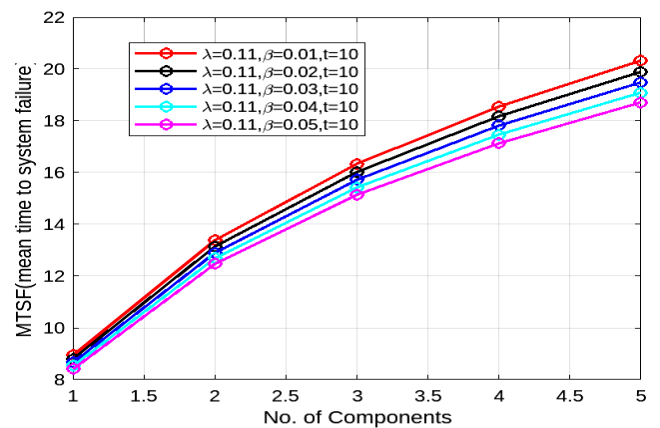

Figure 6. Component Count vs. Reliability

Figure 7. Component Counts versus (MTSF)

6.4 Case: 4

(When the failure rate, operational period, as well as shape factor are independent variables)
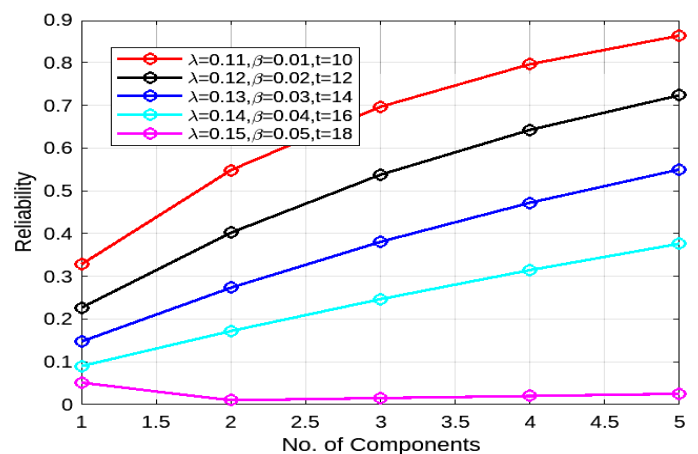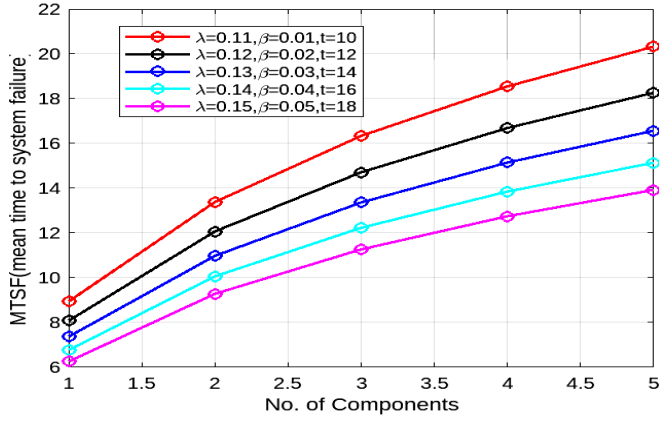

Figure 8. Component Count vs. Reliability
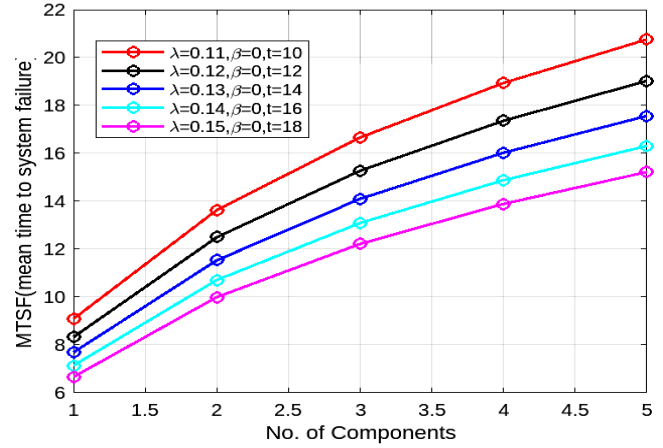
Figure 9. Component Counts versus (MTSF)

Figure 11. Component Counts versus (MTSF)

7. Particular circumstances at what time β=0 furthermore β=1

After doing the contour constraint check for β =0 and β = 1 [20], [21]. We examine the results for the following scenario: So, when β =0 in equation (7) and (8). We may express the component reliability as.

$$Rs\,(t) = 1 - (1 - e^{-\lambda_i}\frac{t^{\theta+1}}{\theta+1})^n$$
$$= 1 - (1 - e^{-\lambda_i t})^n \quad (9)$$

And MTSF

$$= \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\frac{\Gamma(1/\theta+1)}{[n\lambda((\theta+1)^{\wedge}\theta)]^{\wedge}(1/\theta+1)}$$

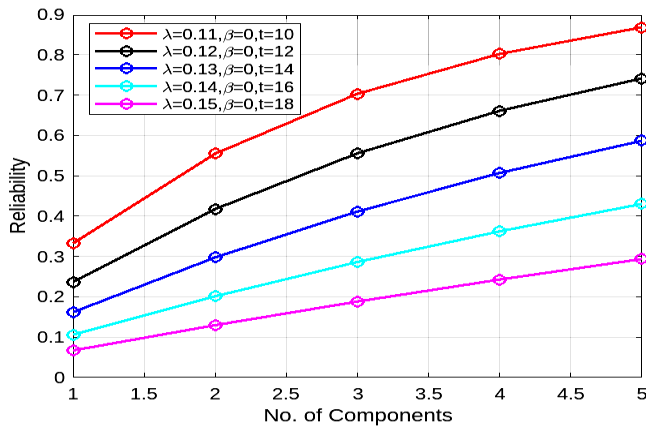$$= \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\frac{1}{n\lambda} \quad (10)$$

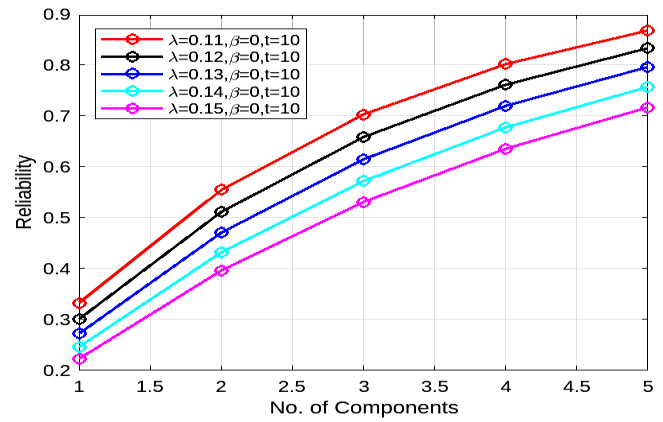**Figure 12.** Component Count vs. Reliability

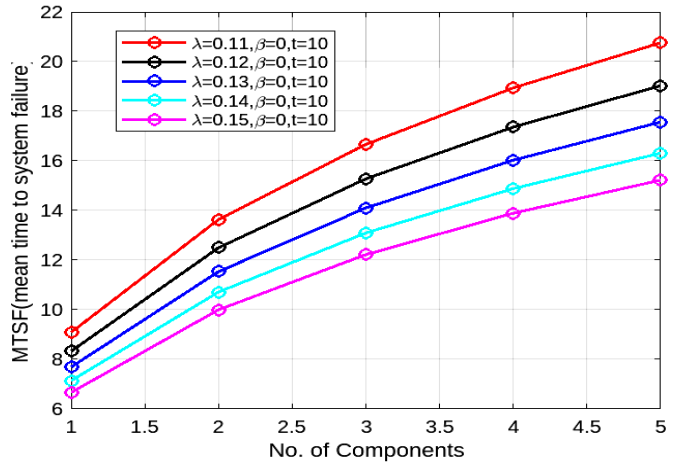

Figure 10. Component Count vs. Reliability

Figure 13. Component Counts versus (MTSF)

When β=1, in equation (7) and (8). In light of this, the element's consistency is provided by [20]

$$R_s(t) = 1 - \left(1 - e^{-\lambda_i \frac{t^{i+1}}{i+1}}\right)^n = 1 - \left(1 - e^{-\lambda_i \frac{t^2}{2}}\right)^n \quad (11)$$

And $MTSF = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\frac{\Gamma(1/i+1)}{[n\lambda((i+1)^n i)]^n (1/i+1)}$

$$= \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\frac{\Gamma(1/2)}{(n\lambda 2)^{1/2}} \quad (12)$$
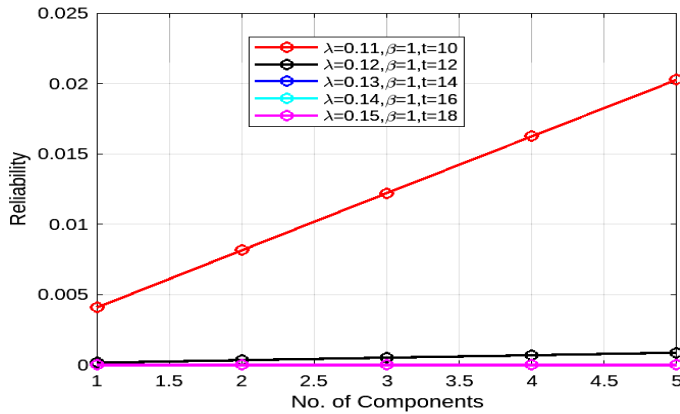

Figure 14. Component Count vs. Reliability
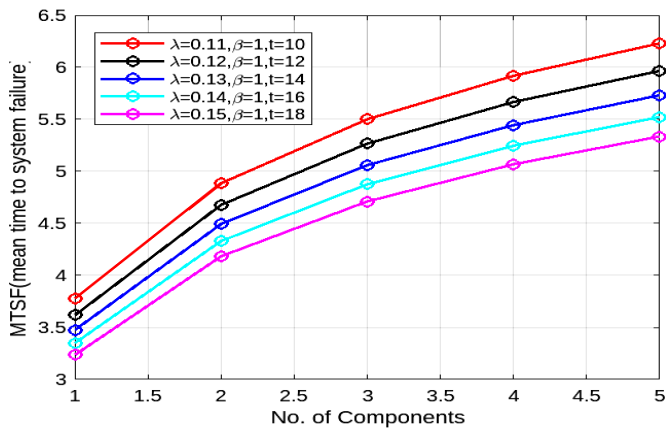

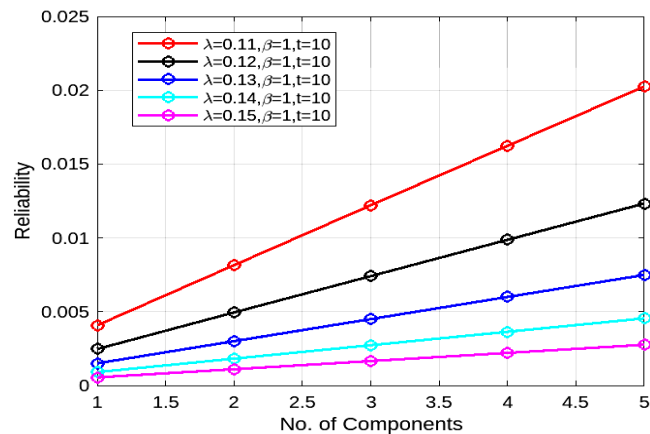**Figure 15.** Component Counts versus (MTSF)

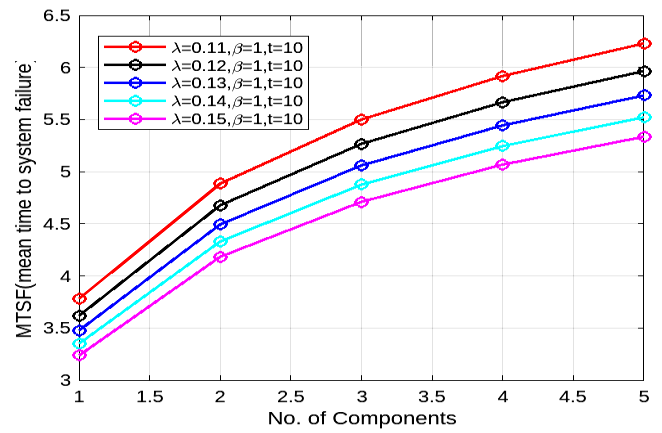**Figure 16.** Component Count vs. Reliability

Figure 17. Component Counts versus (MTSF)

8. Discussion

Basically, we use the Weibull distribution to quantify reliability and use three parameters:
•Rejection rate
•Shape component
•Running time

MATLAB was used to find these conclusions. For every given set of parameters, we can see that as the total amount of modules along with the rate at which they fail increase, the reliability as well as MTSF for the parallel system in its entirety also rises. The operation's time (t) is different in Case 1 even though the component's failure rate and form parameter are identical and the same. If we change the operating duration of the system while keeping the system's failure rate along with form variable at the exact same arbitrary values, the dependability of the system does not change despite the distributions created by component breakdown durations using Table 1. Table 2 illustrates the same condition in the MTSF of a concurrent system, demonstrating that it is not influenced by running time (MTSF). Since every value in this example is the same, the intersecting condition in Figure 2. and Figure 3. causes the final colour to be pink. When the component's running time (t) along with structure parameter (β) the same but its failure rate (λ) is different, Case-2 consistency Figure 4. has a greater value compared with the MTSF about a parallel system Figure 5. The component's failures rate, functioning duration (t), and shape parameter can vary in this third scenario. For increasingly complex values with the shape of the element (0.01, 0.02, 0.03, along with 0.05), we observe that as the value of declines, so do the dependability along with MTSF for a parallel system made up of, say, five comparable modules. The results are provided numerically with illustrative information. Component no. 4 has a distinct operational time (t), shape parameter, and failure rate (λ). The outcome is identical as in Case 2, as demonstrated in Figure 8. as well as Figure 9. When the failure rates as well as operating time are separate, the reliability of the system improves with increasing operating time, despite dispersion based on failure of components durations using Table data

and the identical MTSF of a system that is parallel (results shown in all figures). Here, the Weibull distribution tends to approach the exponential one. If we choose operating duration values that are similar but have a different failure rate, the outcome is the same as what is depicted in Figure 2. and Figure 3. Weibull distribution changes to an exponential distribution in this case at $\beta = 0$. Weibull distribution in this situation converges to Rayleigh distribution when $\beta = 1$.

9. Conclusion

In this study, we randomly pick values for the three variables to calculate the MTSF and dependability of a system that is parallel. As a result, many authors analyze the stability of the system and MTSF using only a single decimal place, but we verify the usage of combinations of one and two decimal places for random variables and points. The following findings are reached: reliability along with MTSF continually rise with constituent quantity, rates of failure, and operational duration. The research shows that employing the fewest possible parts in a parallel structure improve performance. As the numerical value of the shape parameter increases, the reliability and MTSF of the system degrade. Tables and figures provide numerical and visual representations of the findings.

Acknowledgements

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